Towards Modeling Non-Isothermal Sloshing of Liquid Hydrogen

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Context





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Context

Goal: Develop a hydrogen tank design tool for the civil aeronautical industry



Mockup of A380 with LH2 Fuel Tank



Impact of Renewable Aviation Technologies on CO2 emissions



System diagram of LH2 tank



Subprojects of HASTA

1. The Problem

2. Numerical Modeling

- 2.1 Low-Mach Solver
- 2.2 VOF+Embed

3. Isothermal Sloshing in Arbitrary Geometries

4. Conclusion, Next Steps

Physical Phenomena Present in Cryogenic Sloshing Tank



- 1. Sloshing physics (isothermal) in a complex tank geometry
- 2. Consistent numerical modeling of gravity and tank acceleration for multiphase flow
- 3. Densities in both phases are sensitive to temperature variations
- 4. Phase change effects, including the interface velocity jump (Stefan flow) and calculation of the mass transfer rate
- 5. High Jakob number (Ja \approx 70), leading to thin thermal boundary layers
- 6. Natural convection driven boundary layers (Re $\approx 6 \times 10^7$, Ra $\approx 3 \times 10^9$)
- 7. Coupled solid-fluid heat transfer
- 8. Nucleate boiling
- 9. Dynamic contact line in super-heated wall with very small contact angles
- Phase change at small length scales, like droplets and films

Schematic representation of tank and flow physics to be modelled

Large range of length/time scales

Slosh Induced Pressure Drop



Temperature distribution in two-phase system



 $\begin{array}{l} \mathsf{Gentle\ Slosh} \rightarrow \mathsf{Stratification} \rightarrow \mathsf{Slow}/\mathsf{No\ Pressure\ Drop} \\ \mathsf{Violent\ Slosh} \rightarrow \mathsf{Mixing} \rightarrow \mathsf{Fast\ Pressure\ Drop} \end{array}$

$$Pr \approx 0.7$$

$$Ra = Pr \frac{g\rho_0^2(T_h - T_c)L^3}{T_0\mu^2} \approx 10^9$$

$$\epsilon = \frac{T_h - T_c}{T_h + T_c} \approx 0.66$$

$$M \approx 0.01 \text{ (slosh velocity)}$$

Thermal compressible effects must be considered



Low mach solver used in gas phase

Numerical Capabilities: Not Exhaustive!

Problem	Numerical technique	Sandbox
Navier-Stokes Eqs.	Incompressible (liquid) Low-Mach (gas)	centered.h jmaarek/two_phase_low_mach.h
Fluid-Solid Interaction	Moving Reference Frame Embed+VOF	tavares/contact-embed.h
Turbulence Modeling	LES	Antoonvh/vreman.h
Conjugate Heat Transfer	Time-implicit flux-partioned diffusion	Antoonvh/diffusion-pair.h jmaarek/diffusion-three_field.h
Phase Change	Interface Velocity-Jump	ecipriano/
Thin Thermal BL	BL-SGS	maarek/thin_BL/

Liquid Equations Solved: Boussinesq Incompressible NS

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}_{rel}) + \left[-\nabla \rho' + \nabla \cdot \left(2\mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \right) \right] + \mathbf{F}_{\sigma} + \mathbf{g} \cdot \mathbf{x} \nabla \rho \qquad (1)$$
$$\nabla \cdot \mathbf{u} = 0 \qquad (2)$$

$$\partial_t f + \mathbf{u}_{rel} \cdot \nabla f = 0 \tag{3}$$

$$\mathbf{u} = \mathbf{u}_{rel} + \mathbf{u}_{rigid} \qquad \mathbf{u}_{rigid} = \mathbf{u}_{trans} + \Omega \times \mathbf{r}$$
(4)

$$\rho c_{\rho} \frac{\partial T}{\partial t} + \rho c_{\rho} (\mathbf{u}_{rel} \cdot \nabla T) = \nabla \cdot (k \nabla T)$$
(5)

Gas Equations Solved: Conservative Low Mach Equations

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}_{rel}) + \left[-\nabla p' + \nabla \cdot \left(2\mu \left[\nabla \mathbf{u} - (\nabla \mathbf{u})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right] \right) \right] + \mathbf{F}_{\sigma} + \mathbf{g} \cdot \mathbf{x} \nabla \rho$$
(6)
$$\overline{P} = \rho R \overline{T}$$
(7)

$$\frac{d\overline{P}}{dt} = (\gamma - 1) \int_{V} \nabla \cdot (k\nabla T) \, dV \tag{8}$$

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot (\rho \mathbf{u}_{rel} h) = \nabla \cdot (k \nabla T) + \frac{d\overline{P}}{dt}$$
(9)

$$\nabla \cdot \mathbf{u} = \frac{R}{\overline{P}c_{\rho}} \left(\nabla \cdot (k\nabla T) - \frac{1}{\gamma - 1} \frac{d\overline{P}}{dt} \right) - \frac{\zeta}{\gamma \overline{P}} \frac{\rho RT - \overline{P}}{\Delta t} {}^{12}$$
(10)

¹J. Bell, "Amr for low mach number reacting flow," in Adaptive Mesh Refinement-Theory and Applications: Proceedings of the Chicago Workshop on Adaptive Mesh Refinement Methods, Sept. 3–5, 2003, pp. 203–221, Springer

²R. Knikker, "A comparative study of high-order variable-property segregated algorithms for unsteady low mach number flows," International Journal for Numerical Methods in Fluids, vol. 66, no. 4, pp. 403–427, 2011



- tavares/contact-embed.h ³
- easily generalisable to 3D
- Fixed problem of orientation-dependent numerical pinning (zero volumetric flux)





Modification: Perform extrapolation from boundary one cell inside fluid domain

³M. Tavares, C. Josserand, A. Limare, J. M. Lopez-Herrera, and S. Popinet, "A coupled vof/embedded boundary method to model two-phase flows on arbitrary solid surfaces," *Computers Fluids*, vol. 278, p. 106317, 2024

VOF + Embed Validation 1: Sessile Drop Inclined Plane

tavares/test_cases/sessile_embed.c

Validation performed by Theo Witkamp



Better agreement for steady state position, even at shallow angles

VOF + Embed Validation 1: Sessile Drop Inclined Plane



Orientation dependent spreading error signifcantly improved

VOF + Embed Validation 2: Sessile Drop Inclined Plane 3D



Original

Modified

3D asymmetry removed with modification

Low Mach Validation 1: Heated Cavity Benchmark Case

 ${\sf Ra}=10^7$, $\epsilon=$ 0.6, Sutherland's law for viscosity, conductivity, maxlevel 9



⁴Le Quéré, Patrick, Weisman, Catherine, Paillère, Henri, Vierendeels, Jan, Dick, Erik, Becker, Roland, Braack, Malte, and Locke, James, "Modelling of natural convection flows with large temperature differences: A benchmark problem for low mach number solvers. part 1. reference solutions," *ESAIM: M2AN*, vol. 39, no. 3, pp. 609–616, 2005

Low Mach Validation 2: GH2 Passive Pressurisation

Heated at 3.5 $\mathrm{W/m^2}$ for 2 hrs, Sutherland's law for viscosity, conductivity



Isothermal Sloshing 1: Rectangular Domain



⁵H. Rezaei and M. J. Ketabdari, "Numerical modelling of sloshing with vof method," in *The 12th International Conference on Fluidization - New Horizons in Fluidization Engineering* (F. Berruti, X. T. Bi, and T. Pugsley, eds.), (London, Canada), ECI Symposium Series, 2007

Isothermal Sloshing 2: Circular Domain

Air-Water system, corresponding experiments published in ⁶

Simulations performed by Pavan Kumar Kirar



1 Hz $f/f_N \approx 0.8$



1 Hz $f/f_N \approx 0.96$

⁶Ángel Luis Martín López, Estudio Teórico Experimental de la Estabilidad Lateral en Vehículos Cisterna. Metodología para la Determinación del Umbral de Vuelco. Phd thesis, Universidad Politécnica de Madrid, Escuela Técnica Superior de Ingenieros Industriales, Madrid, Spain, 2013

Isothermal Sloshing 2: Circular Domain



Parametric study of X-force response compared with experiment

Horizontal cylinder with rounded ends ⁷

Air-Water system, compared with unpublished COMSOL simulations, maxlevel 9



⁷E. L. Grotle and V. Æsøy, "Numerical simulations of sloshing and the thermodynamic response due to mixing," *Energies*, vol. 10, no. 9, 2017

- 1. Proposed simple correction to existing VOF+Embed that significantly reduces orientation-dependent contact-line mobility
- 2. Developed, validated low-mach solver for gas phase
- 3. Performed 2D/3D isothermal sloshing simulations, relatively good agreement observed for measured forces

- 1. Perform 2-phase NASA self-pressurisation test case $^{\rm 8}$
- 2. Include phase-change, non-ideal effects
- 3. Perform non-isothermal slosh simulations, validation with published numerical simulations ⁹, LH2 experiments performed by group members

in Proceedings of Space Propulsion 2018, (Seville, Spain), p. –, NASA Glenn Research Center / European Space Agency / 3AF, May 2018. Report No. SP-2018-179 (GRC-E-DAA-TN54730)

⁸W. L. Johnson, L. M. Ameen, F. D. Koci, D. Oberg, and J. G. Zoeckler, "Structural heat intercept, insulation and vibration evaluation rig (shiiver),"

⁹E. L. Grotle and V. Æsøy, "Numerical simulations of sloshing and the thermodynamic response due to mixing," *Energies*, vol. 10, no. 9, 2017