IMPERIAL

Coupled 2-D HOS-Basilisk simulations of multiple breaking waves in dispersively-focused wave groups

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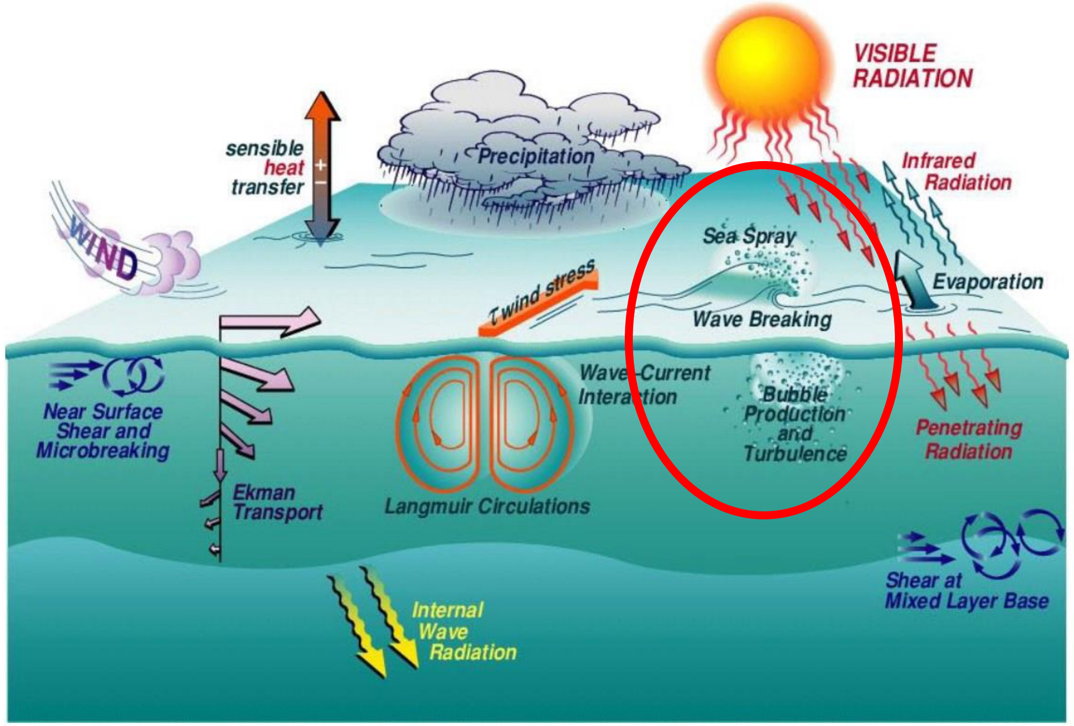
Breaking Waves

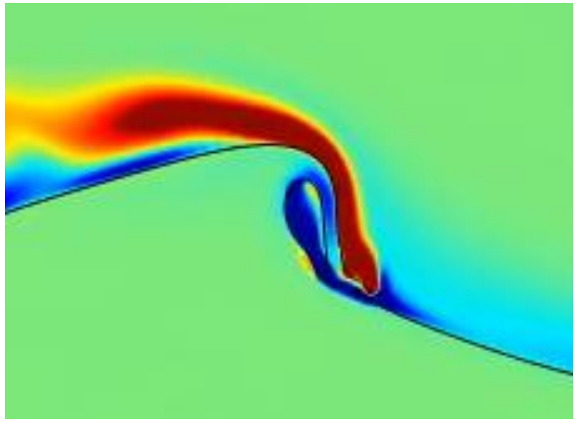


- Limit wave height
- Balance wind input
- Extreme wave load
- Enhance mass, momentum, energy transfer

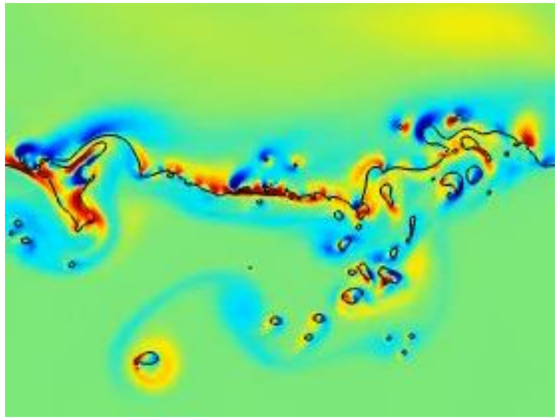
A Multiple-Scale Problem


By Adrian Hlj / Getty Images

Wind Wave Field
O(1km - 100km)



Individual Breaking Wave
O(0.1m - 10m)



Bubbles and Droplets
O(0.1um - 1cm)

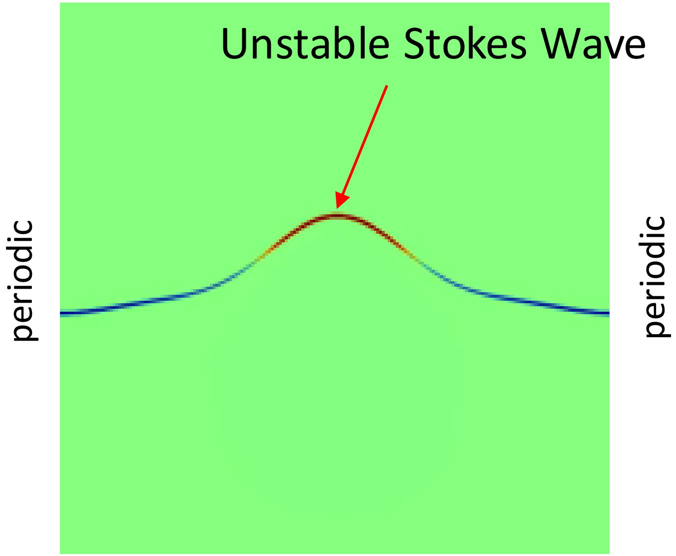
Evolution of wave field:

$$\frac{\partial F}{\partial t} + C_g \cdot \nabla F = S_{in} + S_{nl} + \boldsymbol{S_{diss}}$$

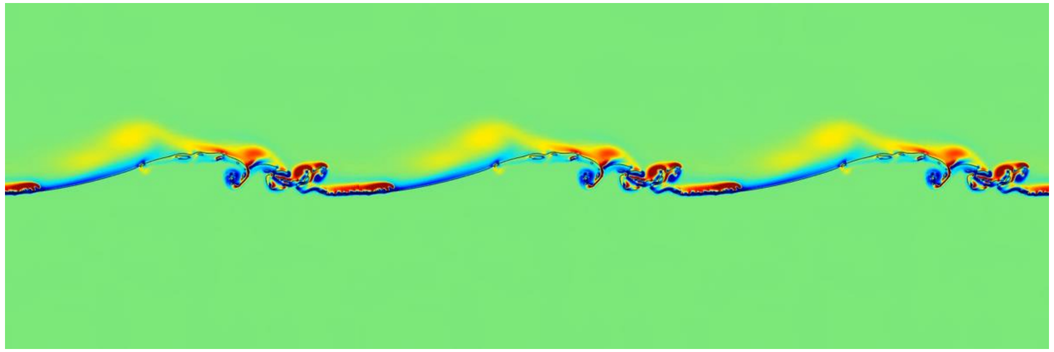
Least Understood
Dominated by Breaking Waves

F: energy density spectrum

Most Direct Numerical Simulation: Unstable Stokes Wave



e.g. Deike 2015, 2016



Unstable initial condition: practical mechanism leading to breaking

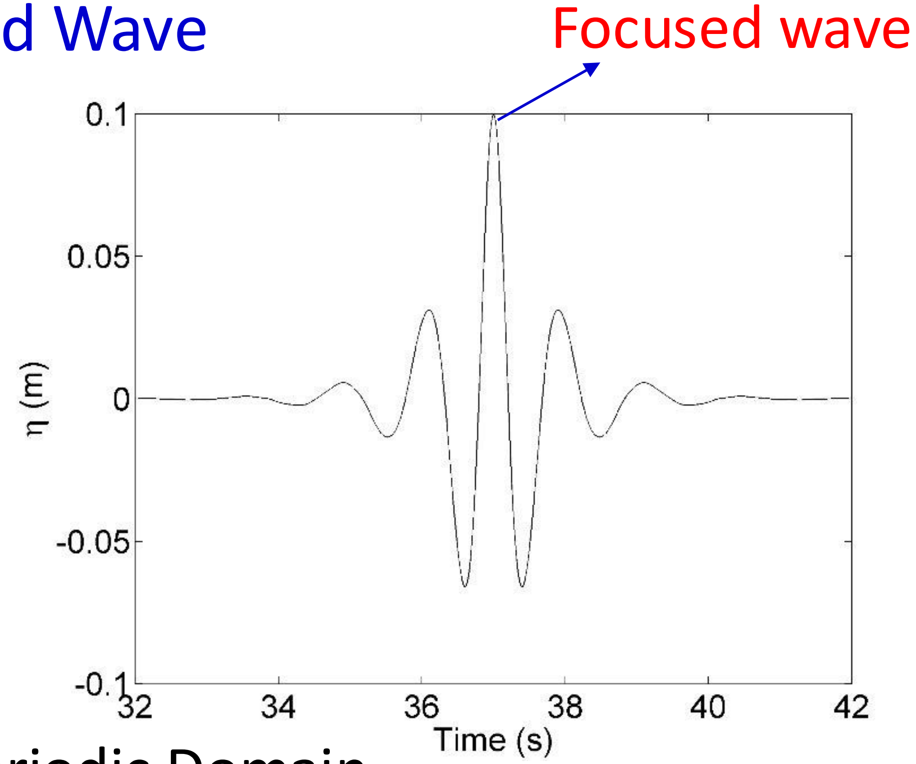
Close breaking wave interval: interact during active breaking

How to create a more realistic breaking wave ?

A More Realistic Breaking Wave: Dispersively-focused Wave

$$\eta = \sum a_i \cos(\omega_i t + k_i x + \phi_i)$$

Widely used both experimentally and numerically
Adjusting ϕ_i to create a large wave at focusing point



- A numerical wave tank

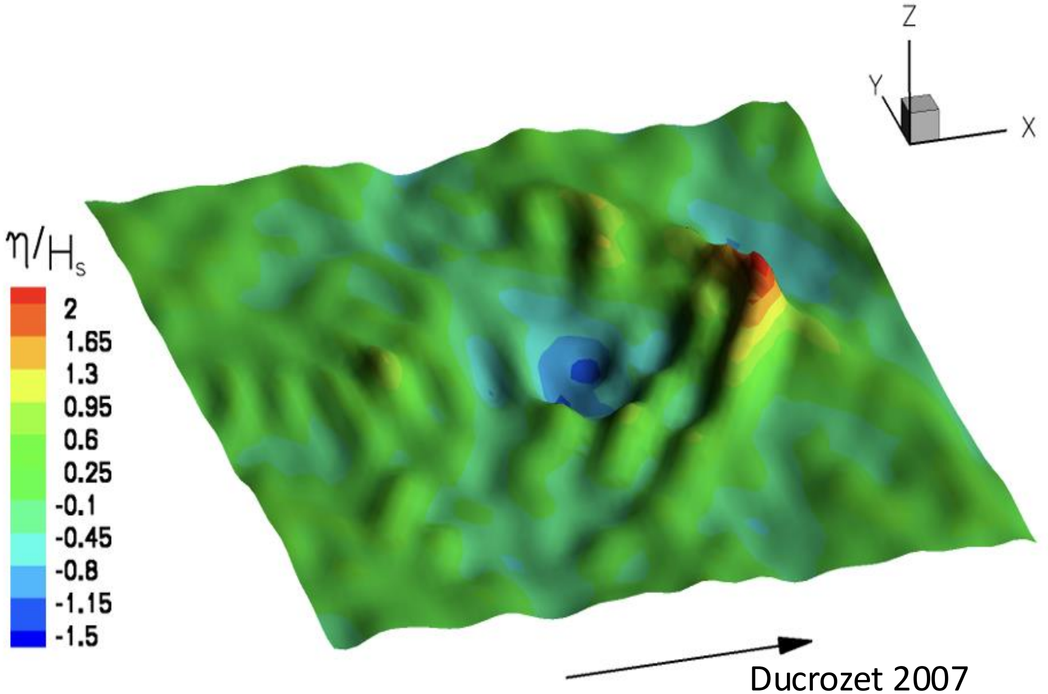
or simpler →

- Periodic Domain
- Initialise with multiple wave components

How to initialize a fully developed non-linear wave field in Basilisk?

To Generate Fully Non-linear Wave Field: HOS-ocean

HOS-ocean, a software based on High-Order Spectral (HOS) method which enables the simulation of highly non-linear wave fields.



Ducrozet 2007

Governing Equation:

$$\Delta\phi = 0$$

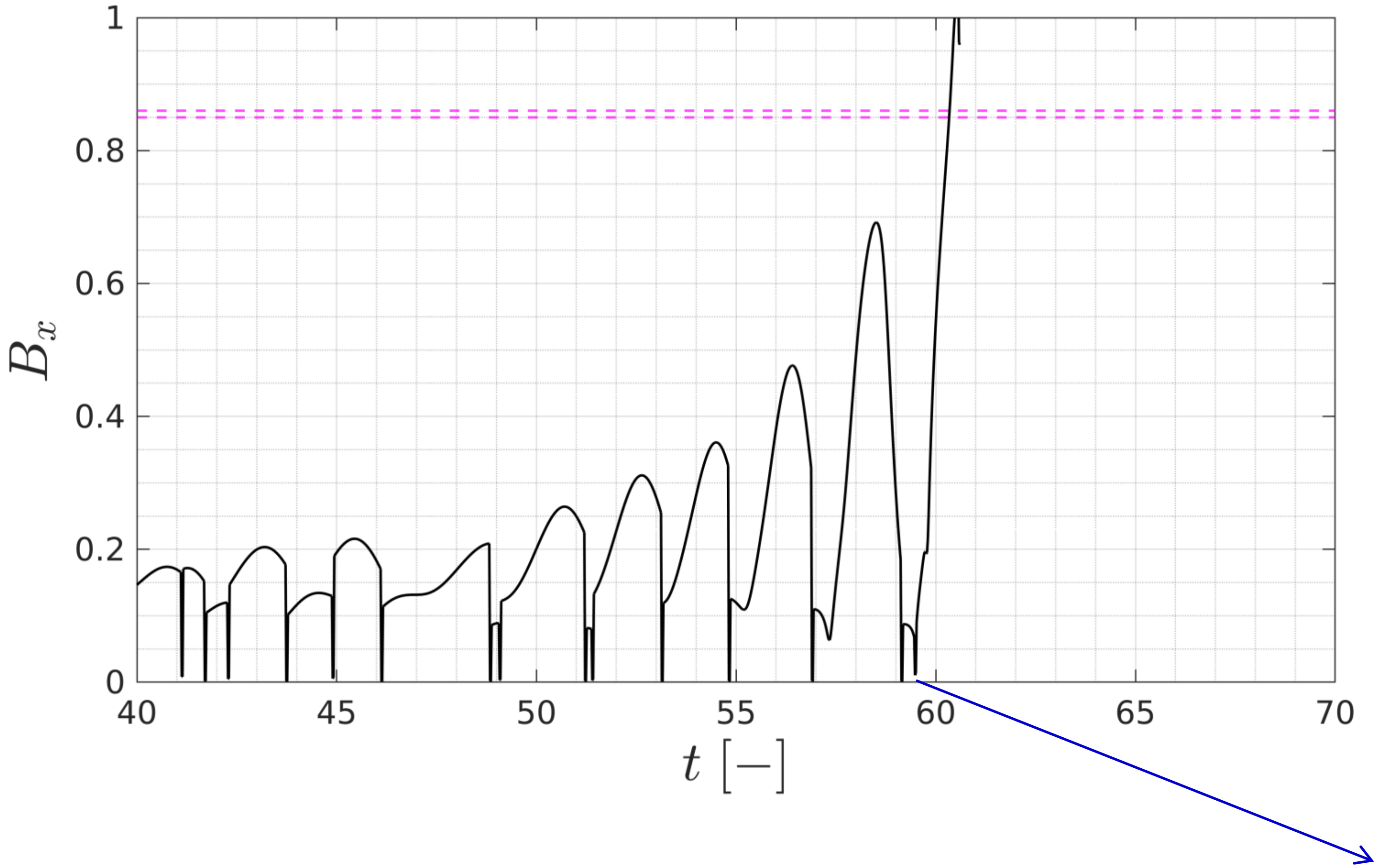
Free Surface Boundary Condition:

$$\frac{\partial \phi^S}{\partial t} = -g\,\eta - \frac{1}{2}|\nabla\phi^S|^2 + \frac{1}{2}(1+|\nabla\eta|^2)\left(\frac{\partial\phi}{\partial z}\right)^2$$

$$\frac{\partial\eta}{\partial t} = (1+|\nabla\eta|^2)\frac{\partial\phi}{\partial z} - \nabla\phi^S\cdot\nabla\eta$$

Linearised wave field $\xrightarrow[\text{smooth transition}]{\text{HOS-Ocean}}$ Fully Developed Nonlinear Wave Field

Time from HOS-ocean to Basilisk

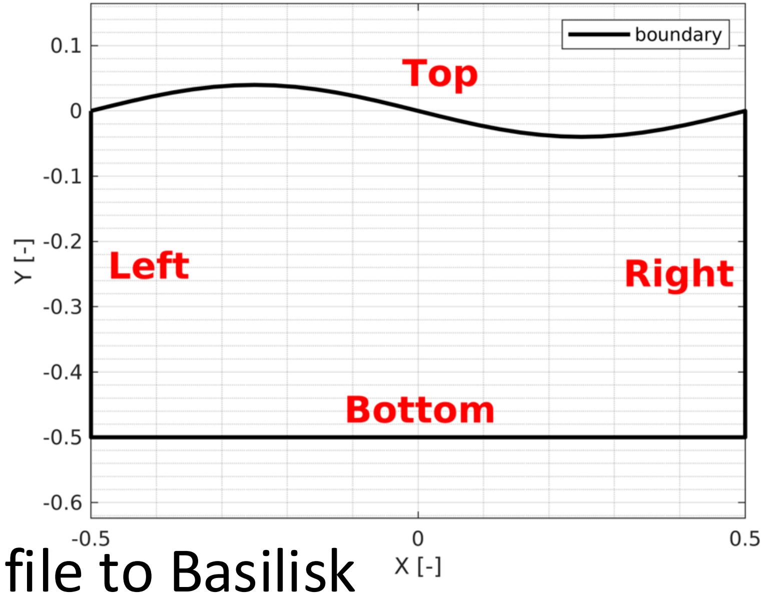


$B_x = \frac{u_x}{c_x}$: a diagnostic parameter for wave breaking

(Barthelemy 2018)

Last trough before $B_x > 0.85$
HOS result to Basilisk

HOS-Ocean to Basilisk: Boundary Element Method (BEM)



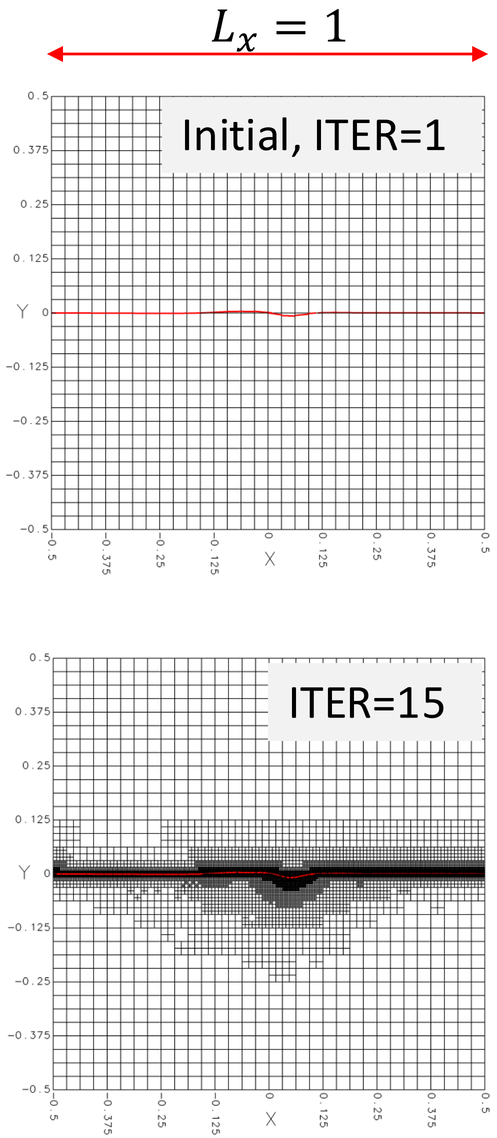
- Output of HOS-ocean: ϕ^S and η (Surface information)

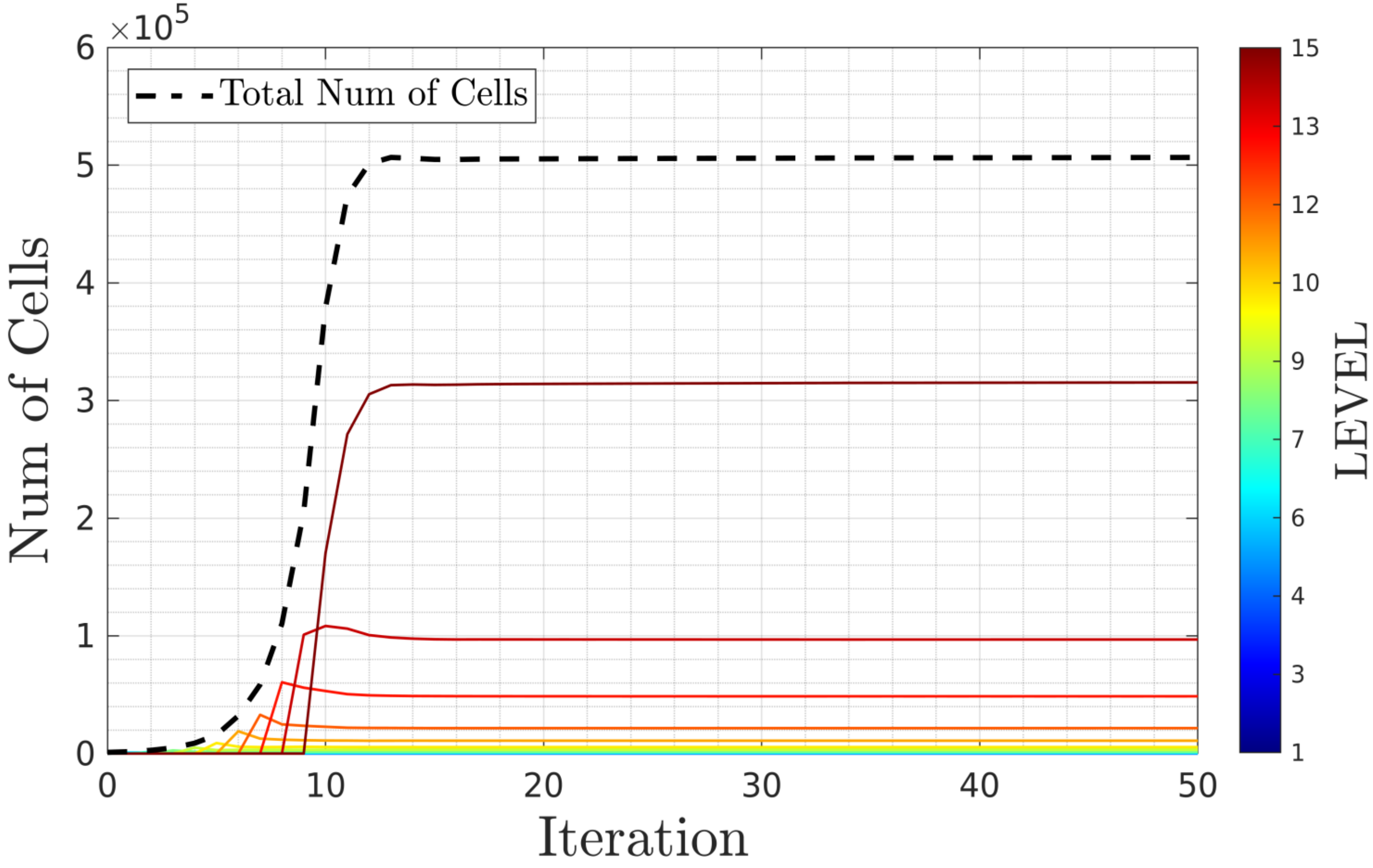
$\Downarrow$ BEM

- Basilisk Initial Condition: whole velocity field $\vec{u}(x, y)$
- Adaptive mesh: the mesh is unknown when preparing input file to Basilisk
- Boundary information (known): ϕ and $\frac{\partial \phi}{\partial n}$ input to Basilisk

Minimize the input file while maintain high-accuracy velocity field

Initial Basilisk AMR Iteration

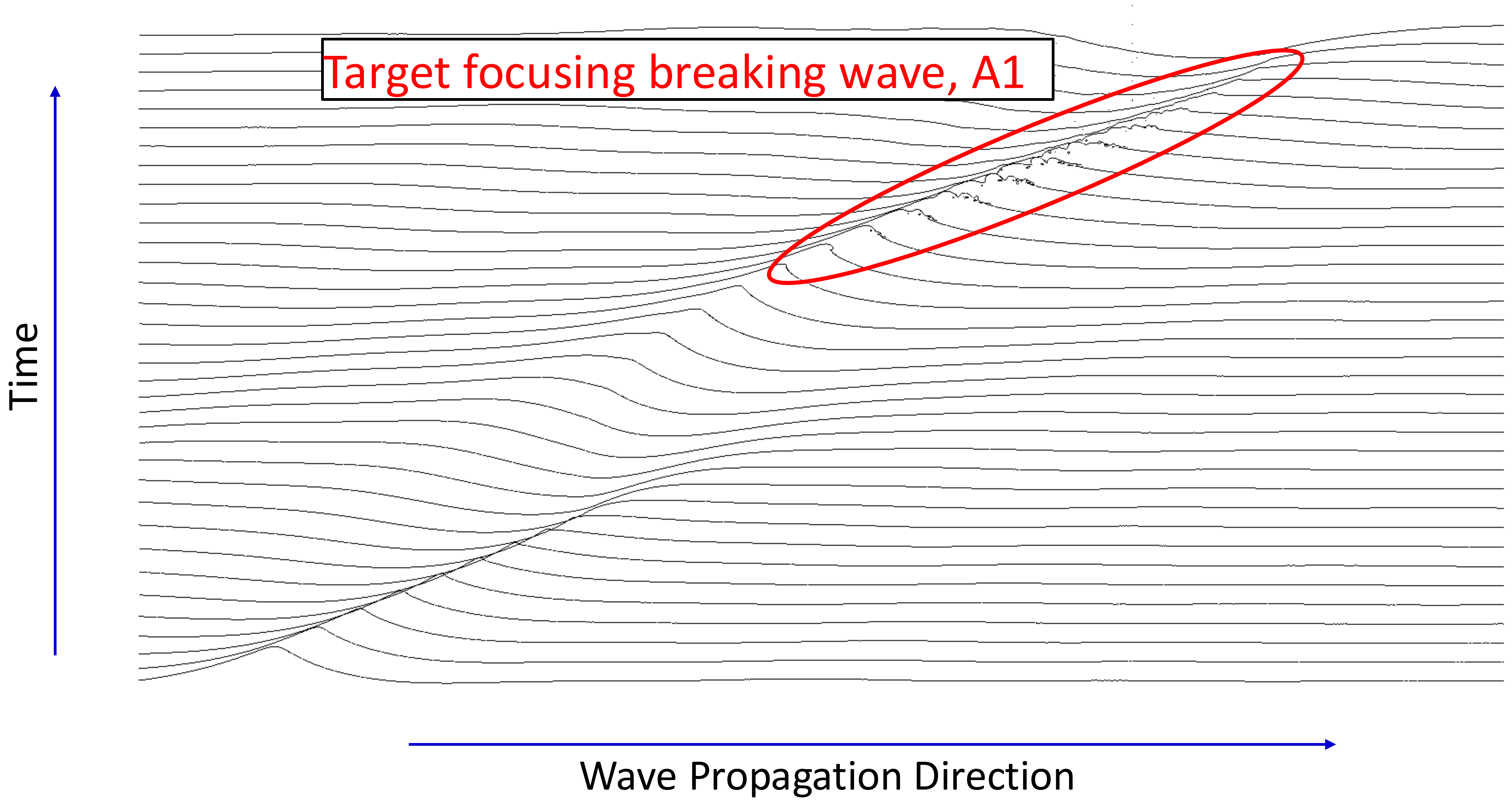




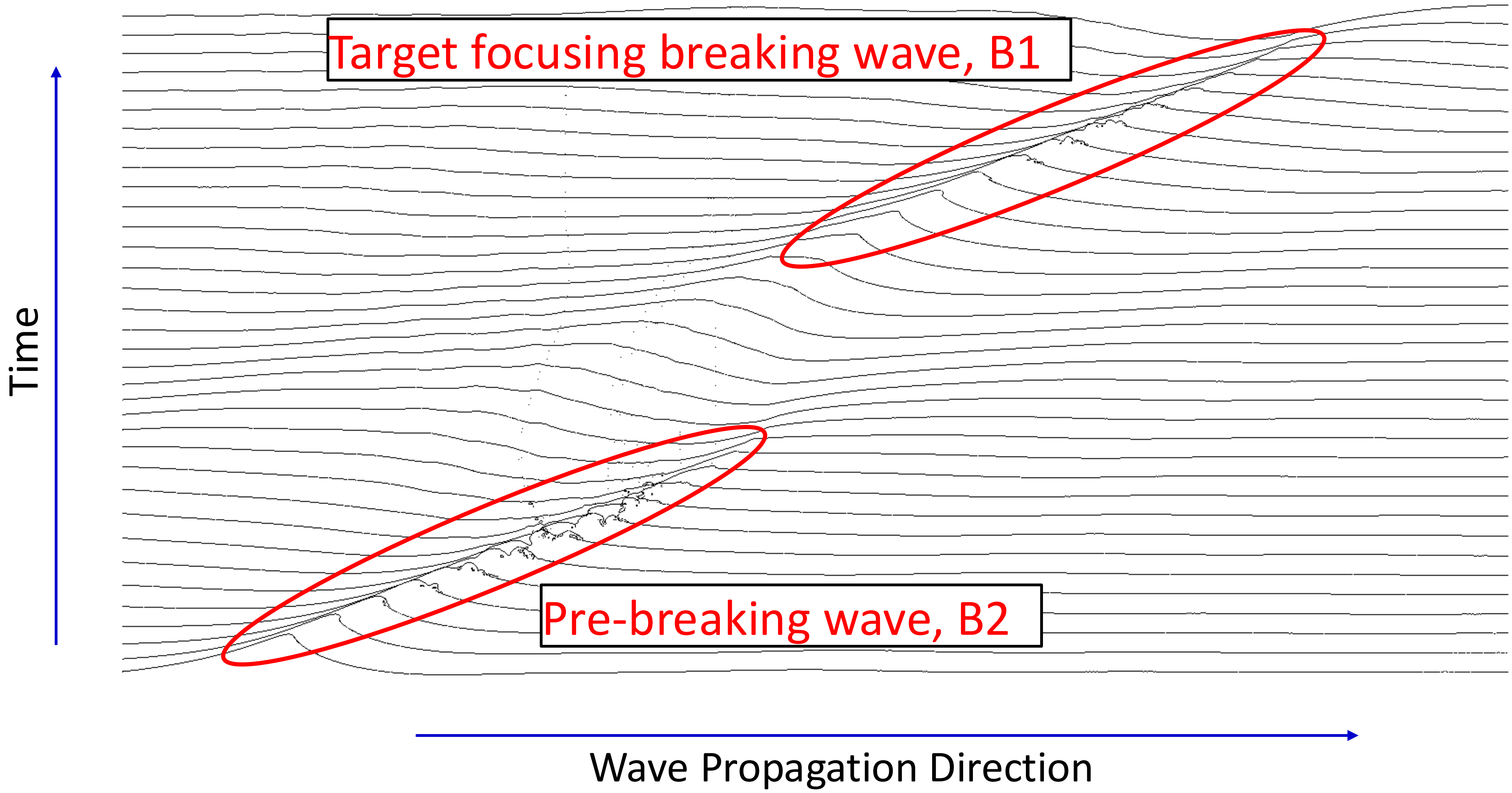
AMR: $\xi = 10^{-5}$ for $\vec{u}$ and f
Initial LEVEL: 5
Finest LEVEL: 15

Non-dimensional parameter
$\rho_w = 1$
$g = 1$
$L_x = 1$

Scenario A


Target focusing breaking wave, A1
Time
Wave Propagation Direction

Scenario B



Target focusing breaking wave, B1

Pre-breaking wave, B2

Breaking Scenario V.S. Global Spectral Wave Slope

- Recurrent wave group (non-breaking)

⇨

Scenario A

- Single breaking of target focusing wave A1

⇨

Scenario B

- Pre-breaking B2
- Target focusing breaking B1

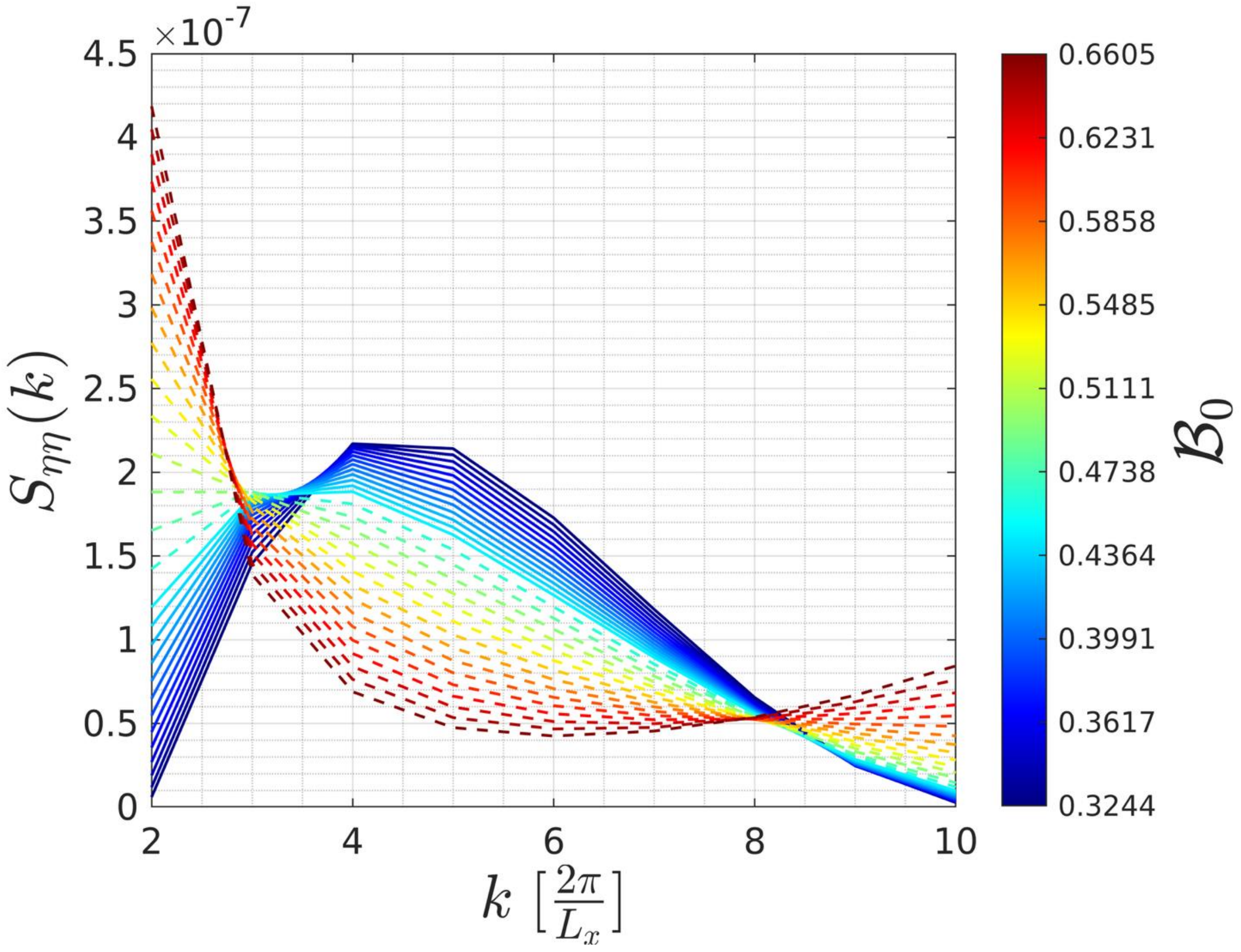
Increasing Wave Slope

Any difference of energy dissipation on Breaking Event A1 and B2?

Are the properties of the breaking wave influenced by bandwidth?

Bandwidth: a parameter describing how energy are distributed over a spectrum

A Wave Spectrum Controlling Slope, Energy and Bandwidth



Global Wave Slope: $S_0 = \sum a_i k_i$

Overall Spectral Energy: $E_0 = \frac{1}{2}\rho_w g \sum a_i^2$

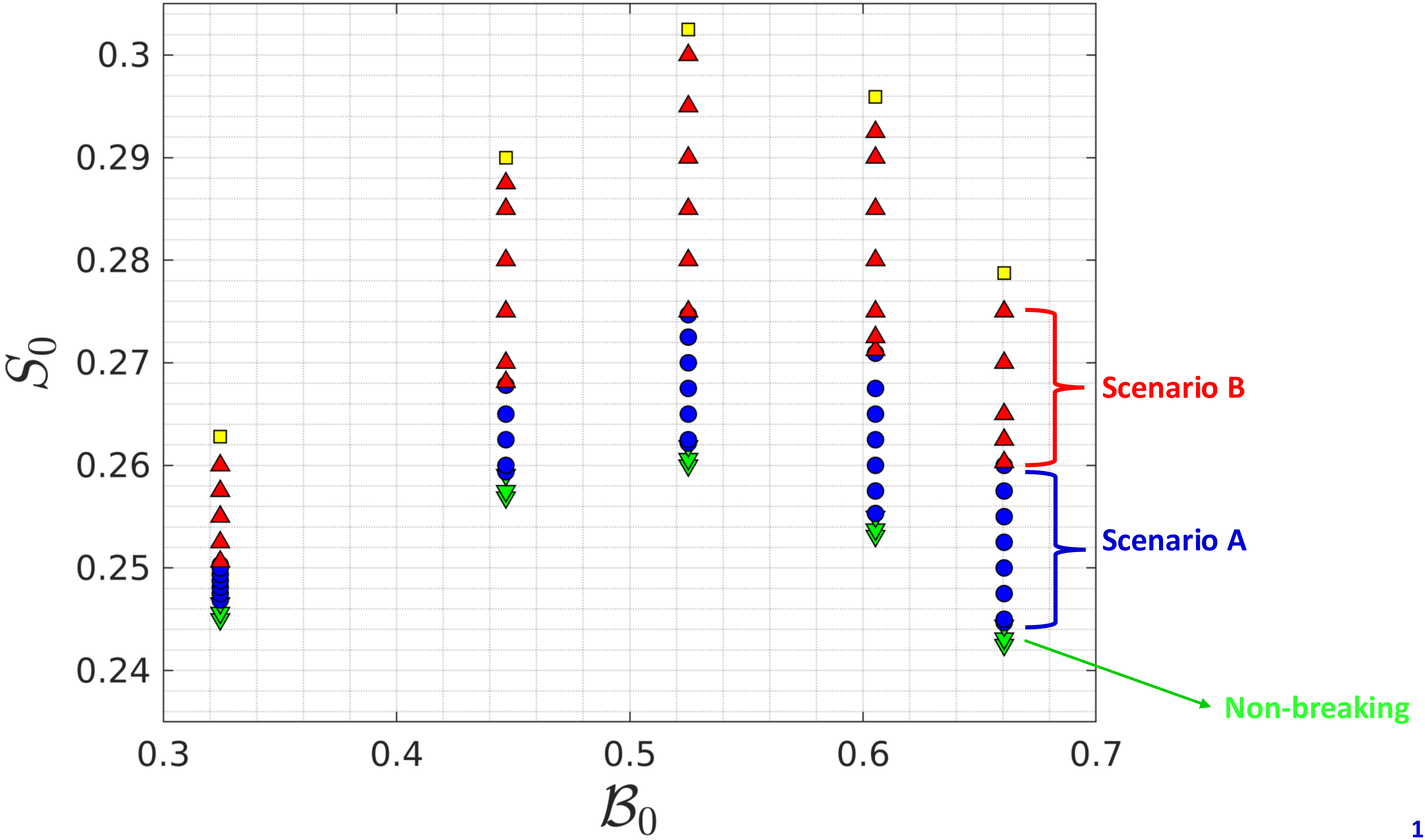
Bandwidth: $\mathcal{B}_0 = \sqrt{\frac{m_1 m_{-1}}{m_0^2} - 1}$

Smith, Venugopal & Wolfram (2006)

$$a_i = C_0 \frac{1}{k_i} + C_1 + C_2 \cdot k_i$$

Spectra of same energy, global slope, but different bandwidth

Breaking Onset Spectral Slope Is Non-Monotonically Dependent on Bandwidth

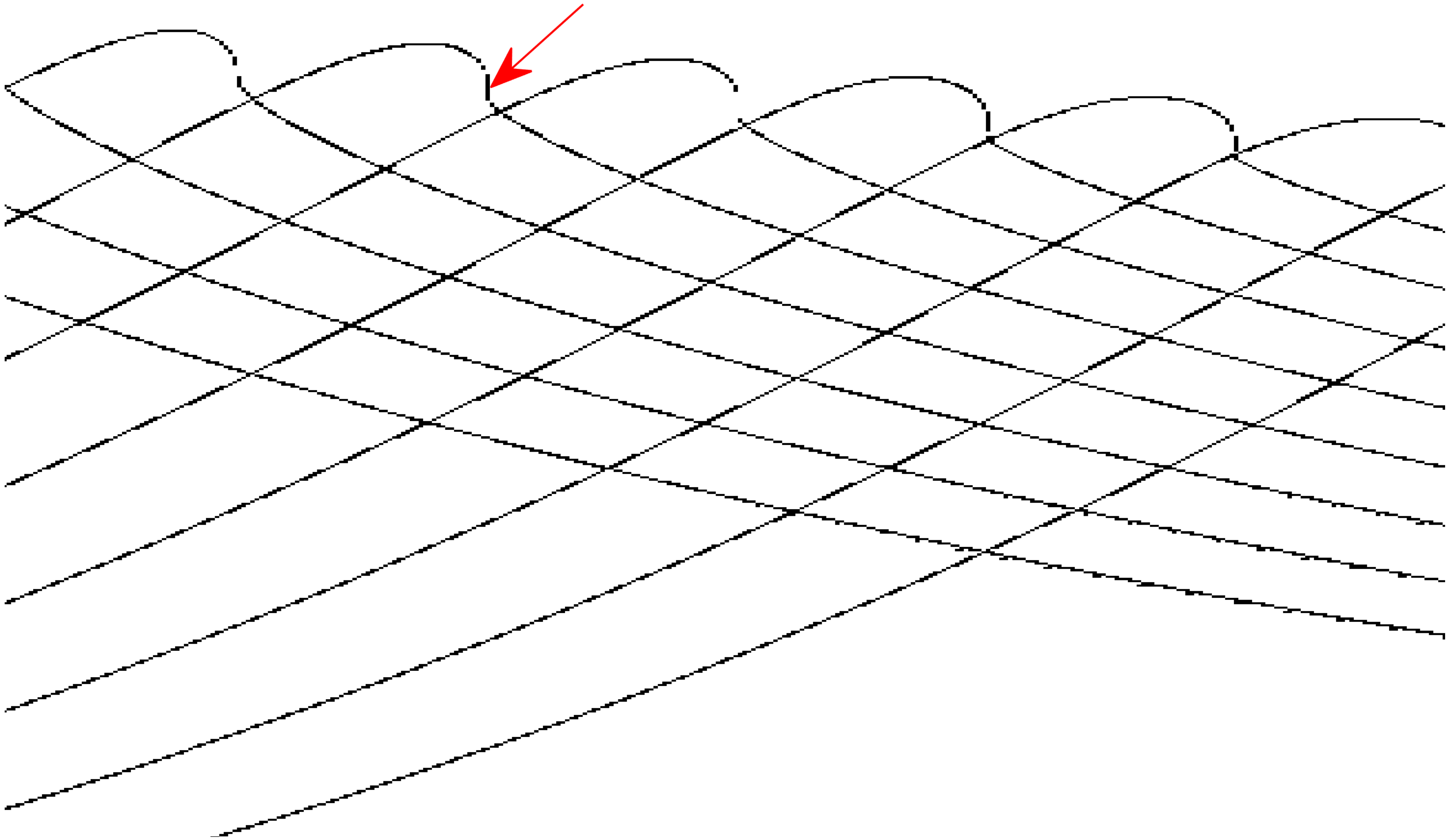


Scenario B

Scenario A

Non-breaking

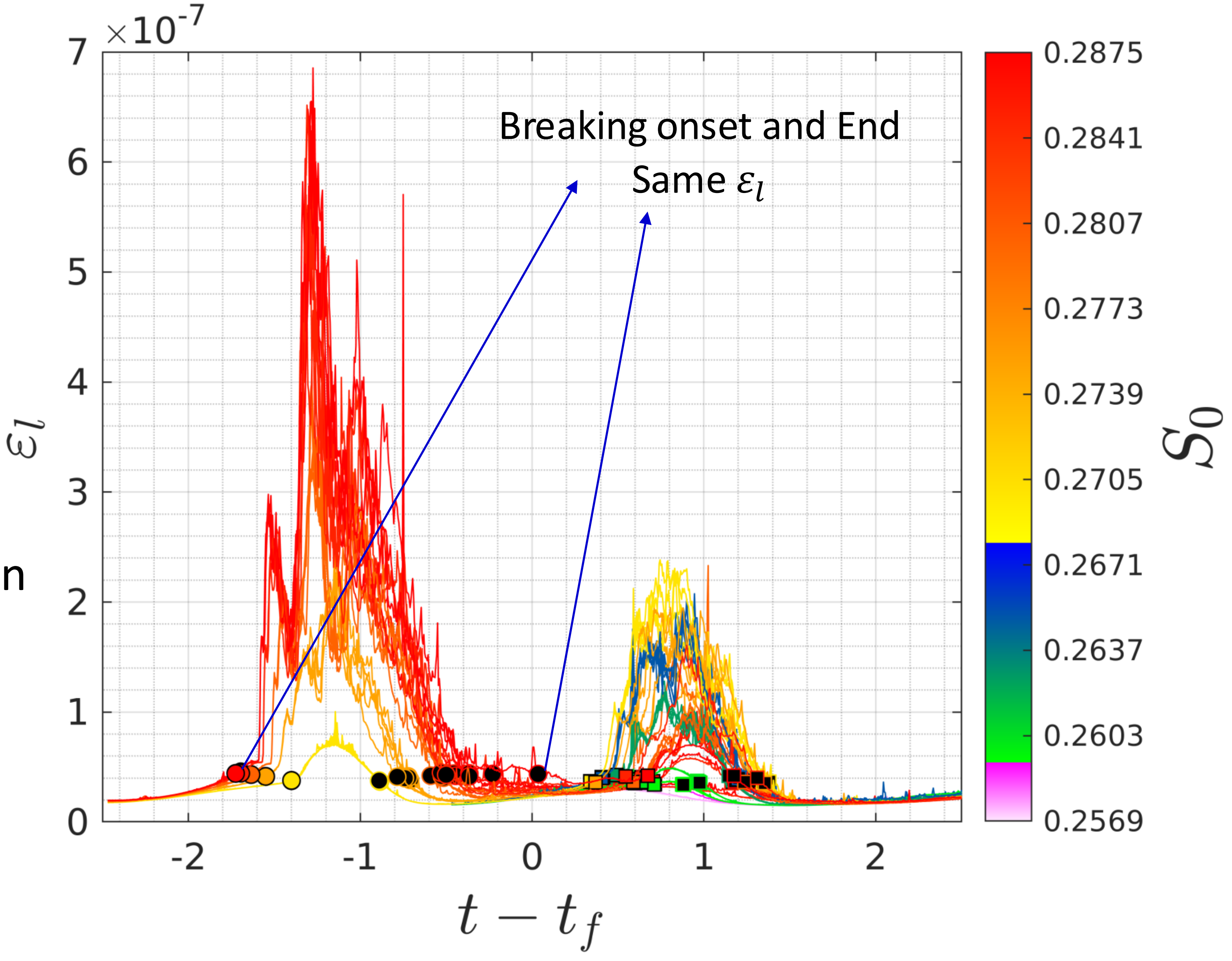
Definition of Breaking Start and End



Occurrence of multi-valued free surface

Definition of Breaking Start and End

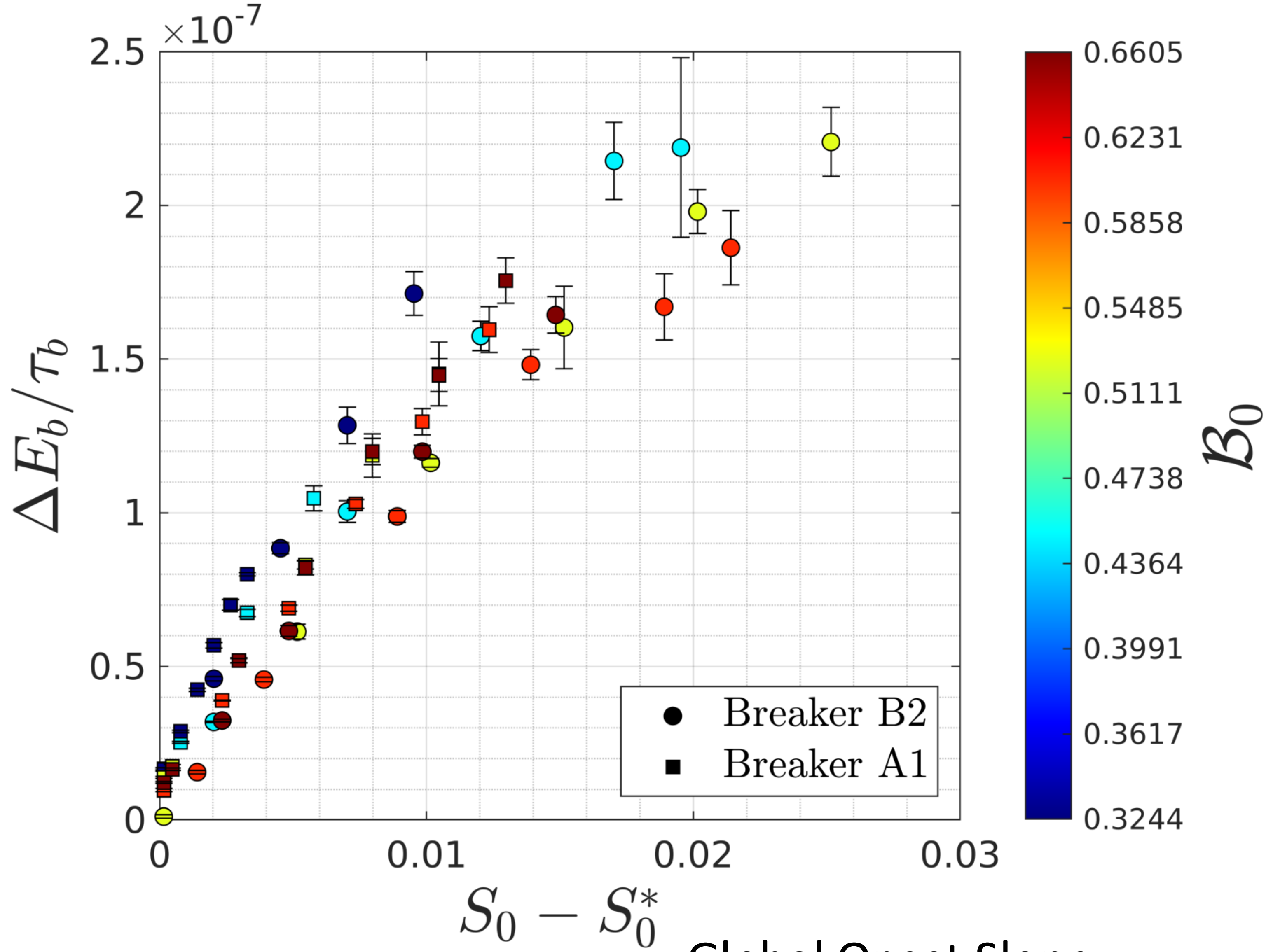
ε_l Energy dissipation rate



Breaking onset and End

Same ε_l

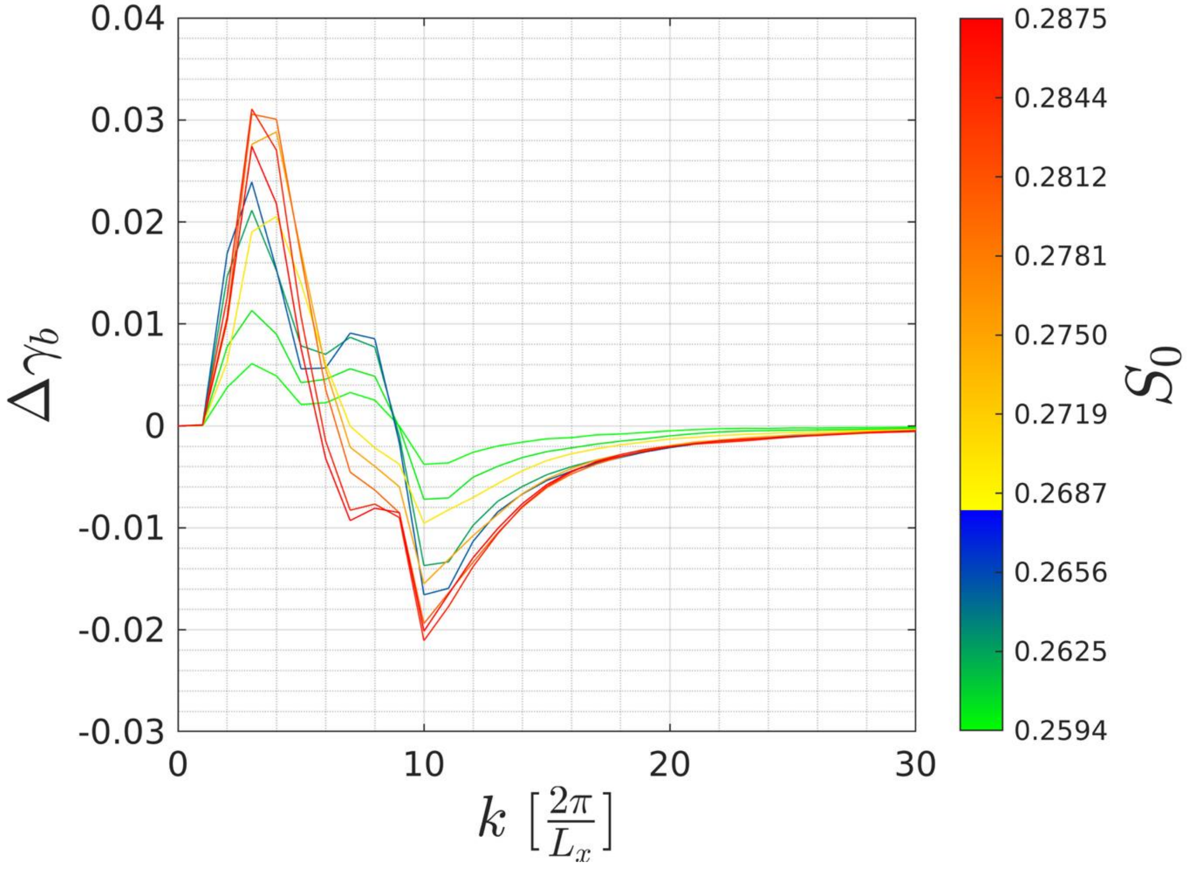
Energy Dissipation Rate: Slightly Influenced by Breaking Events and Bandwidth

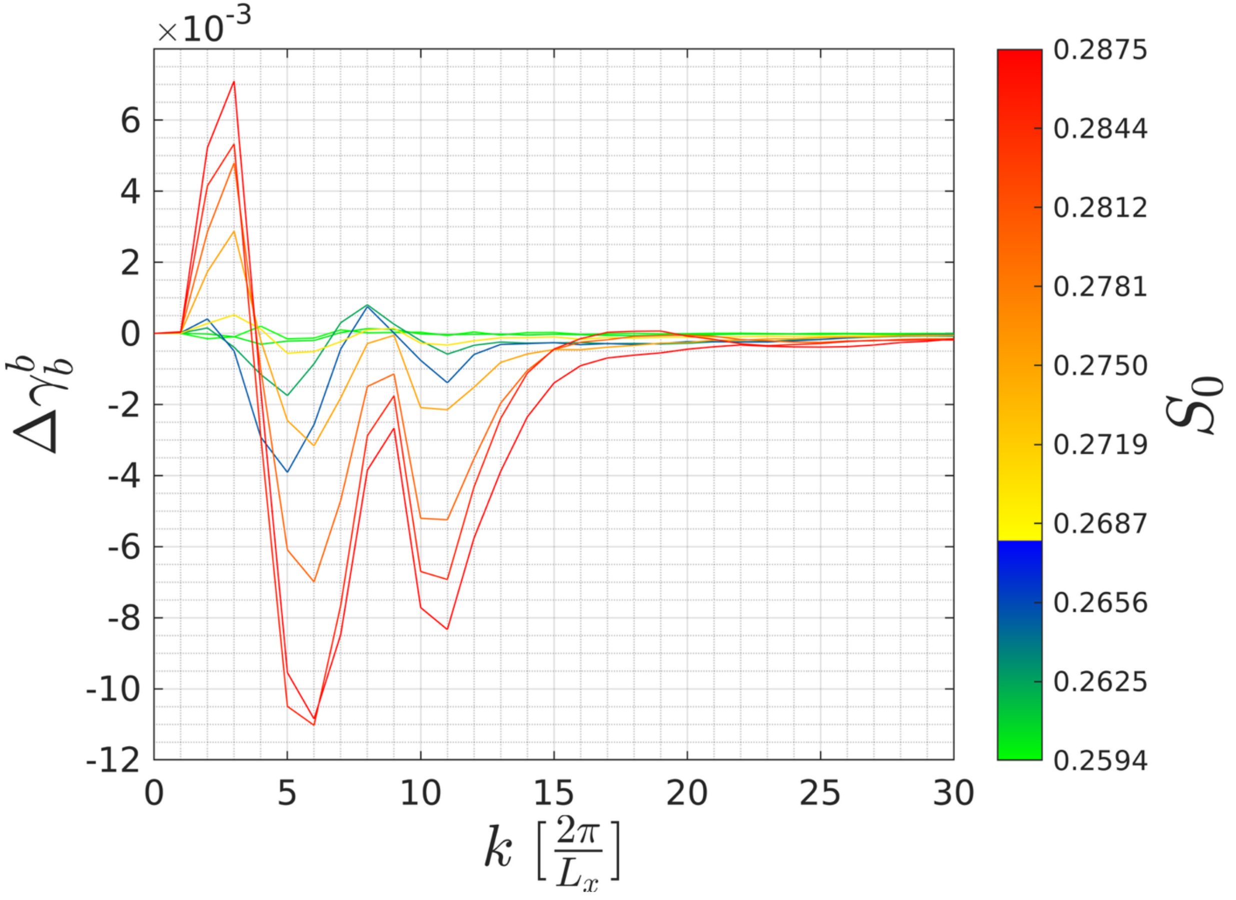


Global Onset Slope

Imperial College London

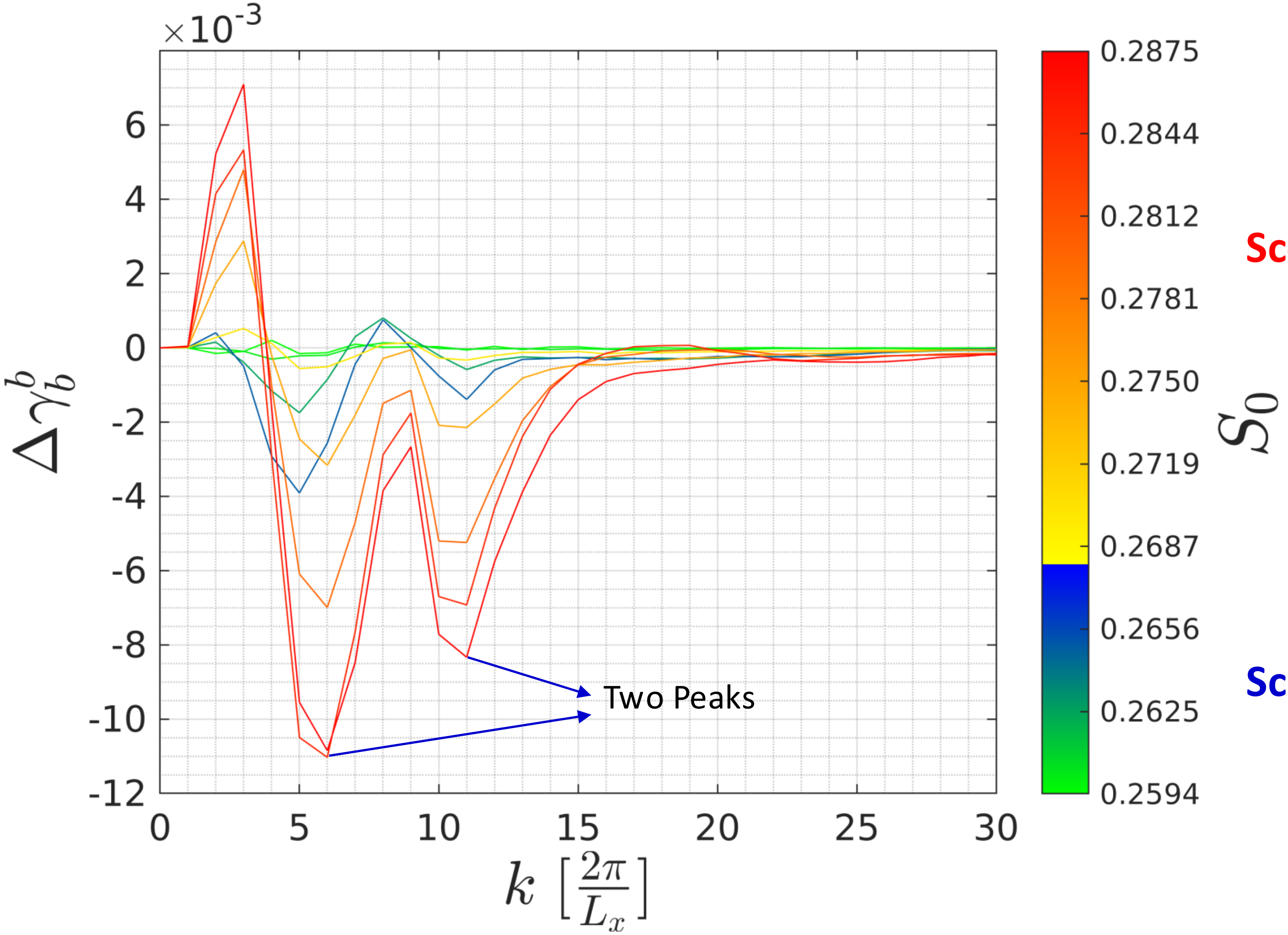
Spectral change significantly influenced by non-breaking nonlinear wave interaction





Normalized spectral change: $\Delta\gamma_b = \frac{a^2_{i,t_{end}} - a^2_{i,t_{start}}}{\sum a^2_{i,input}}$

Spectral Energy Dissipation: Similar Pattern for Different Breaking Events



Scenario B

Scenario A

Summary

- Non-monotonical dependence of breaking onset spectral slope on bandwidth
- Slight influence of breaking events and bandwidth on energy dissipation
- Spectral change over breaking duration is still significantly influenced by non-breaking wave non-linear interaction
- The spectral energy dissipation by breaking of different breaking events shows similar pattern

END