Viscosity as a regularization mechanism for conical cavity collapse like bursting bubbles

N. CAILLER†, R. WUNENBURGER† & A. ANTKOWIAK†

†Institut Jean Le Rond ∂'Alembert, CNRS UMR 7190, Sorbonne Université, Paris 75005, France

BASILISK (GERRIS) USERS' MEETING

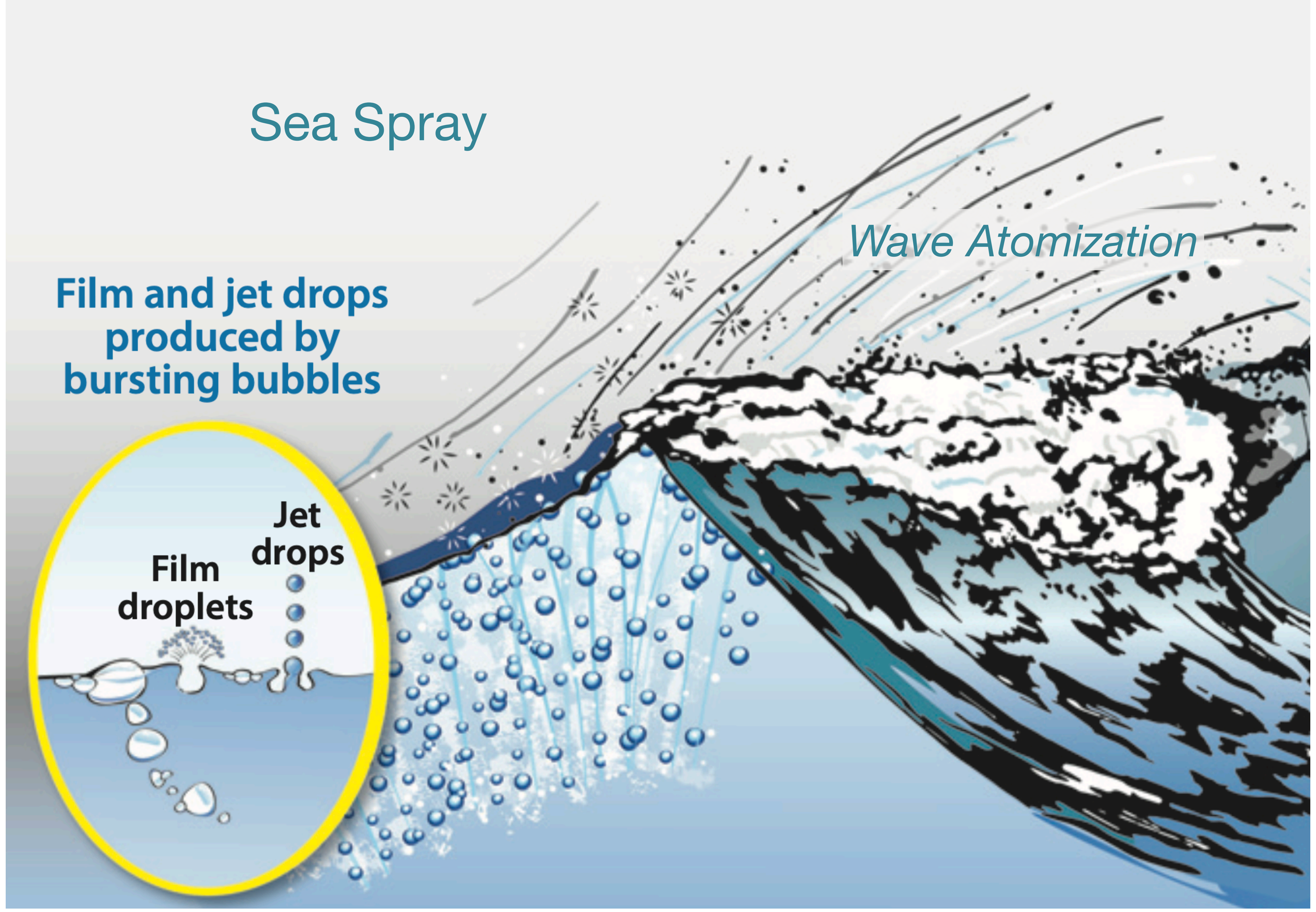
* * *

7-9th July 2025

Introduction

1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

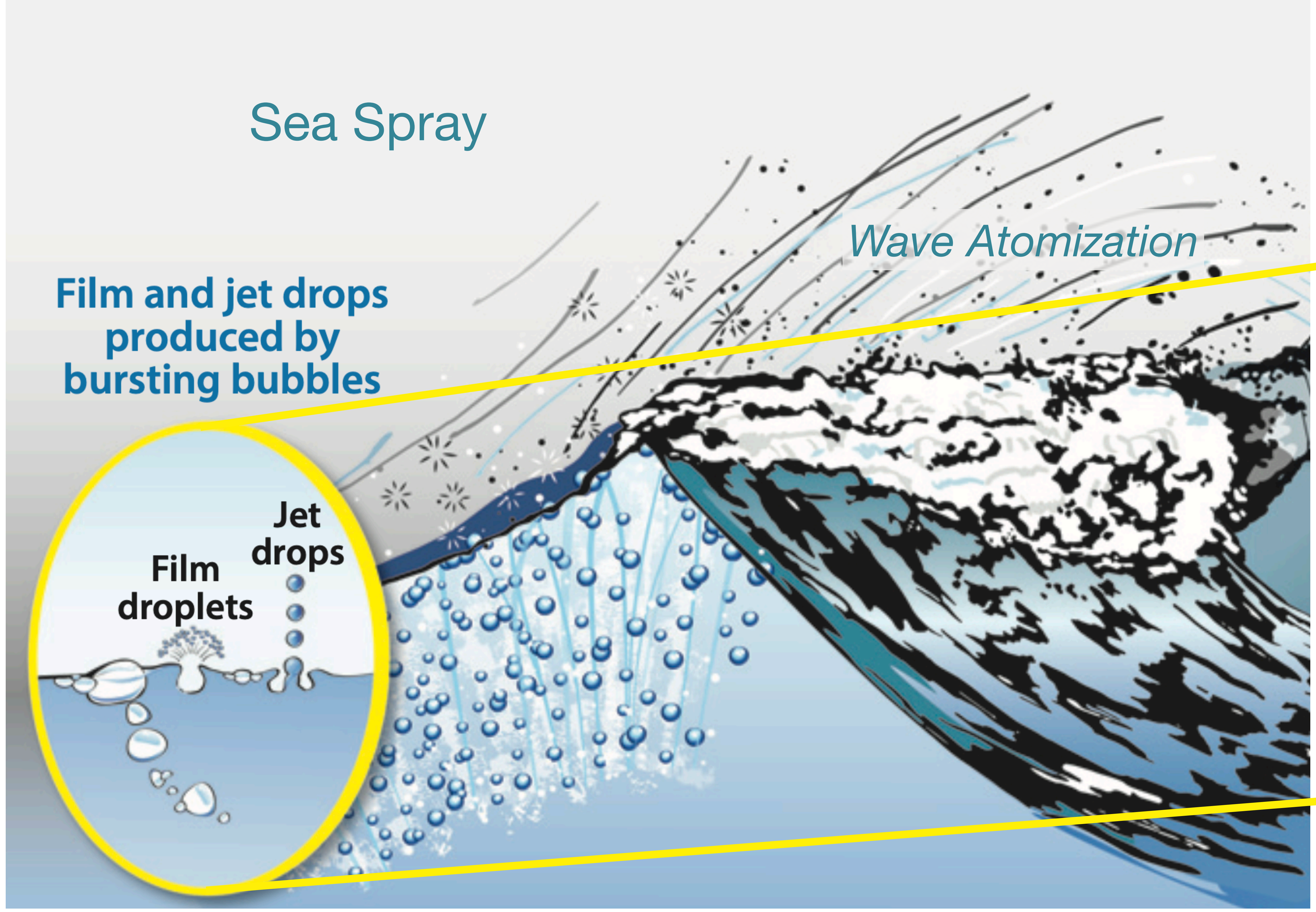
1. Wave breaking: entrapped air pockets → large amount of bubbles rising

2. Conical Collapse

3. Jets: droplets ⩽ 1 µm propelled

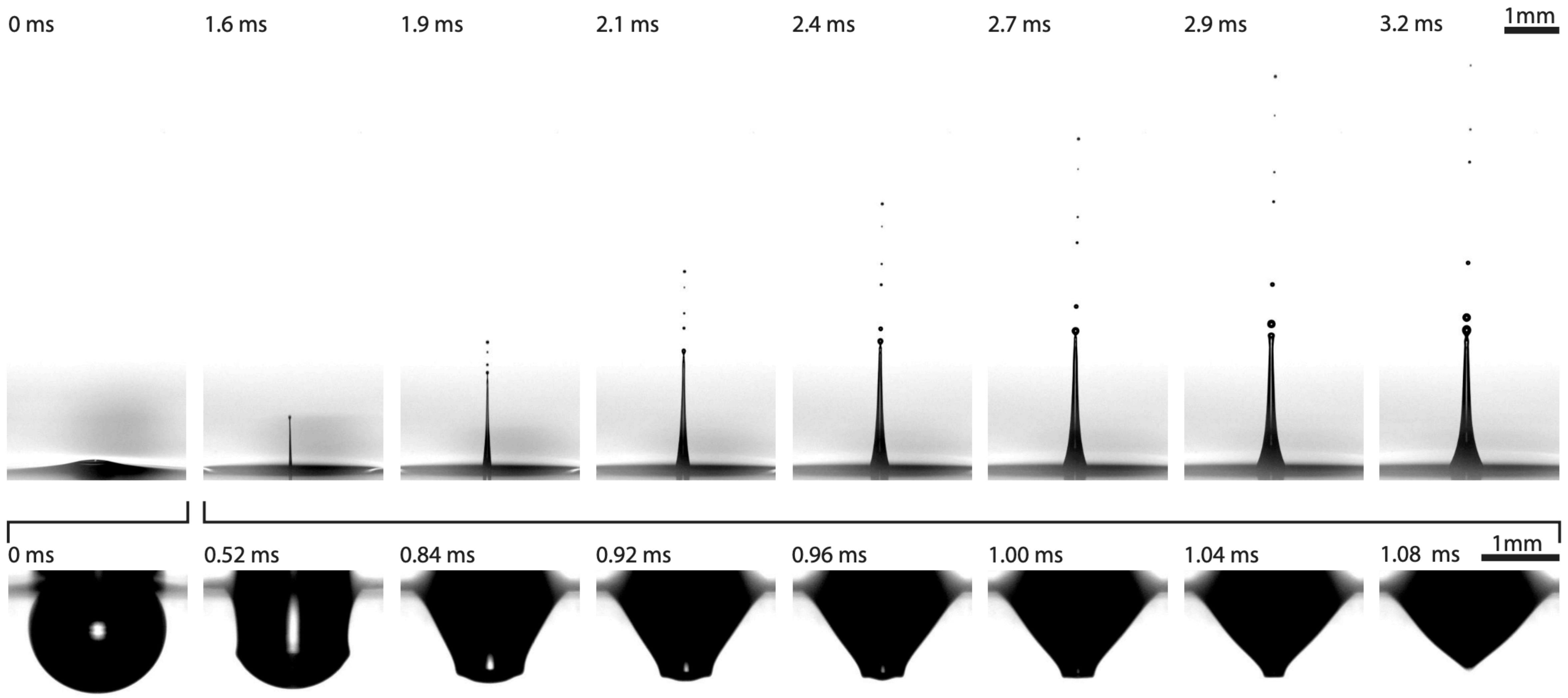
4. Aerosols + Atomization:
heat and mass transfers between ocean/atmosphere

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Ghabache (2015)



μ = 6 mPa.s, R = 883μm, Bo = 0.13
2500x

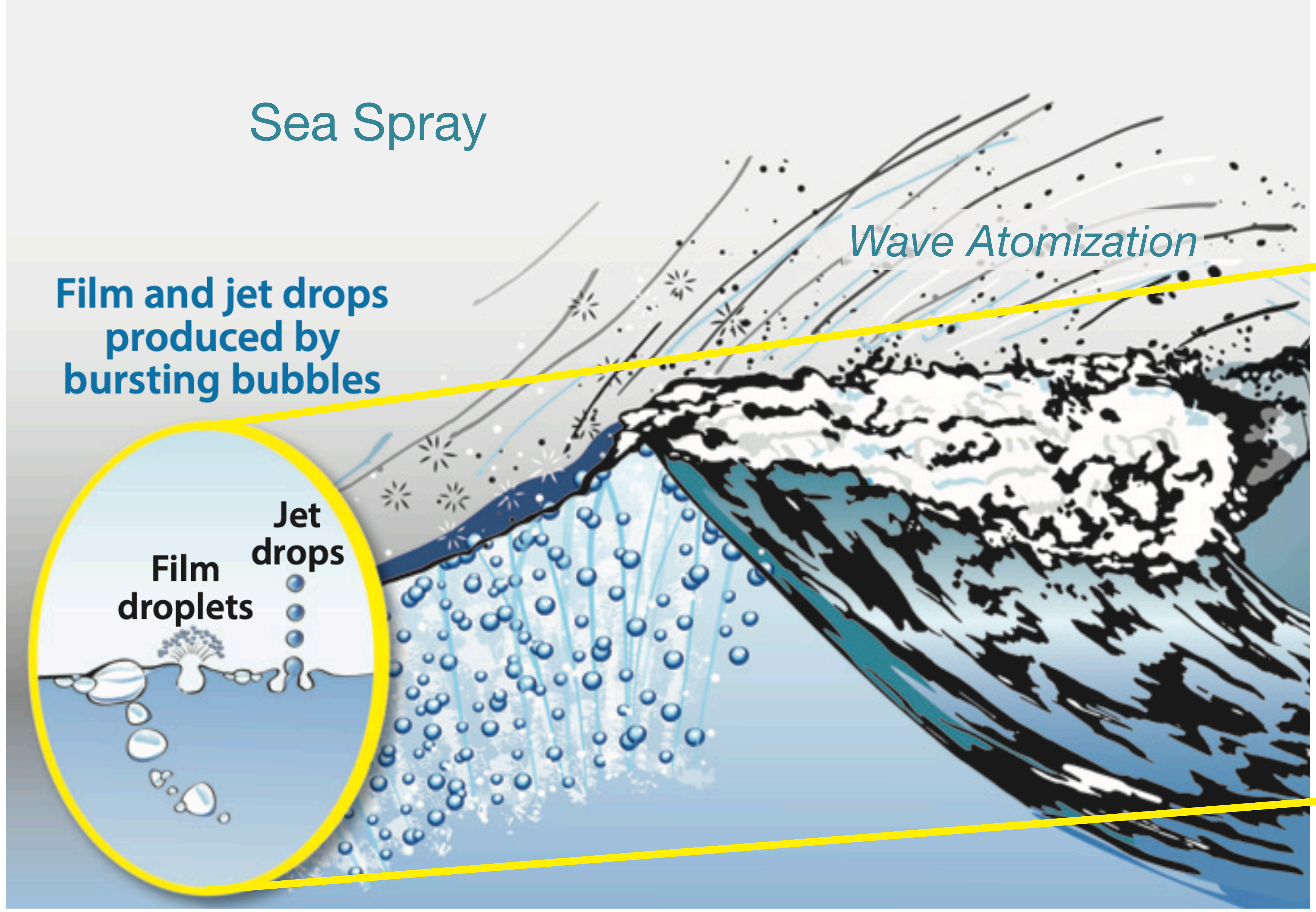
1. Wave breaking: entrapped air pockets → large amount of bubbles rising

2. Conical Collapse

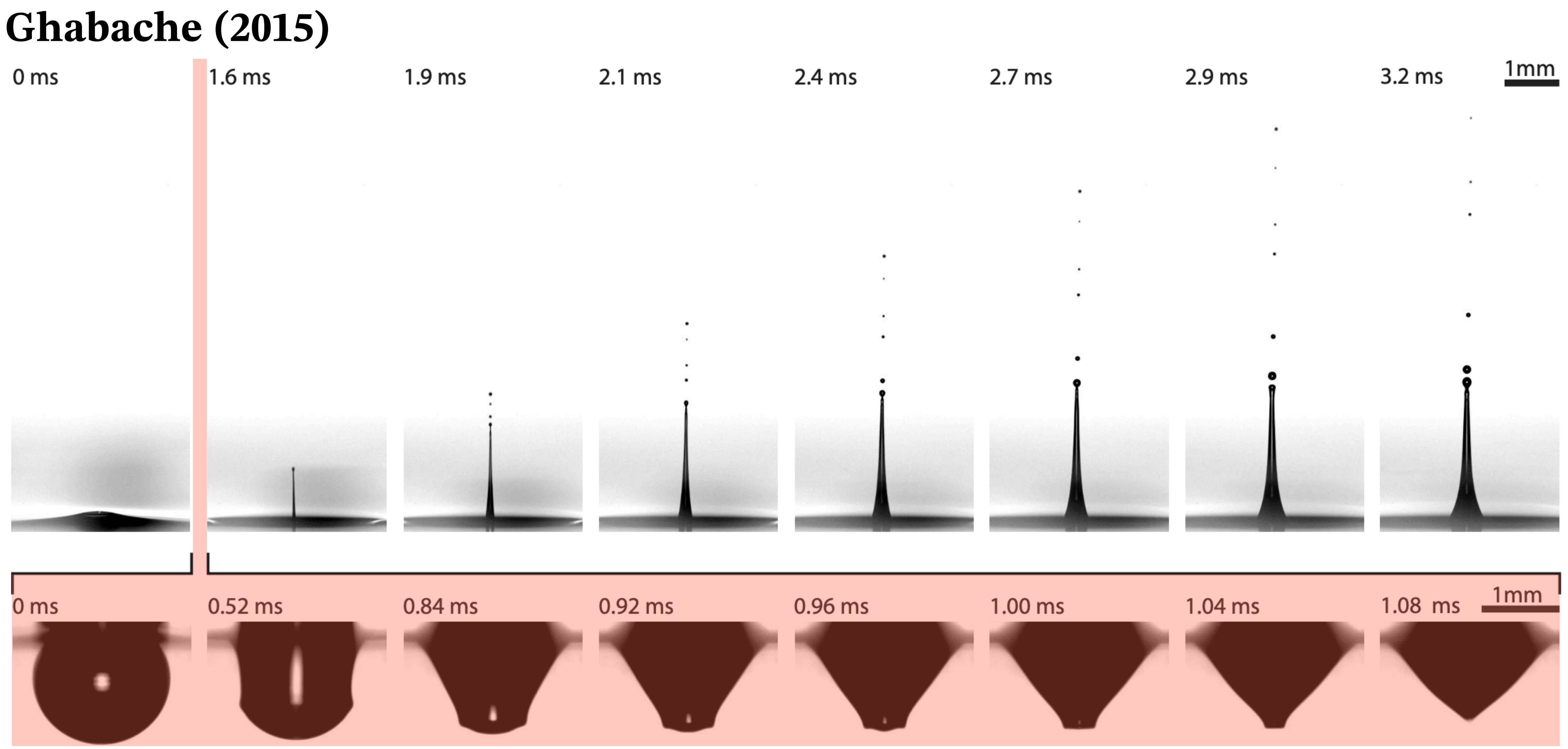
3. Jets: droplets ⩽ 1 μm propelled

4. Aerosols + Atomization: *heat and mass transfers between ocean/atmosphere*

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*



μ = 6 mPa.s, R = 883μm, Bo = 0.13
2500x

1. Wave breaking: entrapped air pockets → large amount of bubbles rising

2. Conical Collapse

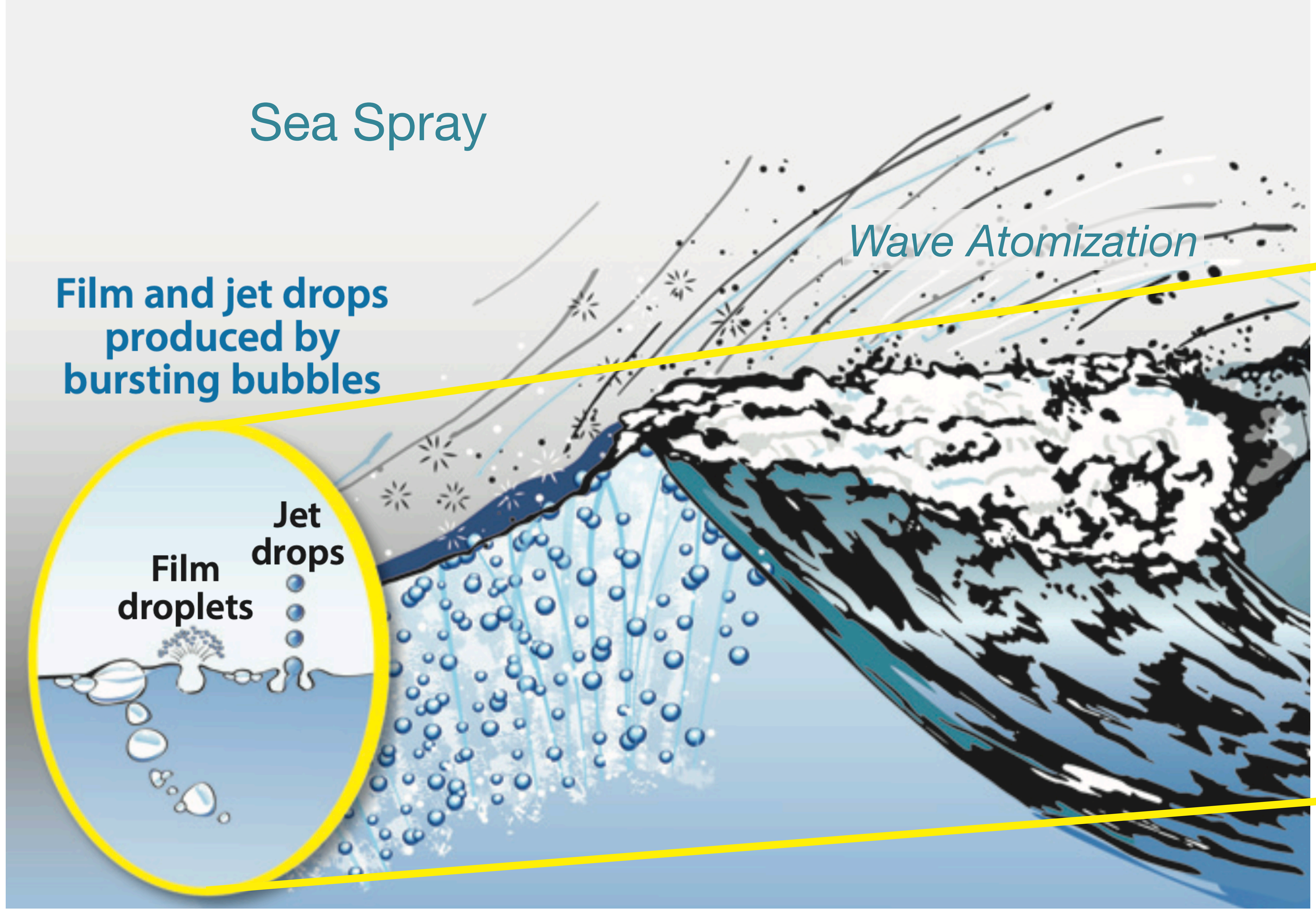
3. Jets: droplets ⩽ 1 μm propelled

4. Aerosols + Atomization:
heat and mass transfers between ocean/atmosphere

Introduction

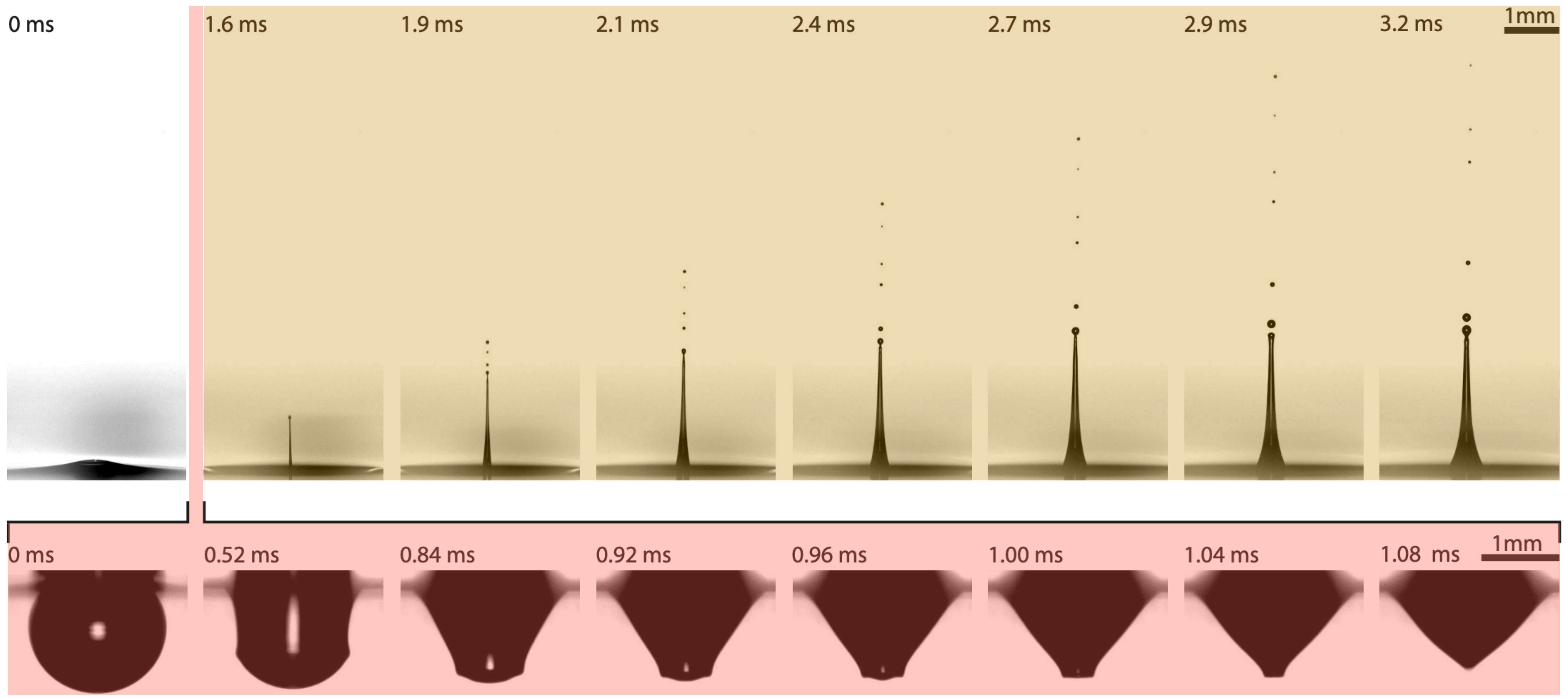
1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Ghabache (2015)



μ = 6 mPa.s, R = 883μm, Bo = 0.13
2500x

1. Wave breaking: entrapped air pockets → large amount of bubbles rising

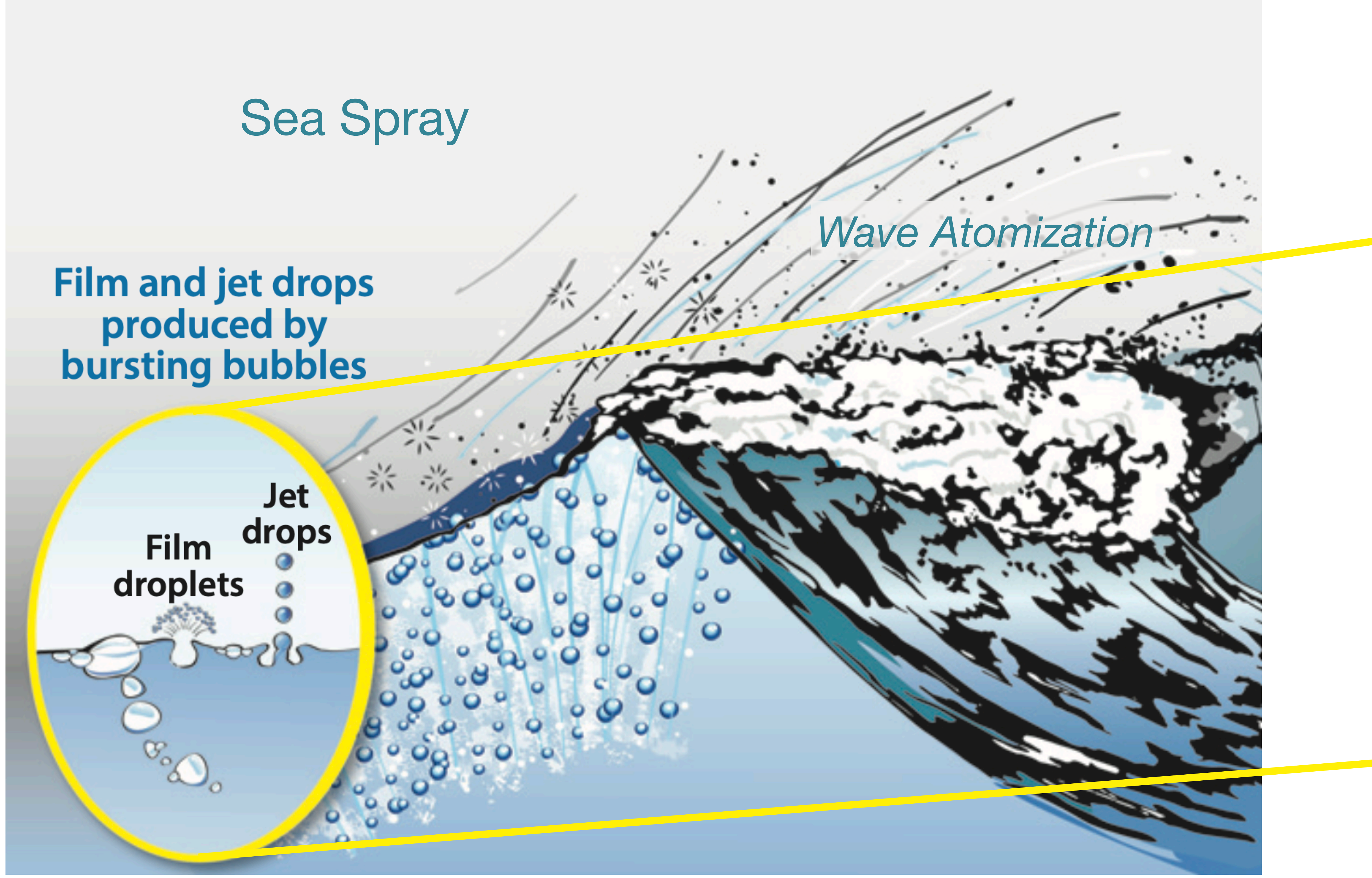
2. Conical Collapse

3. Jets: droplets ⩽ 1 μm propelled

4. Aerosols + Atomization:
heat and mass transfers between ocean/atmosphere

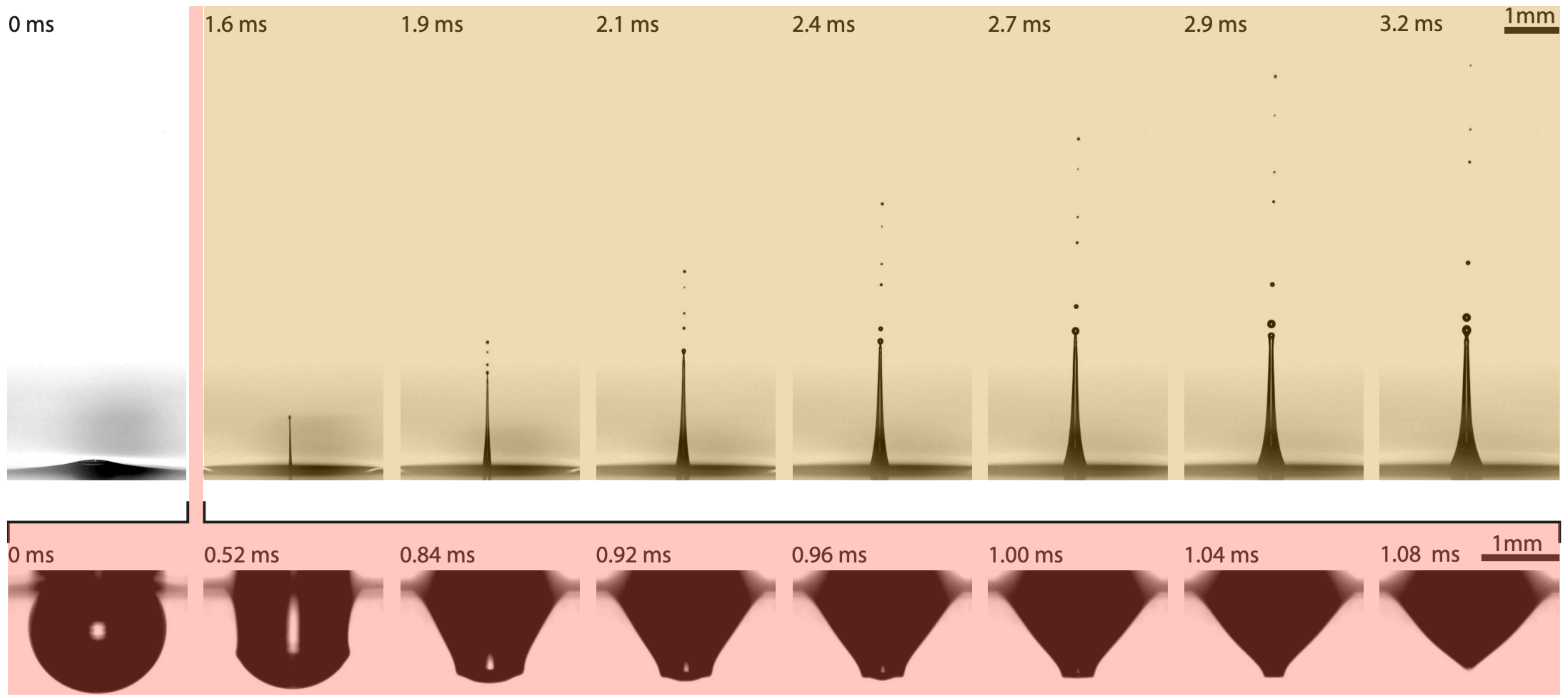
1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Ghabache (2015)



μ = 6 mPa.s, R = 883μm, Bo = 0.13
2500x

1. Wave breaking: entrapped air pockets
→ large amount of bubbles rising

2. Conical Collapse

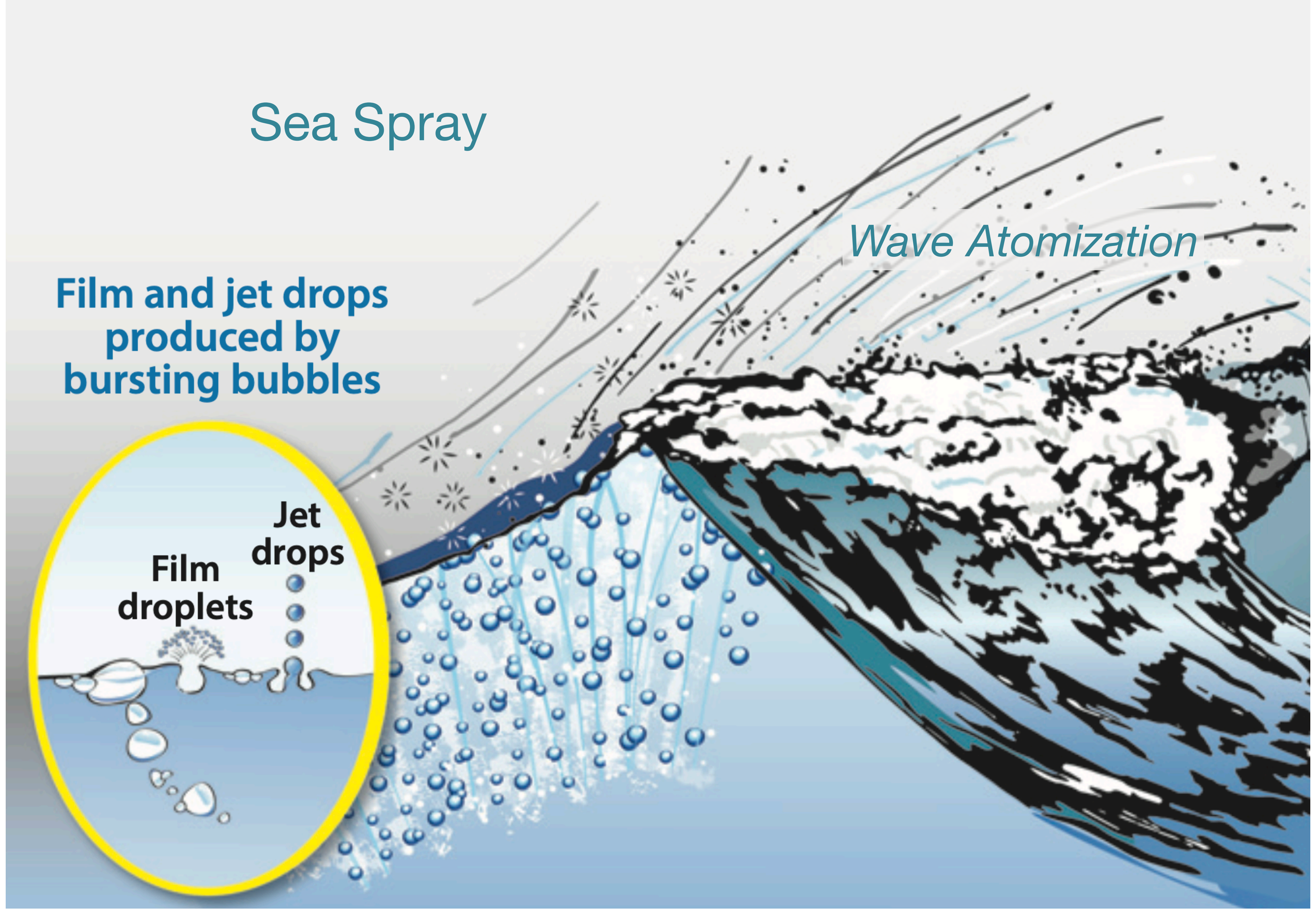
3. Jets: droplets ≲ 1 μm propelled

4. Aerosols + Atomization:
heat and mass transfers between ocean/atmosphere

***Theoretical* & *numerical* descriptions of the hydrodynamics?**

1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

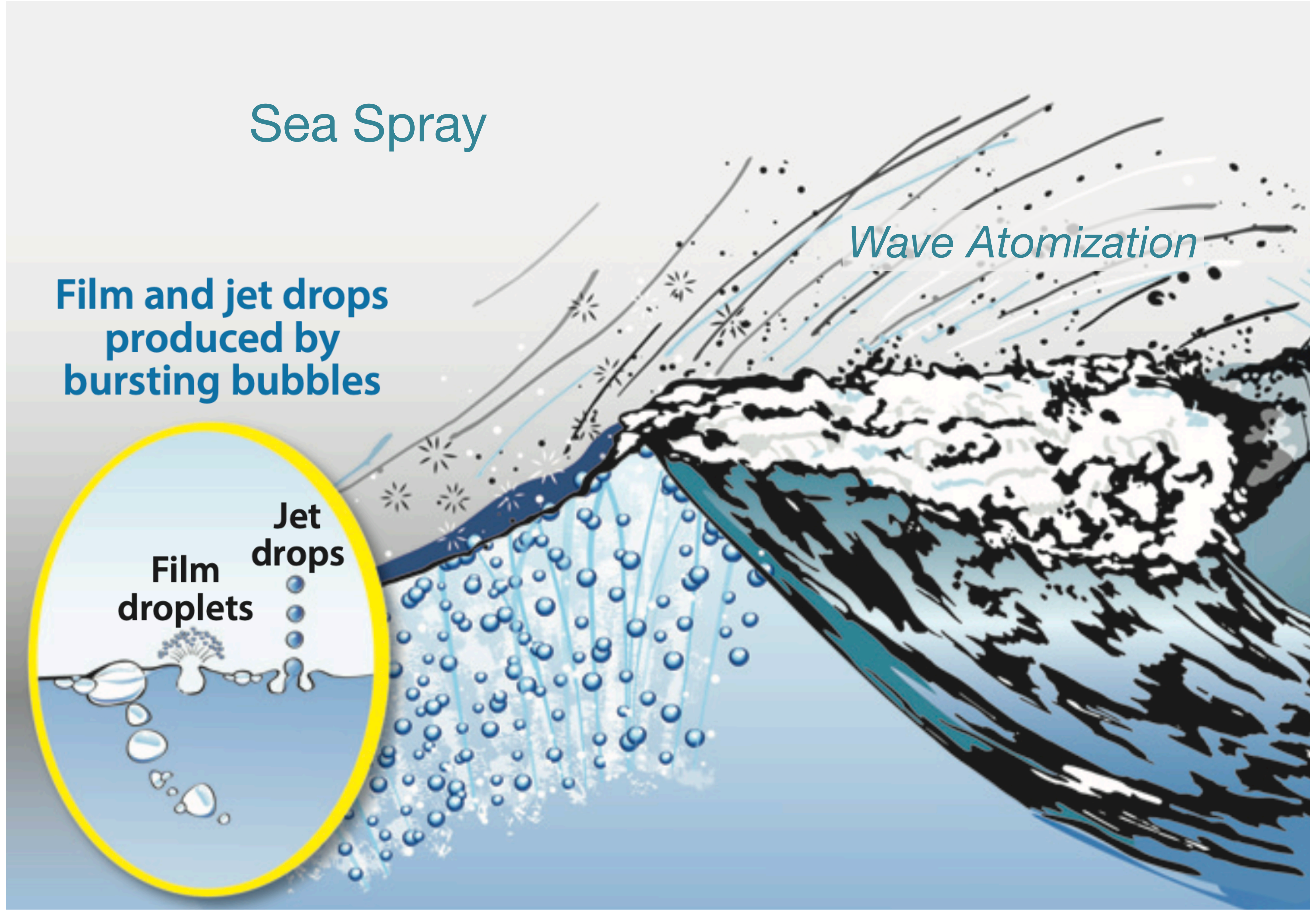
Conical Collapse

μ = 6 mPa.s,
R = 883μm,
Bo = 0.13,
2500x

Ghabache (2015)

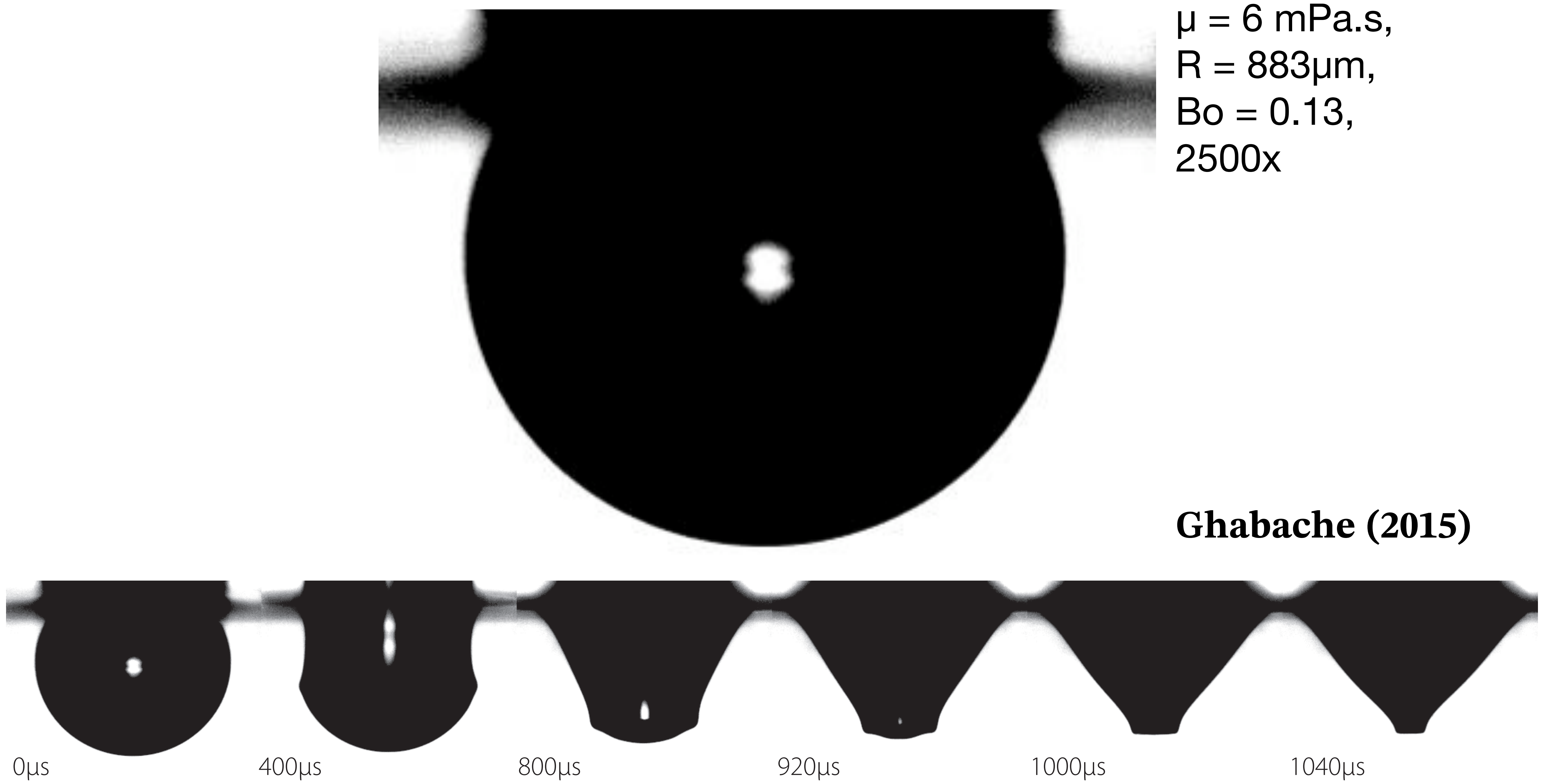
1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface



Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Conical Collapse

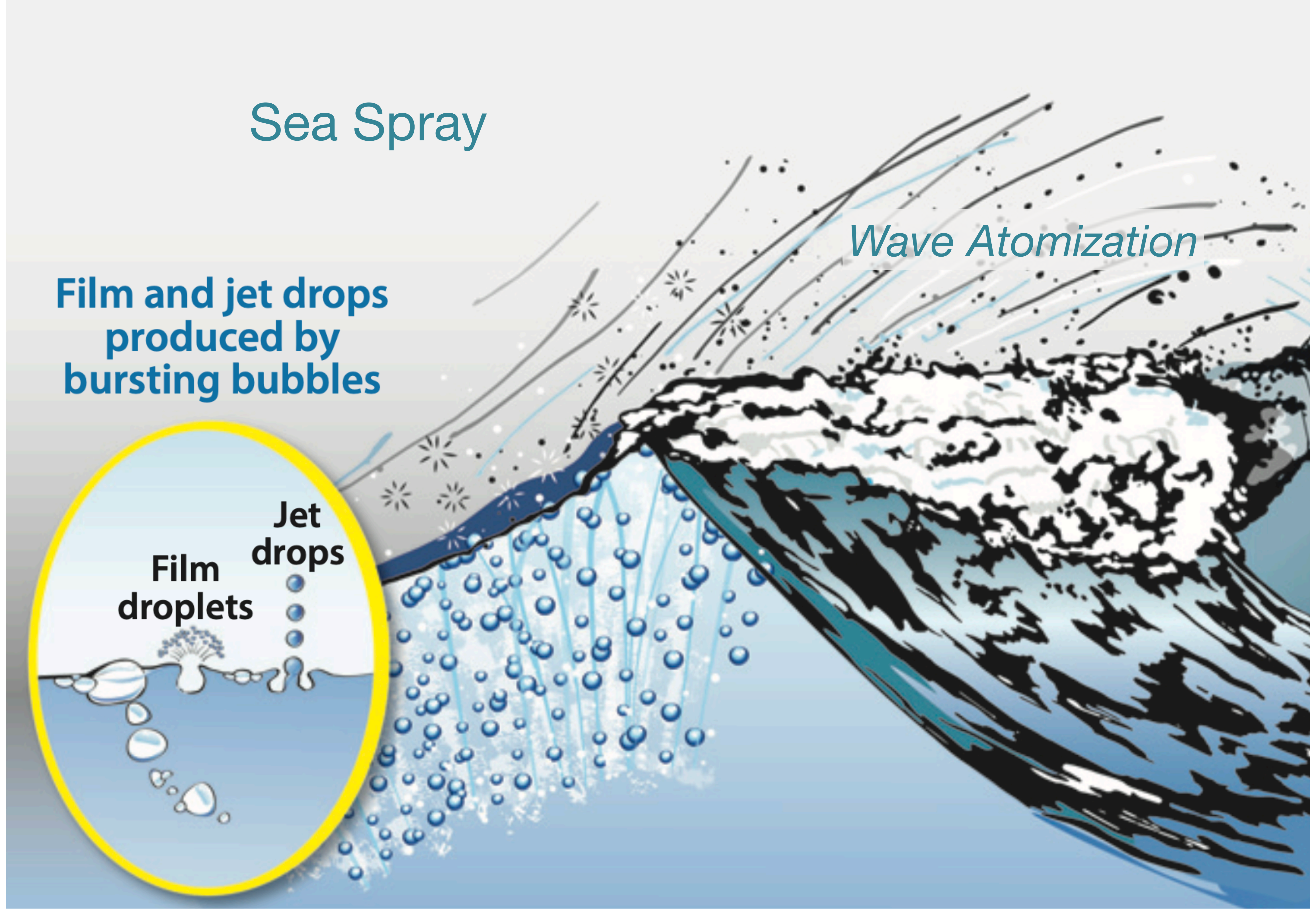


μ = 6 mPa.s,
R = 883μm,
Bo = 0.13,
2500x

Ghabache (2015)

1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface

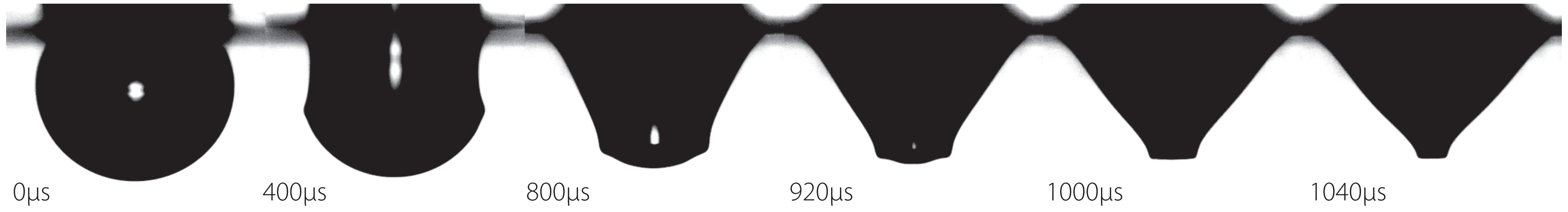


Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Conical Collapse

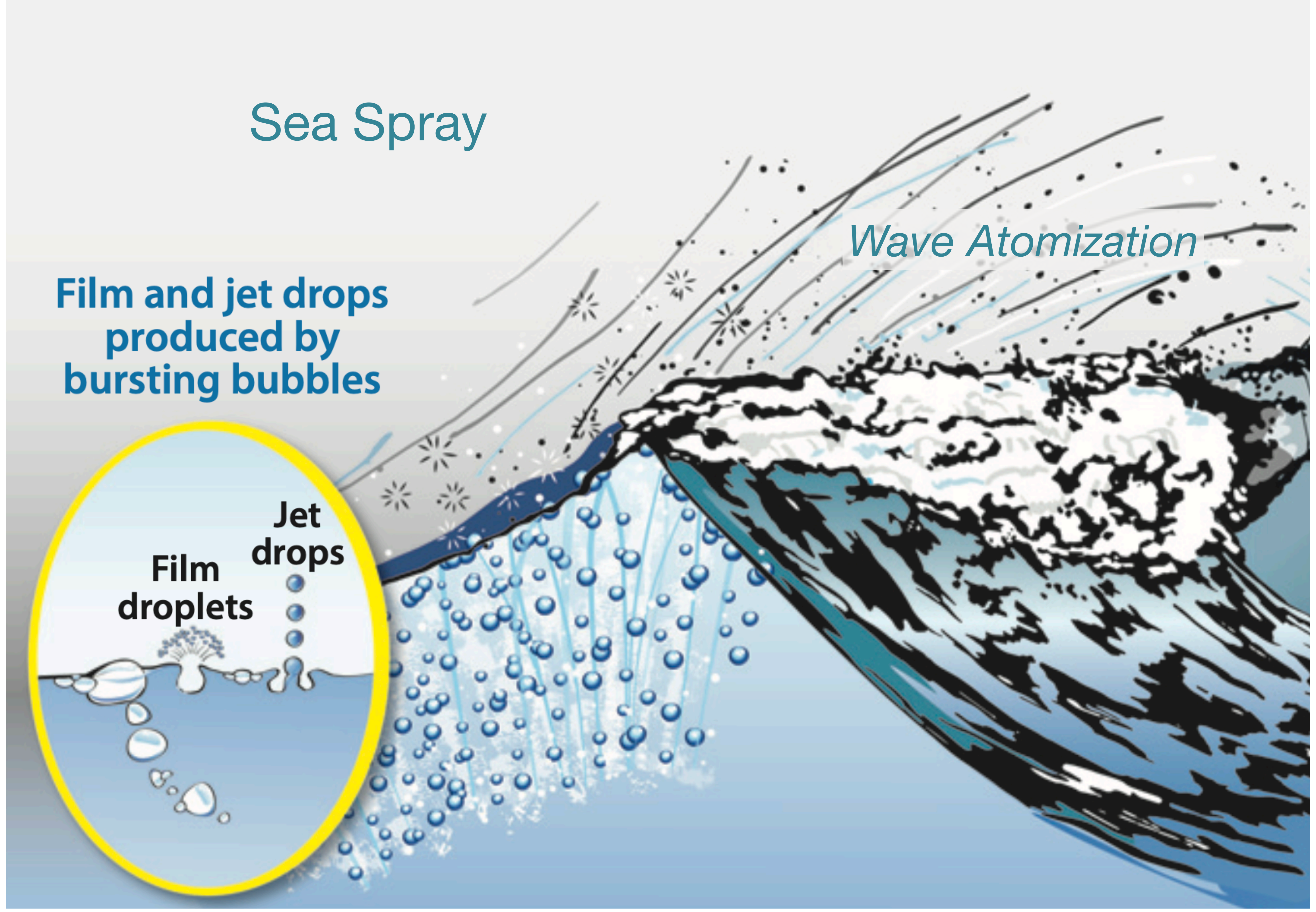
μ = 6 mPa.s,
R = 883μm,
Bo = 0.13,
2500x

Ghabache (2015)



1 - How jets of bubble bursting are emitted?

Mass transfers at the ocean-atmosphere interface

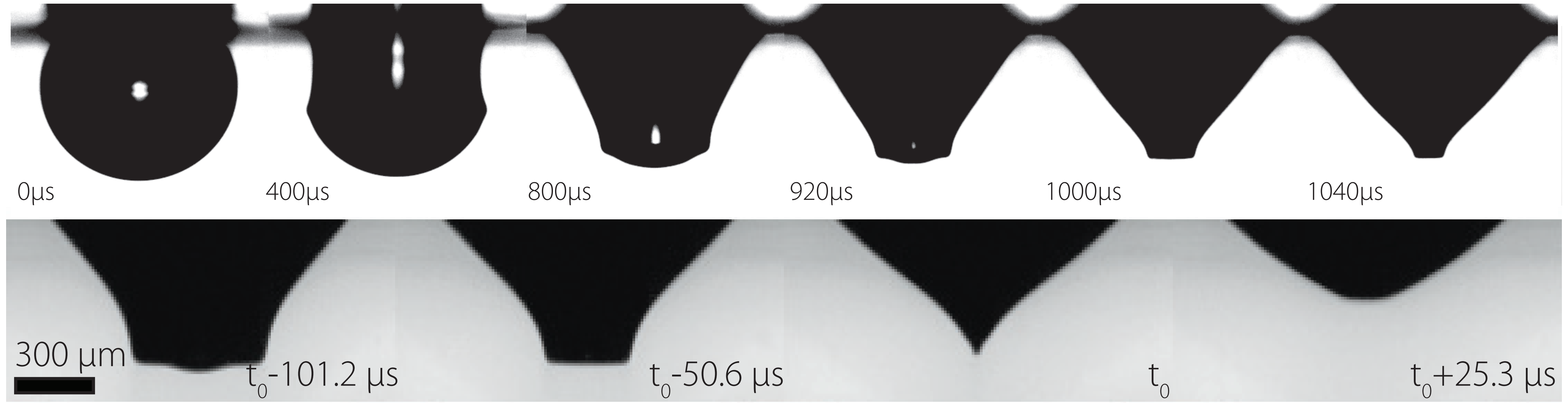


Veron, F. (2015). *Ocean Spray.* Annu. Rev. Fluid Mech. **642** *147-157.*

Conical Collapse

μ = 6 mPa.s,
R = 883μm,
Bo = 0.13,
2500x

Ghabache (2015)

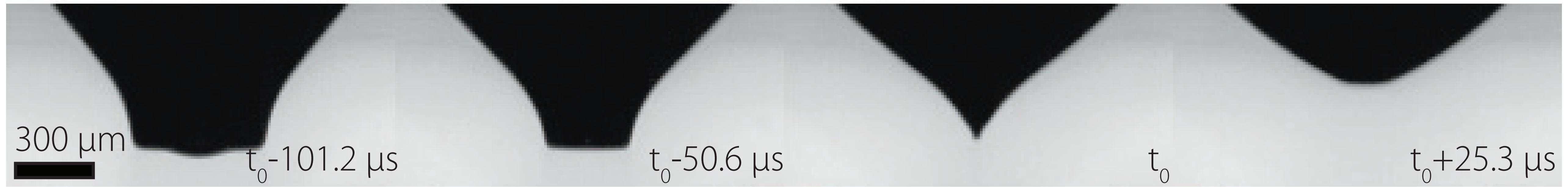


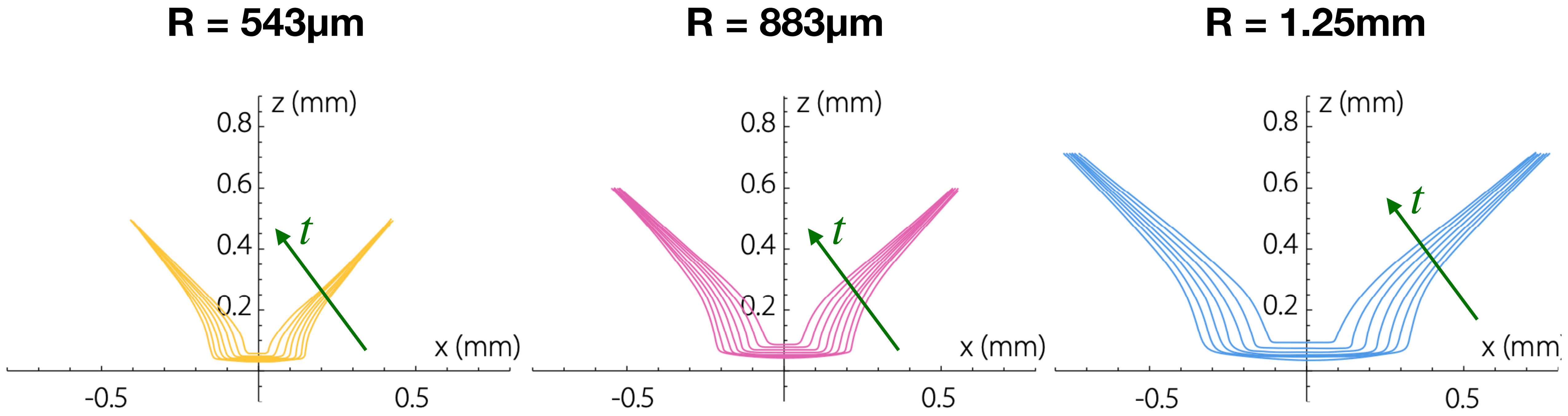
Finite-time singularity

$t_0 \rightarrow$ time of singularity

Introduction

Ghabache (2015)



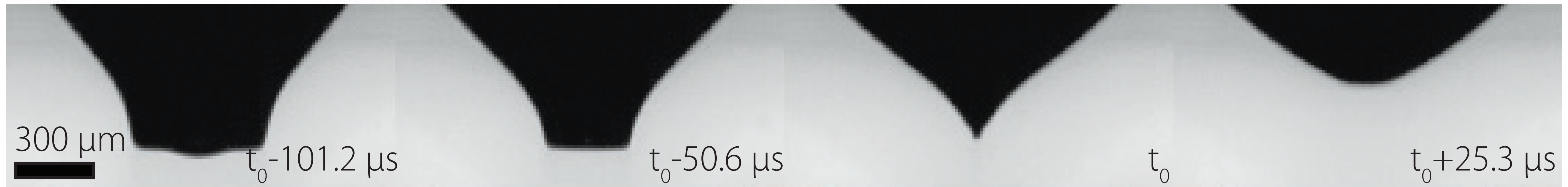


µ = 6 mPa.s

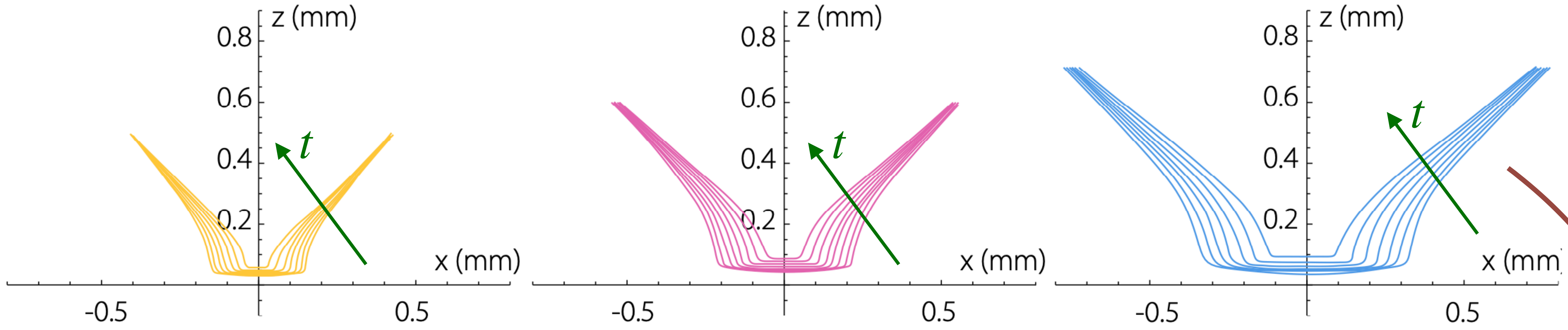
Introduction

1 - How jets of bubble bursting are emitted?

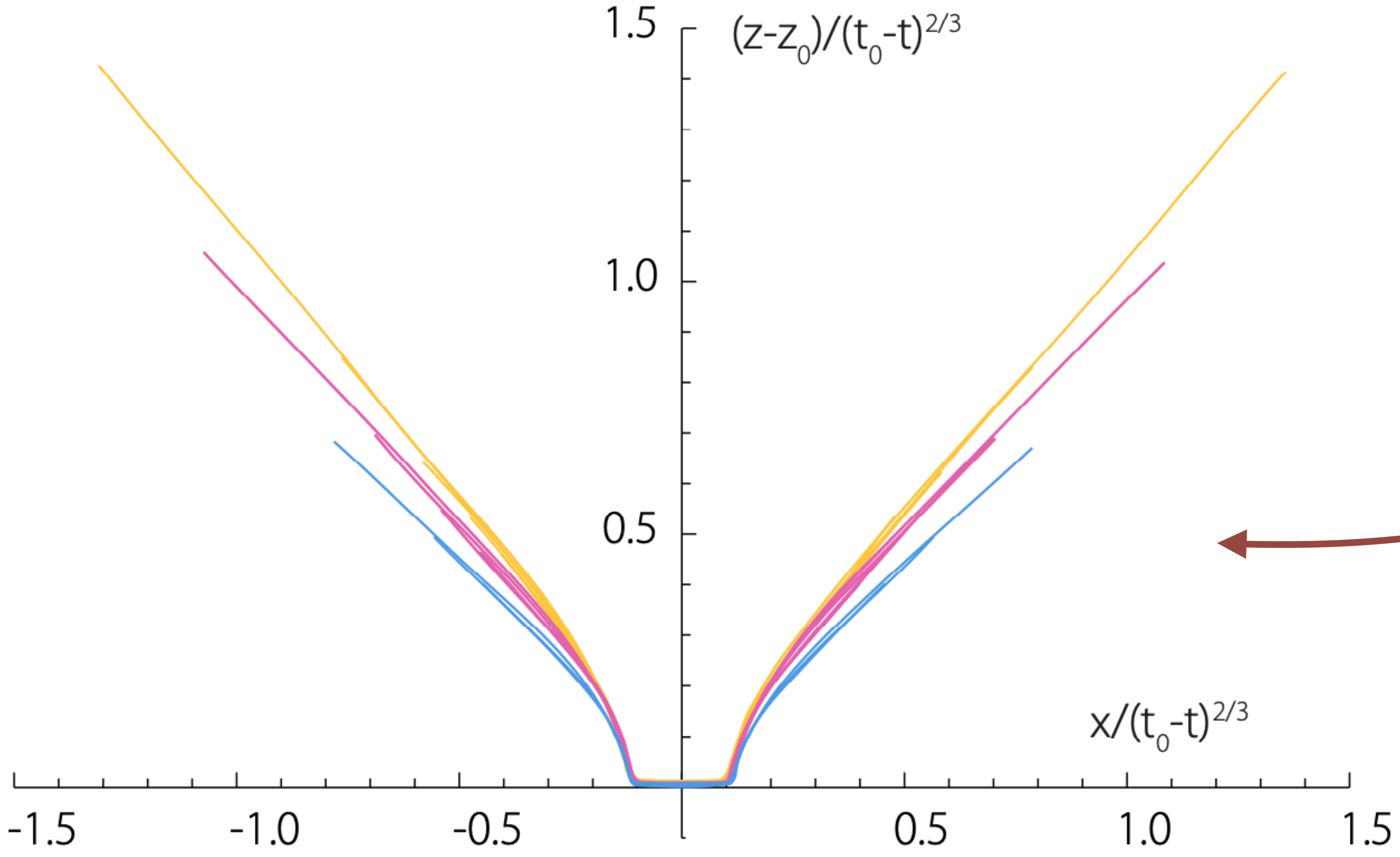
Ghabache (2015)



R = 543µm **R = 883µm** **R = 1.25mm**



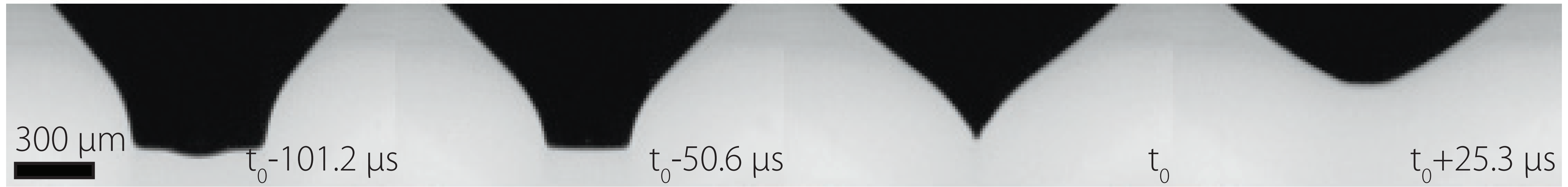
µ = 6 mPa.s



Rescaling by $\boldsymbol{\xi} = \mathbf{x}\,(t_0 - t)^{-2/3}$ *shows that these time-dependent profiles correspond to* ***a single one****: the problem becomes* ***steady!***

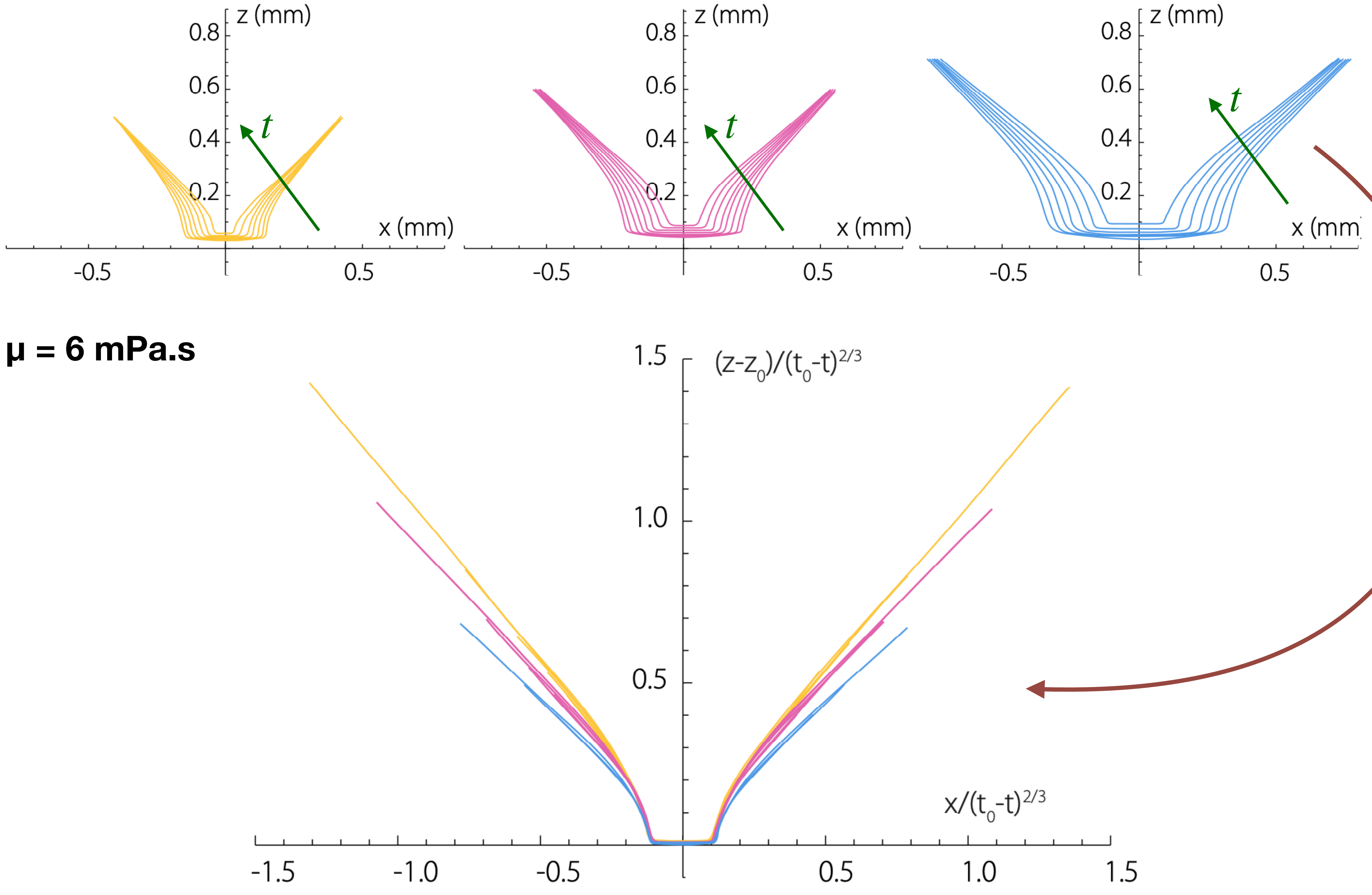
1 - How jets of bubble bursting are emitted?

Ghabache (2015)



R = 543μm **R = 883μm** **R = 1.25mm**

μ = 6 mPa.s



Rescaling by $\boldsymbol{\xi} = \mathbf{x}\,(t_0 - t)^{-2/3}$ *shows that these time-dependent profiles correspond to* ***a single one****: the problem becomes* ***steady!***

Self-similar phenomenon:
solution $y(\mathbf{x}, t)$ is in the form of the **power law**:

$$y(\mathbf{x}, t) = t^{\alpha}\, \tilde{y}\left(\frac{\mathbf{x}}{t^{\beta}}\right) = t^{\alpha}\, \tilde{y}\left(\boldsymbol{\xi}\right)$$

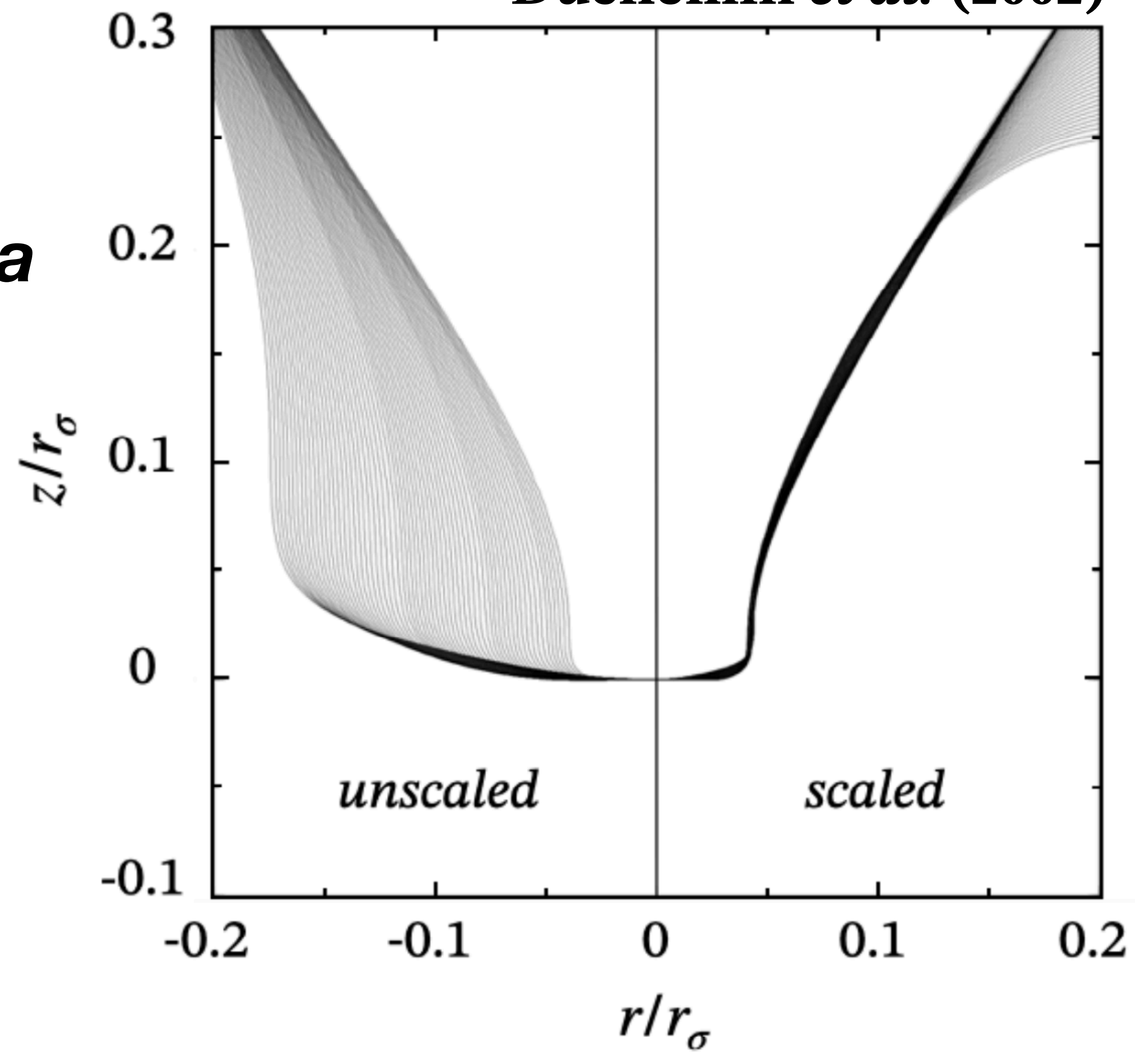
Self-similar evolution

Balance between ***inertia*** *and* ***surface tension****.*

Lengths divided by $(t_0 - t)^{2/3}$

Keller & Miksis (1983)
Zeff *et al.* (2000)

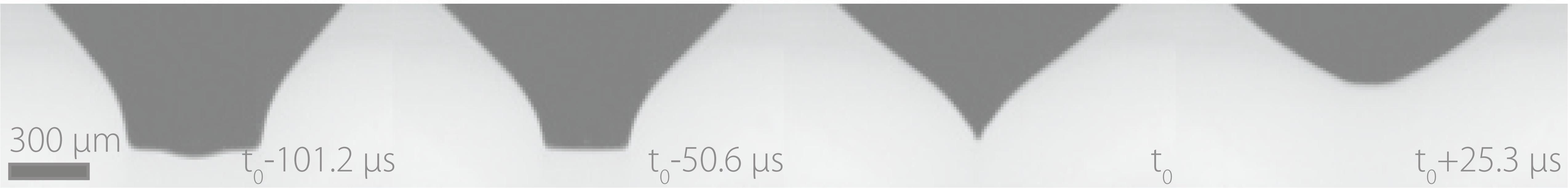
Duchemin *et al.* (2002)



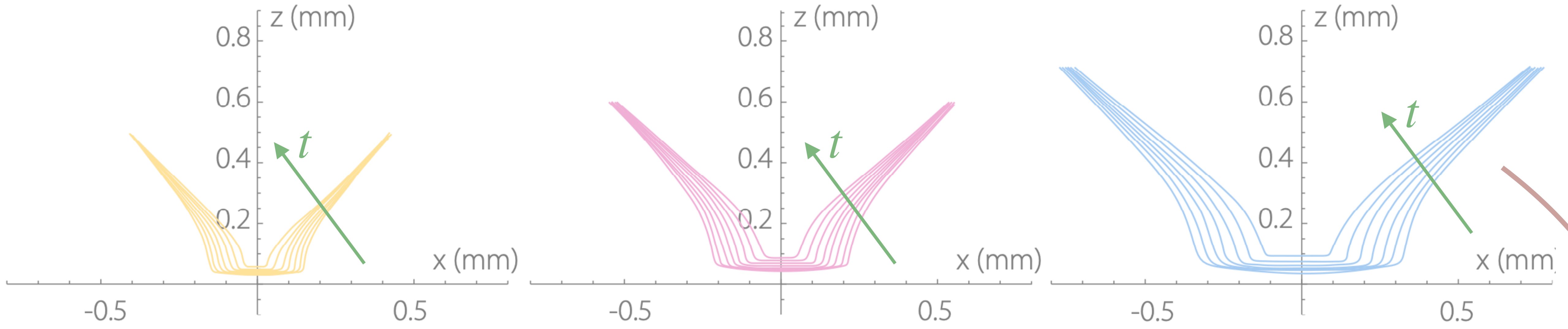
Introduction

1 - How jets of bubble bursting are emitted?

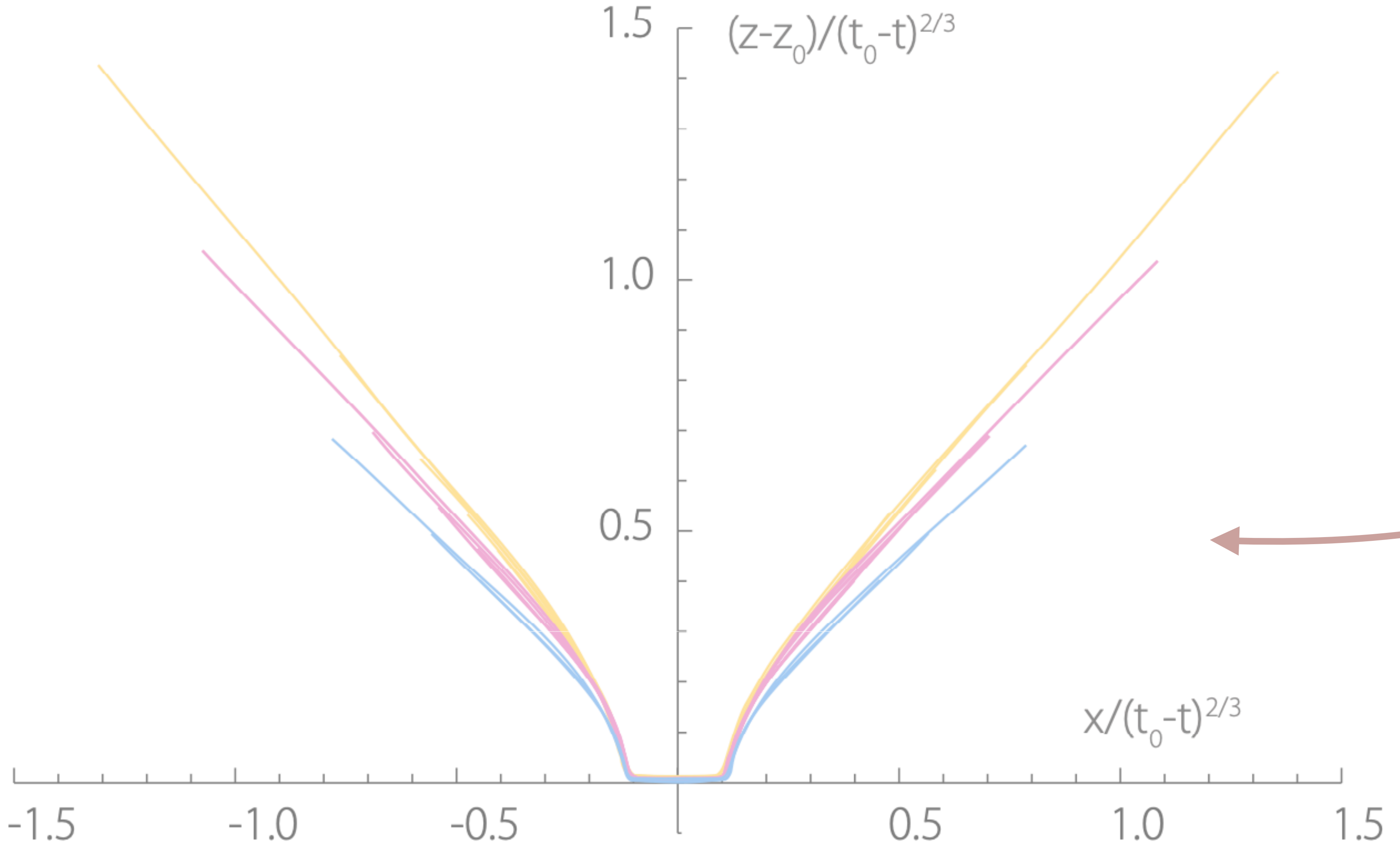
Ghabache (2015)



R = 543µm **R = 883µm** **R = 1.25mm**



µ = 6 mPa.s



Rescaling by $\boldsymbol{\xi} = \mathbf{x}\,(t_0 - t)^{-2/3}$ *shows that these time-dependent profiles correspond to* ***a single one****: the problem becomes* ***steady!***

Self-similar phenomenon:
solution $y(\mathbf{x}, t)$ is in the form of the **power law**:

$$y(\mathbf{x}, t) = t^{\alpha}\, \tilde{y}\left(\frac{\mathbf{x}}{t^{\beta}}\right) = t^{\alpha}\, \tilde{y}\left(\boldsymbol{\xi}\right)$$

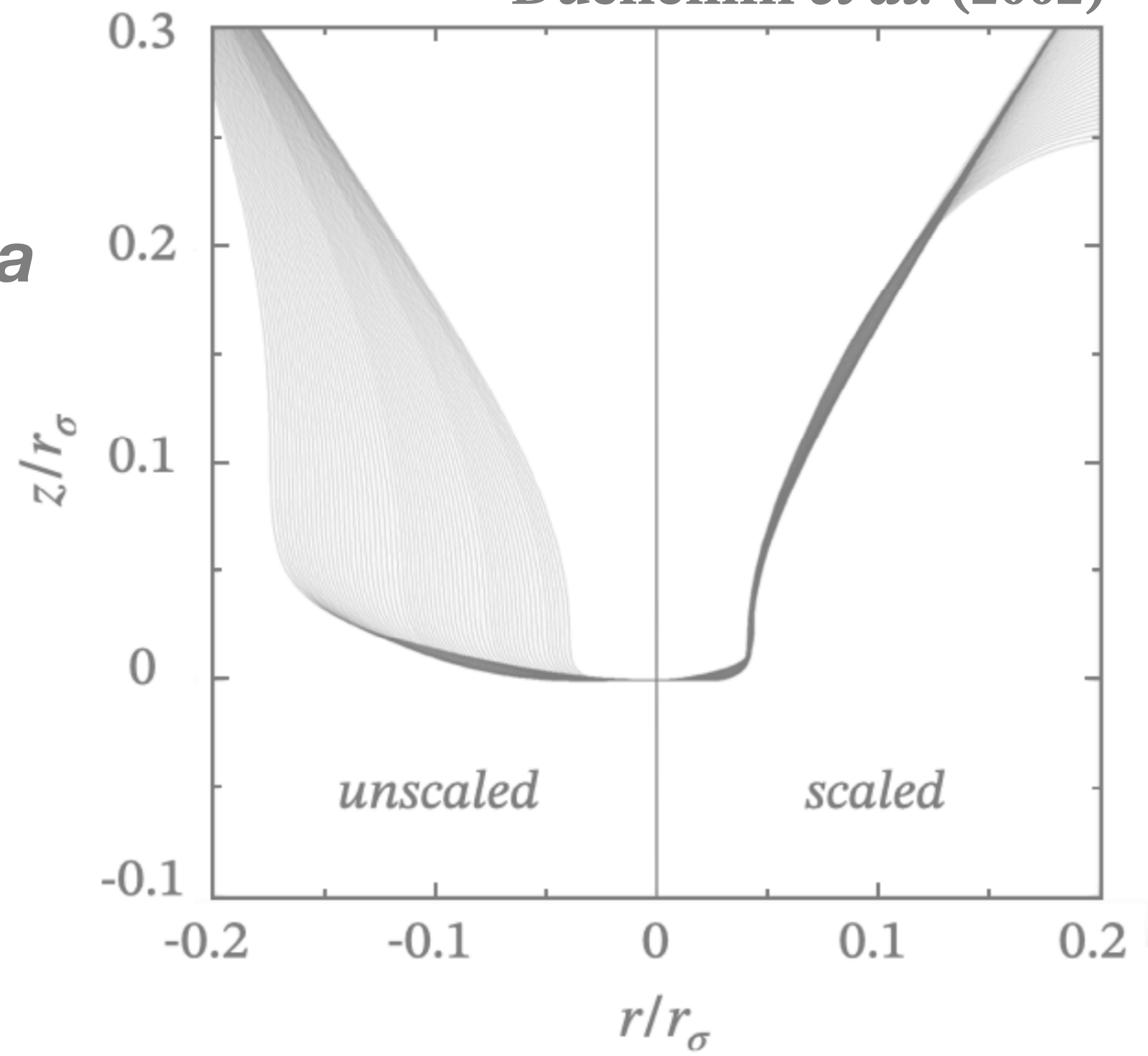
Self-similar evolution

Balance between ***inertia*** *and* ***surface tension****.*

Lengths divided by $(t_0 - t)^{2/3}$

Keller & Miksis (1983)
Zeff *et al.* (2000)

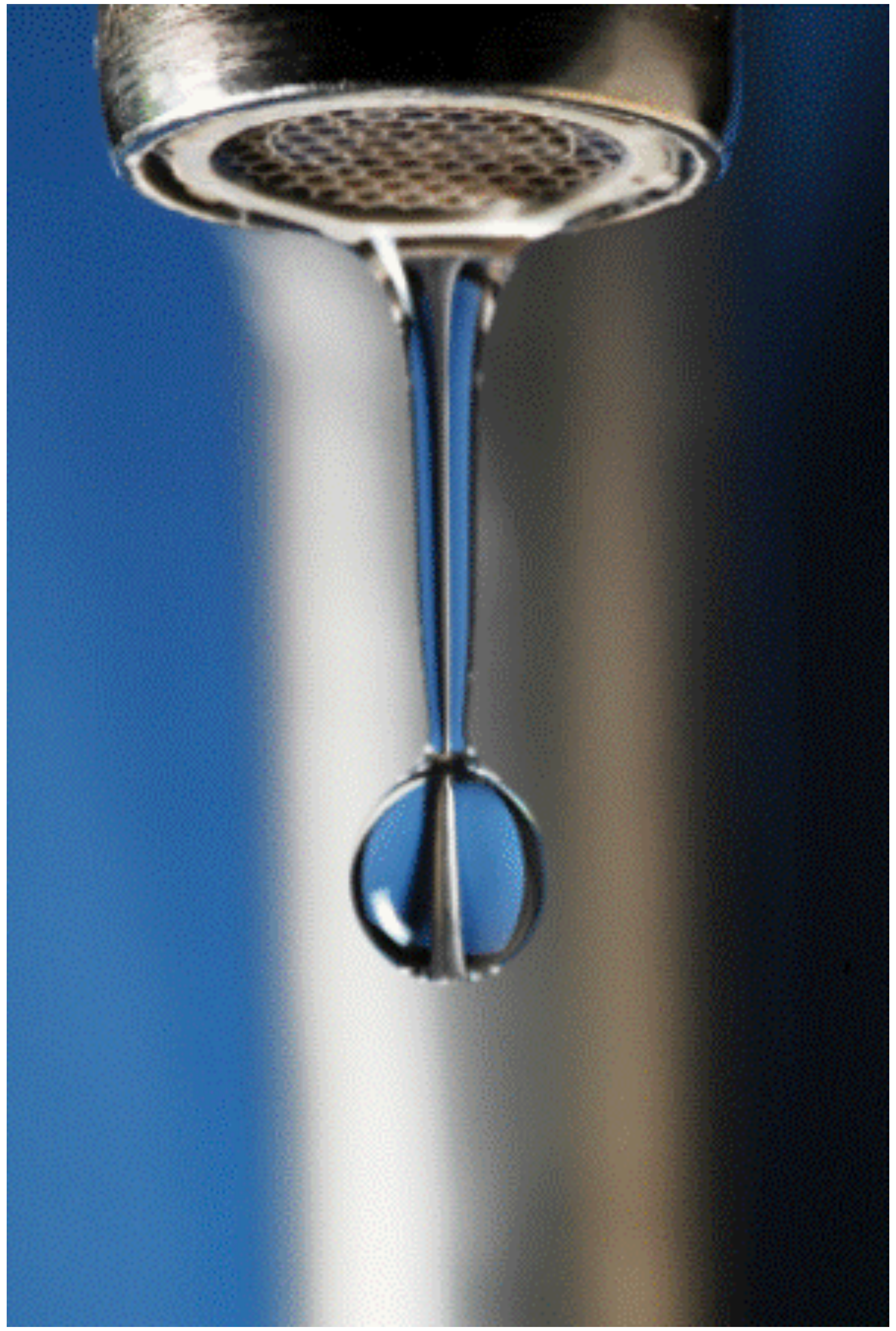
Duchemin *et al.* (2002)



Regularization Mechanism: *Viscosity?*

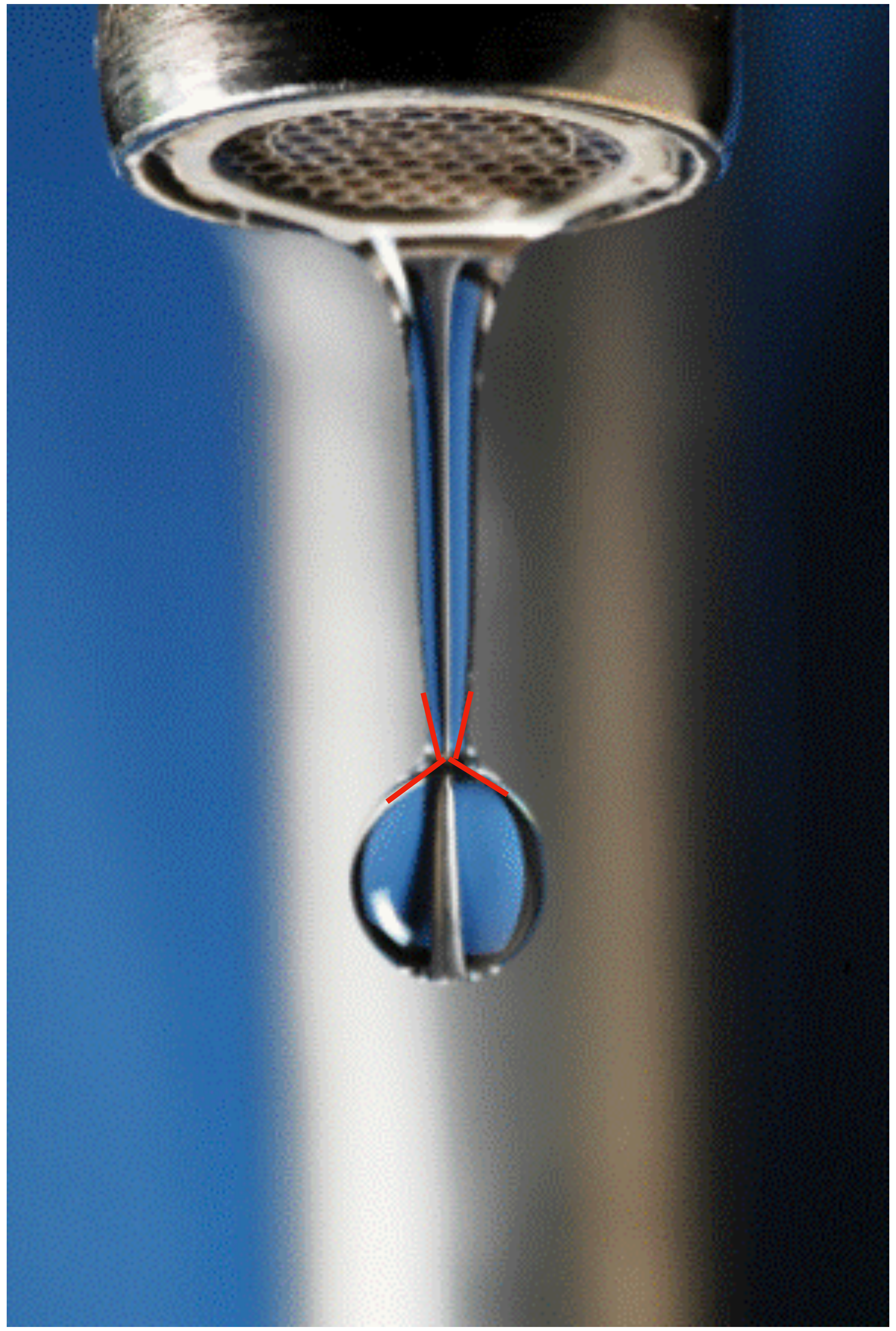
II - Keller & Miksis (1983) problem

II.1 - A 2D inviscid modelization...



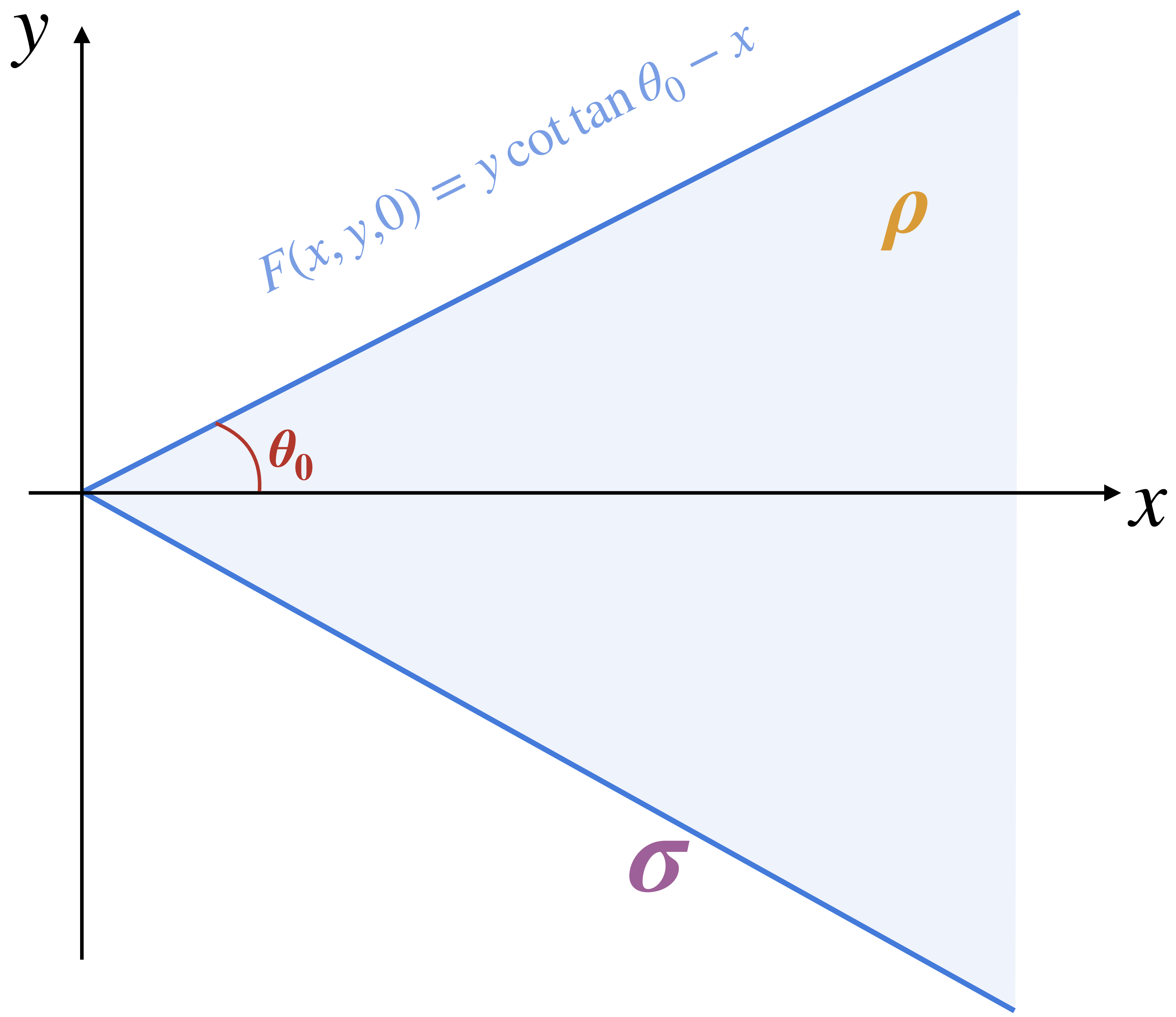
Keller and Miksis (1983). *Surface Tension Driven Flows.* SIAM **43**

II.1 - A 2D inviscid modelization...



Keller and Miksis (1983). *Surface Tension Driven Flows.* SIAM 43

II.1 - A 2D inviscid modelization...



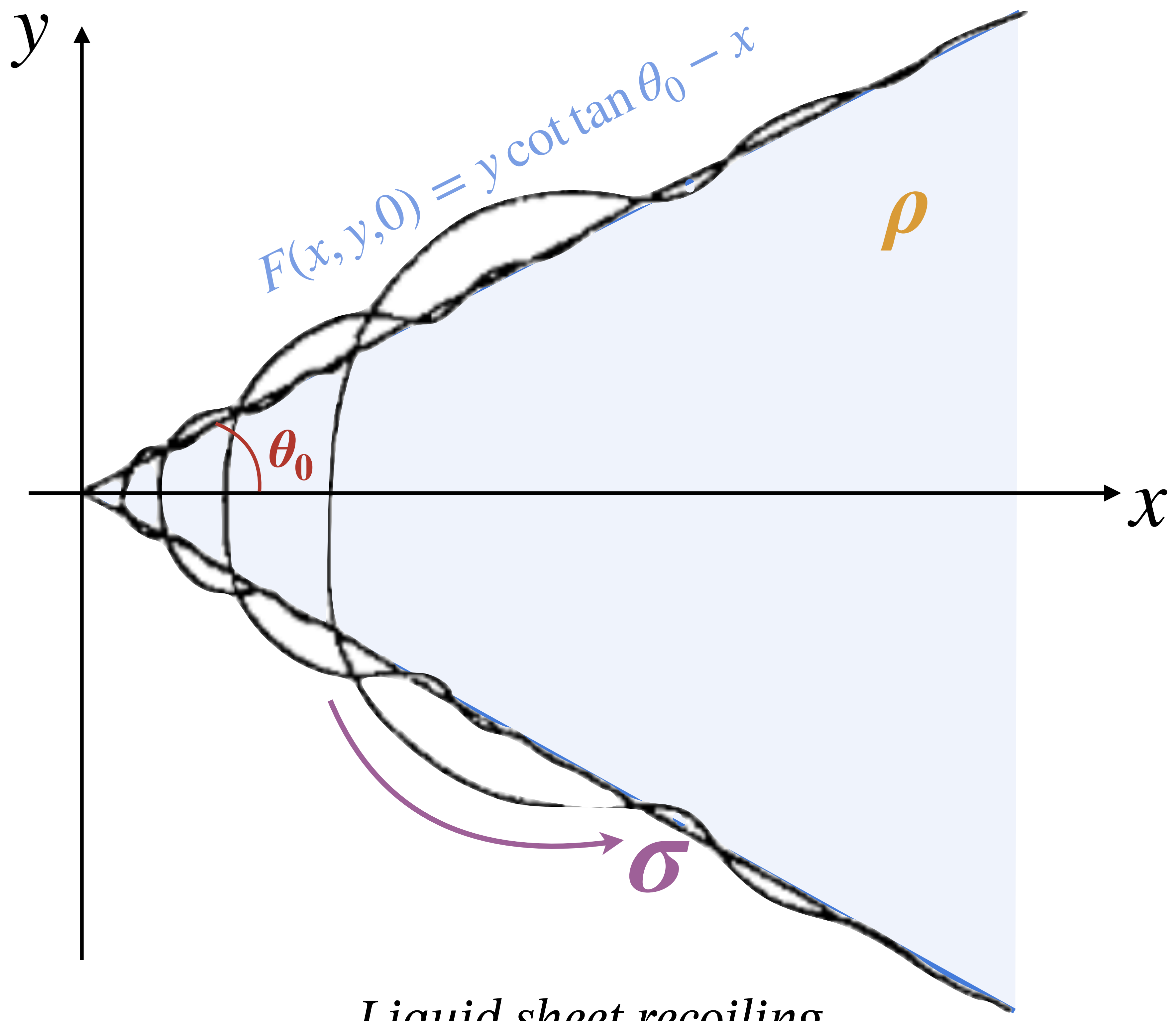
<u>Hypotheses on the flow:</u>
inviscid, irrotational, isochoric

Keller and Miksis (1983). *Surface Tension Driven Flows.* <u>SIAM</u> **43**

II - Keller & Miksis (1983) problem

II.1 - A 2D inviscid modelization...

Recoil near the vertex of the wedge under surface tension effects.

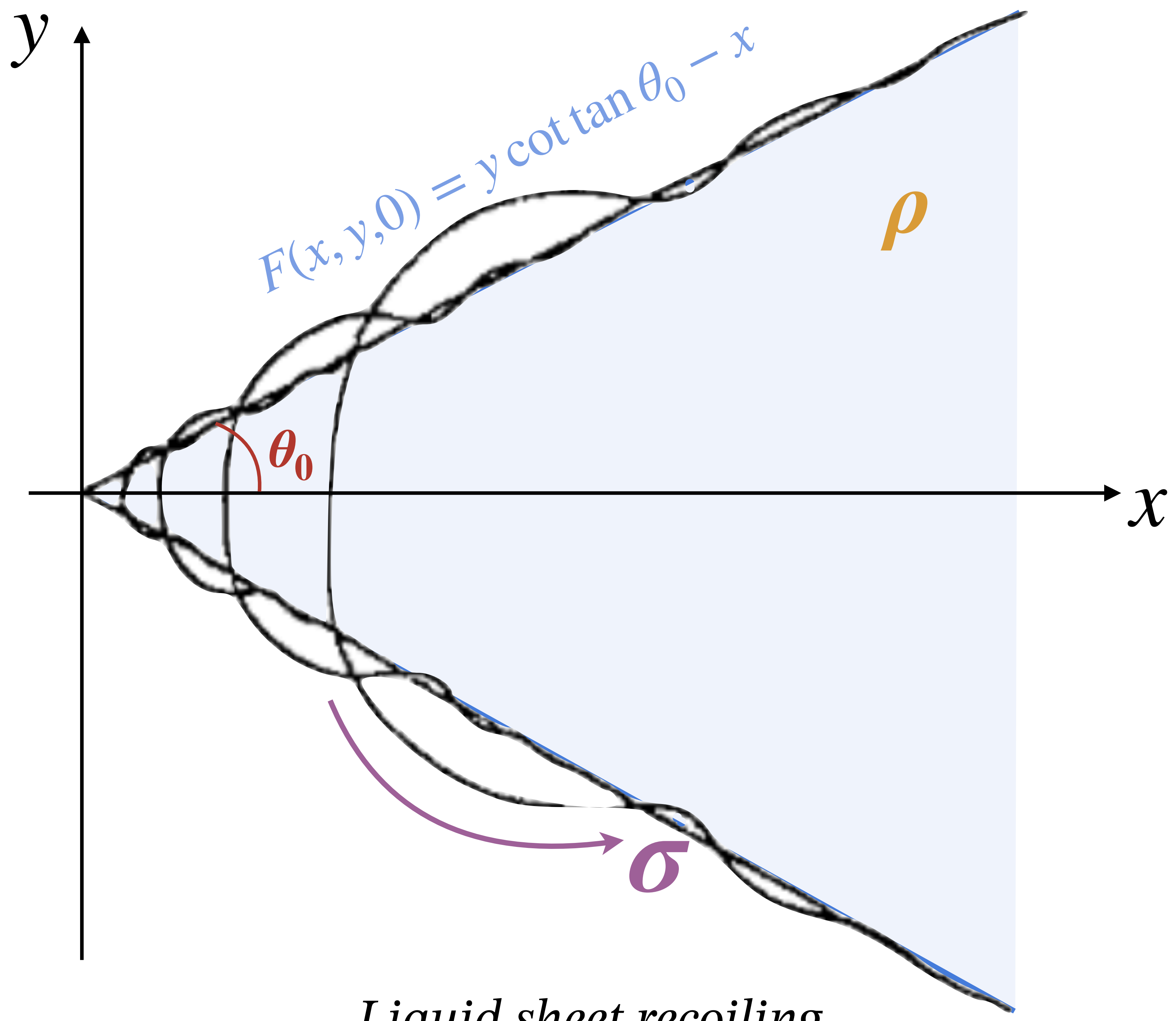


Liquid sheet recoiling
[Sketch from **Peregrine et al. (1990)**]

Hypotheses on the flow:
inviscid, irrotational, isochoric

Keller and Miksis (1983). *Surface Tension Driven Flows.* SIAM **43**

Recoil near the vertex of the wedge under surface tension effects.



Liquid sheet recoiling
[Sketch from **Peregrine et al. (1990)**]

Hypotheses on the flow:
inviscid, irrotational, isochoric

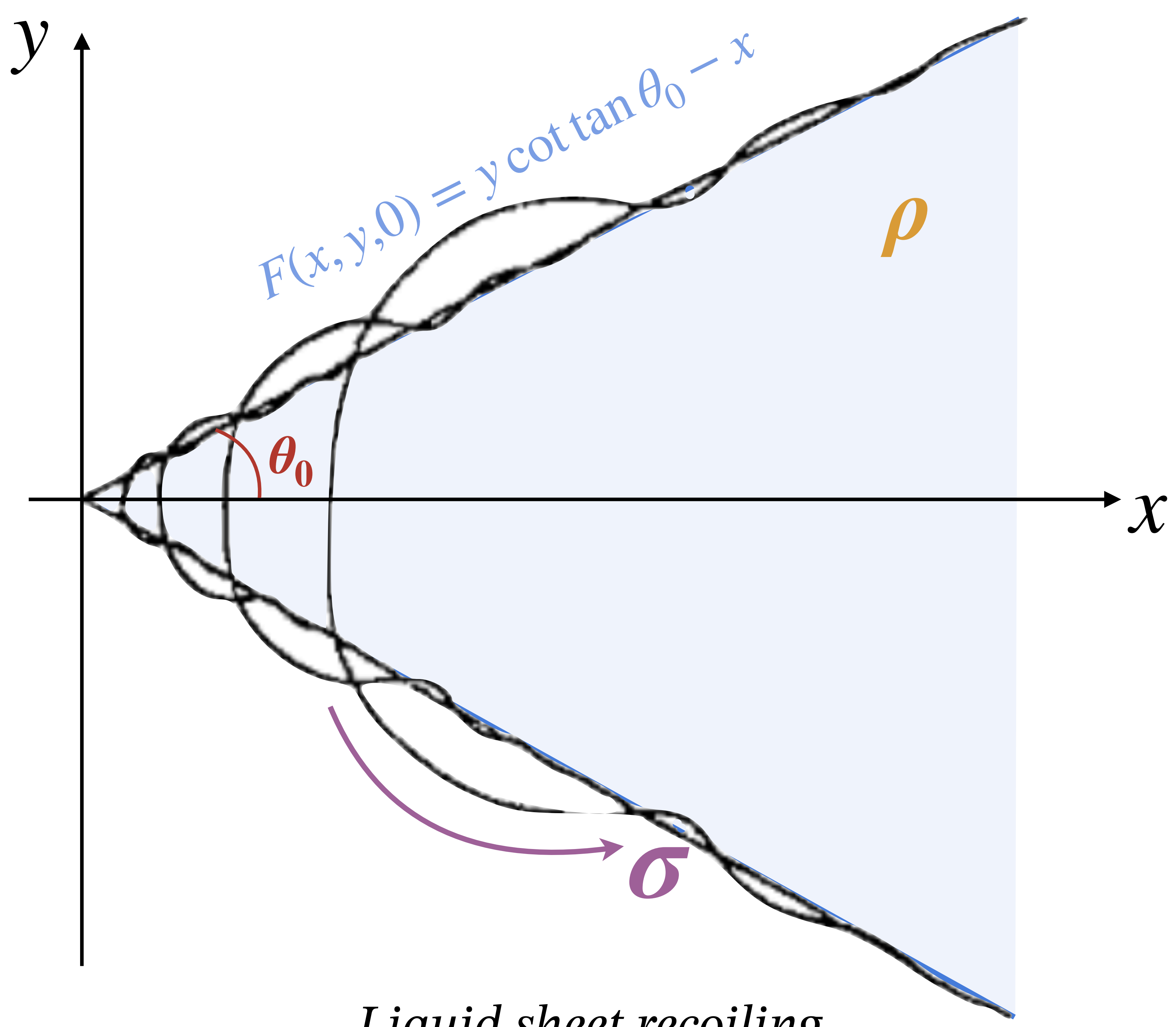
From *dimensional analysis*:

$$\xi = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} x \quad , \quad \eta = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} y$$

are the ***self-similar variables.***

Keller and Miksis (1983). *Surface Tension Driven Flows.* SIAM **43**

Recoil near the vertex of the wedge under <u>surface tension</u> effects.



Liquid sheet recoiling
[Sketch from **Peregrine et al. (1990)**]

<u>Hypotheses on the flow:</u>
inviscid, irrotational, isochoric

From *dimensional analysis*:

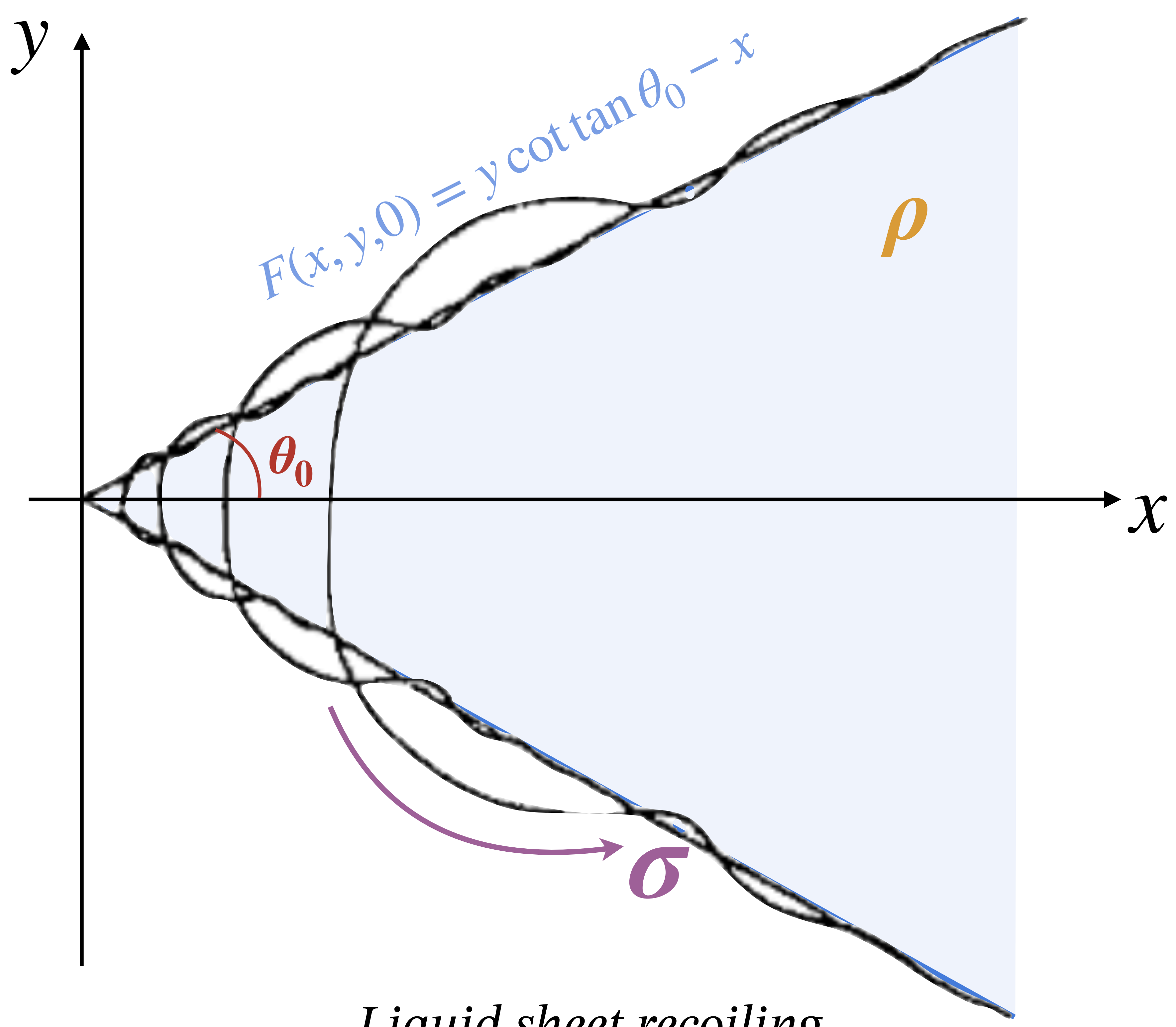
$$\xi = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} x \quad , \quad \eta = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} y$$

*are the **self-similar variables.***

- **Lengths** $\propto t^{2/3}$

Keller and Miksis (1983). *Surface Tension Driven Flows.* <u>SIAM</u> **43**

Recoil near the vertex of the wedge under surface tension effects.



Liquid sheet recoiling
[Sketch from **Peregrine et al. (1990)**]

Hypotheses on the flow:
inviscid, irrotational, isochoric

From *dimensional analysis*:

$$\xi = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} x \quad , \quad \eta = \left(\frac{\rho}{\sigma t^2}\right)^{1/3} y$$

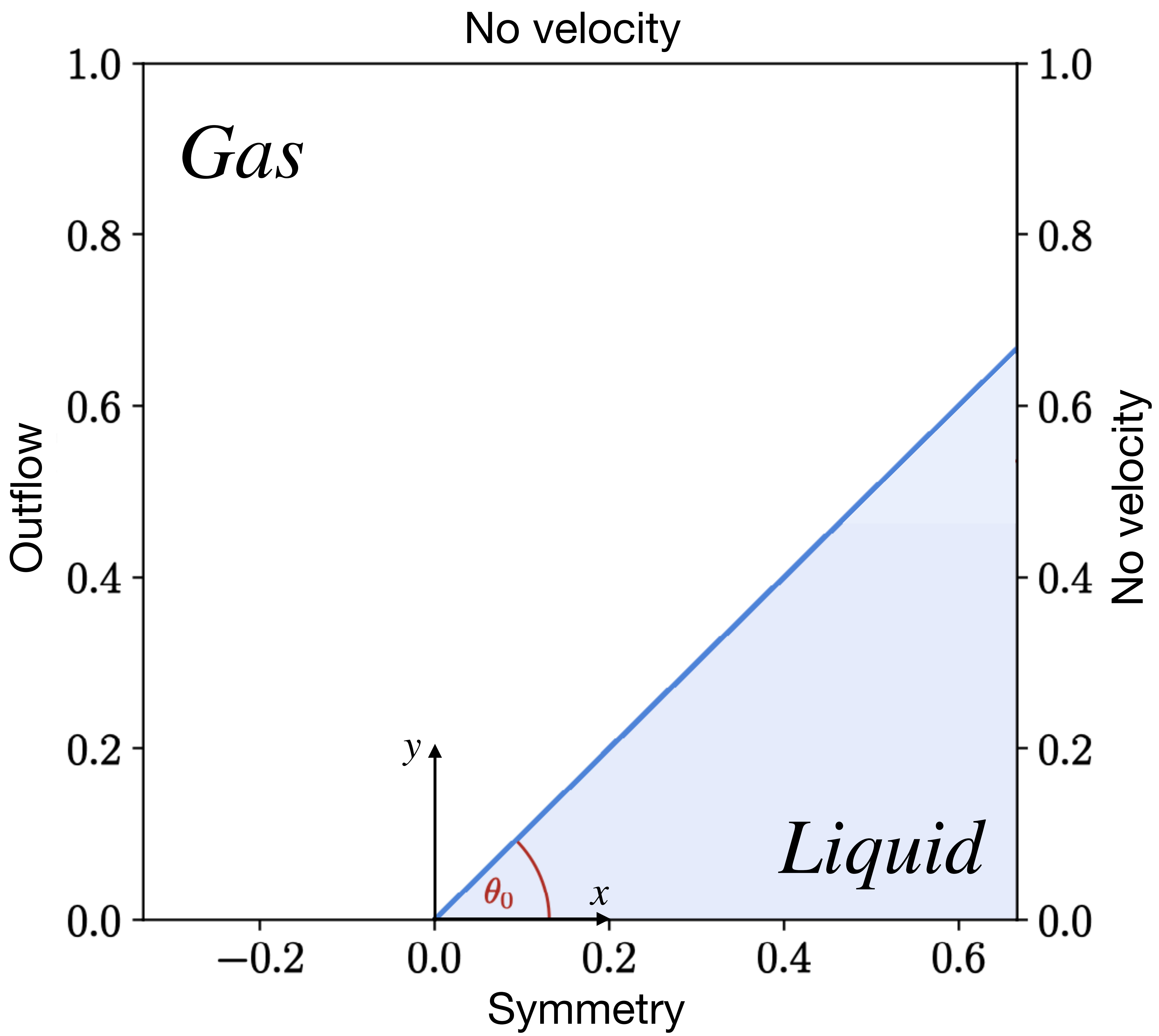
*are the **self-similar variables.***

- **Lengths** $\propto t^{2/3}$
- **Velocity** $\propto t^{-1/3} \underset{t \to 0}{\longrightarrow} +\infty \Rightarrow$ **Finite-time singularity**

Flow self-similar at all times.

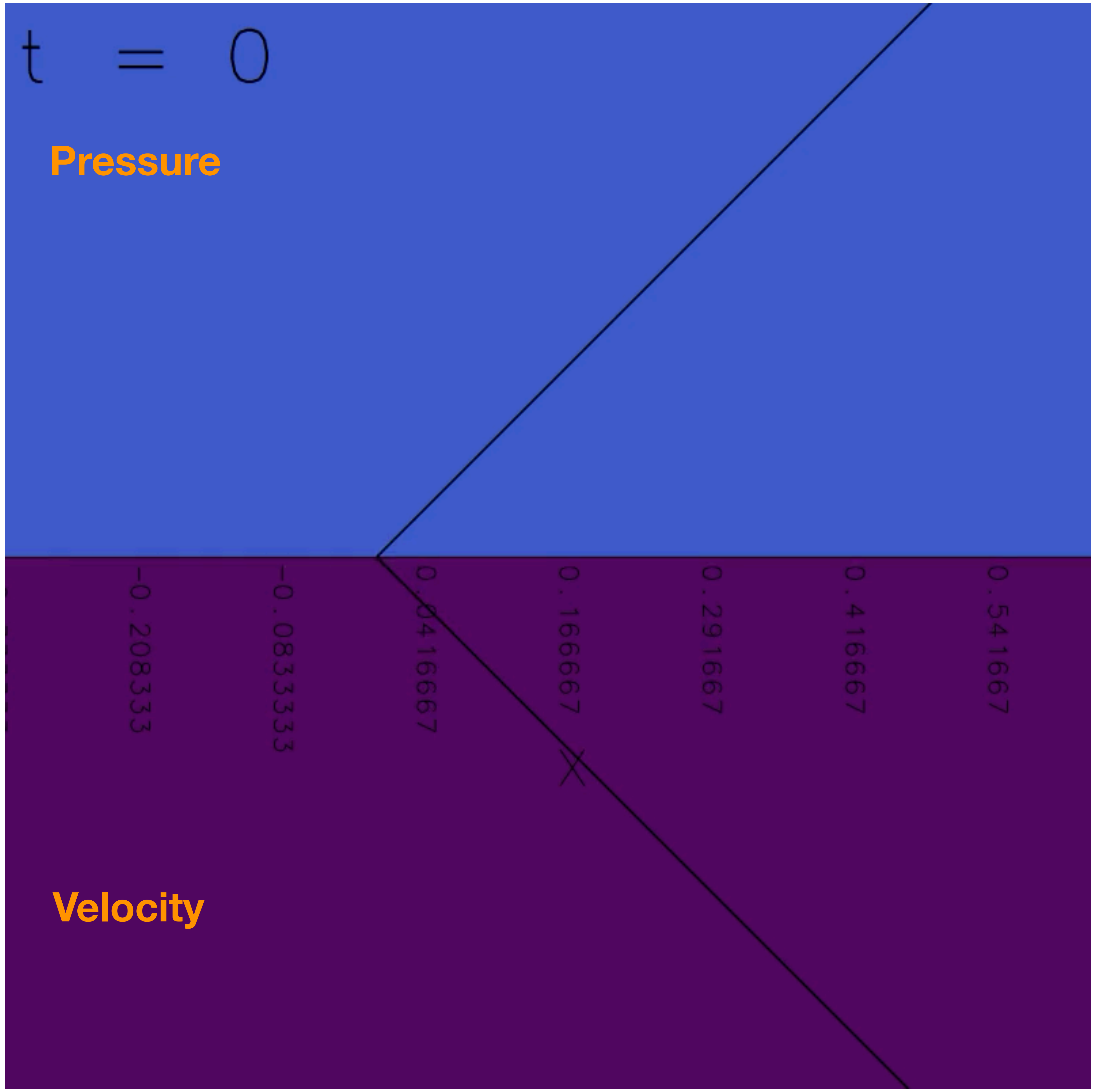
Keller and Miksis (1983). *Surface Tension Driven Flows.* SIAM **43**

Initial Configuration & Boundary Conditions

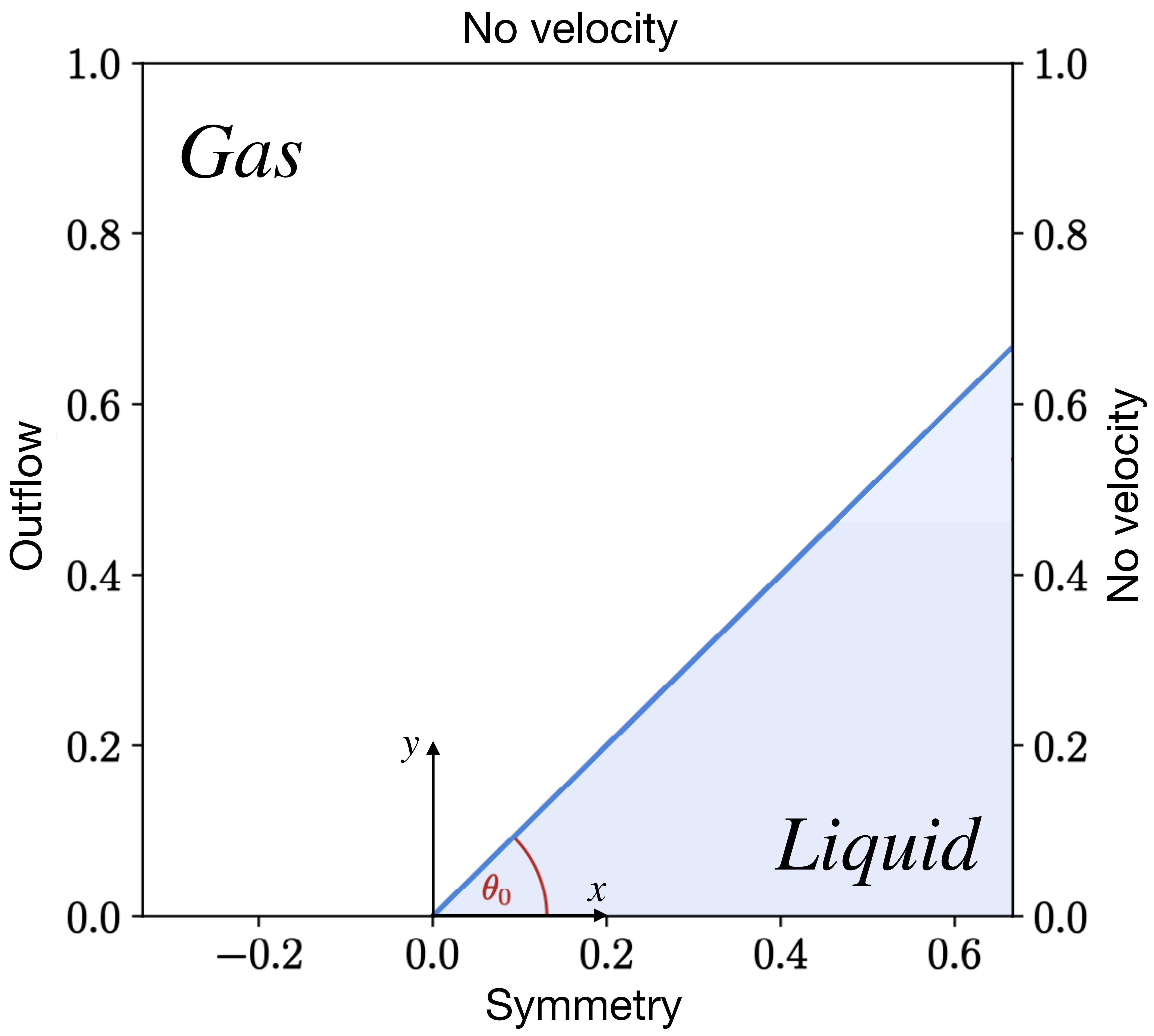


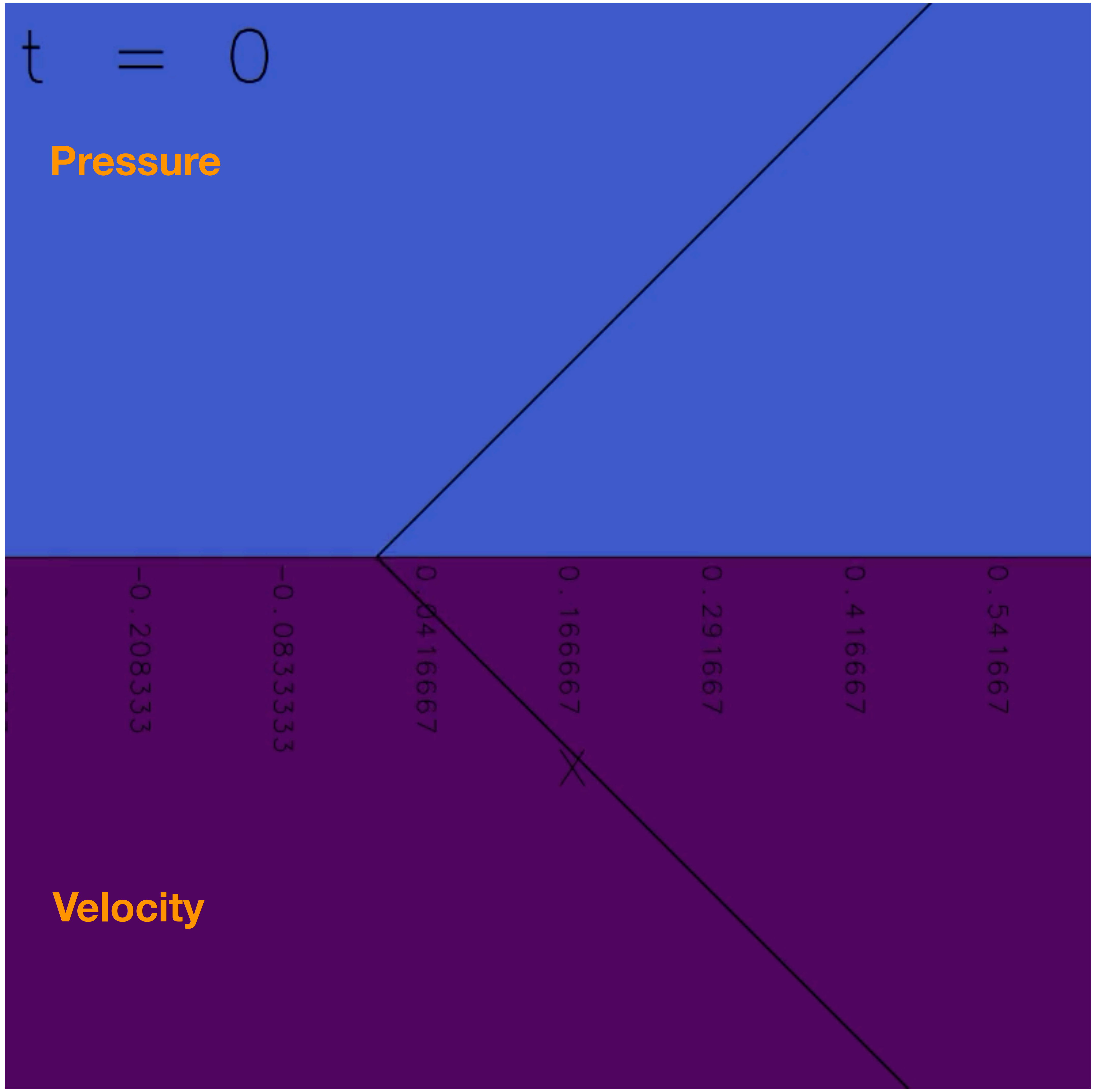
Non-dimensionalized *Euler* eqns solved

$$L_0 = 1 \quad ; \quad \rho_{liq} = 1 \quad ; \quad \rho_{gas} = 10^{-3} \quad ; \quad \sigma^{num} = 1$$



Initial Configuration & Boundary Conditions

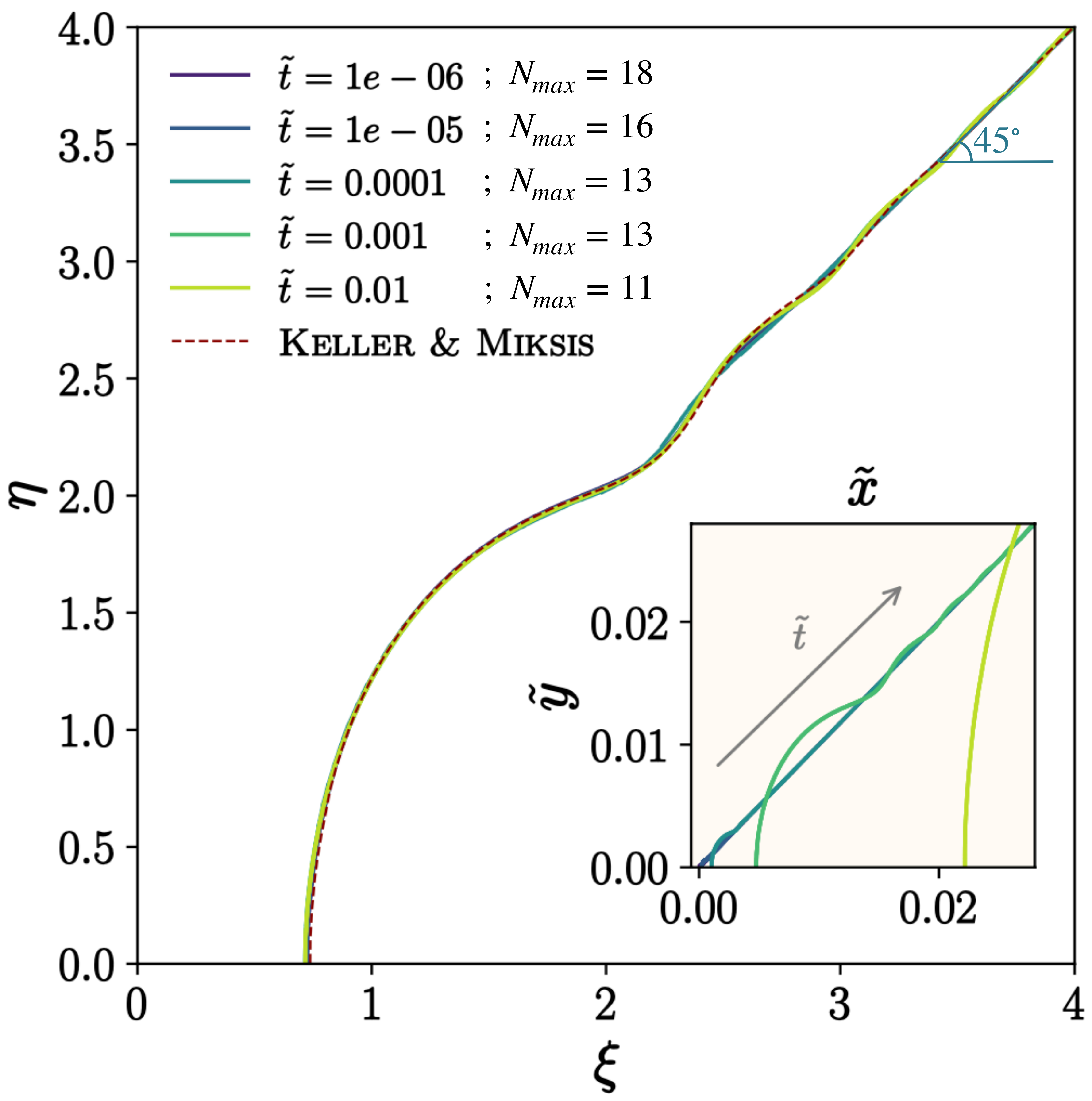


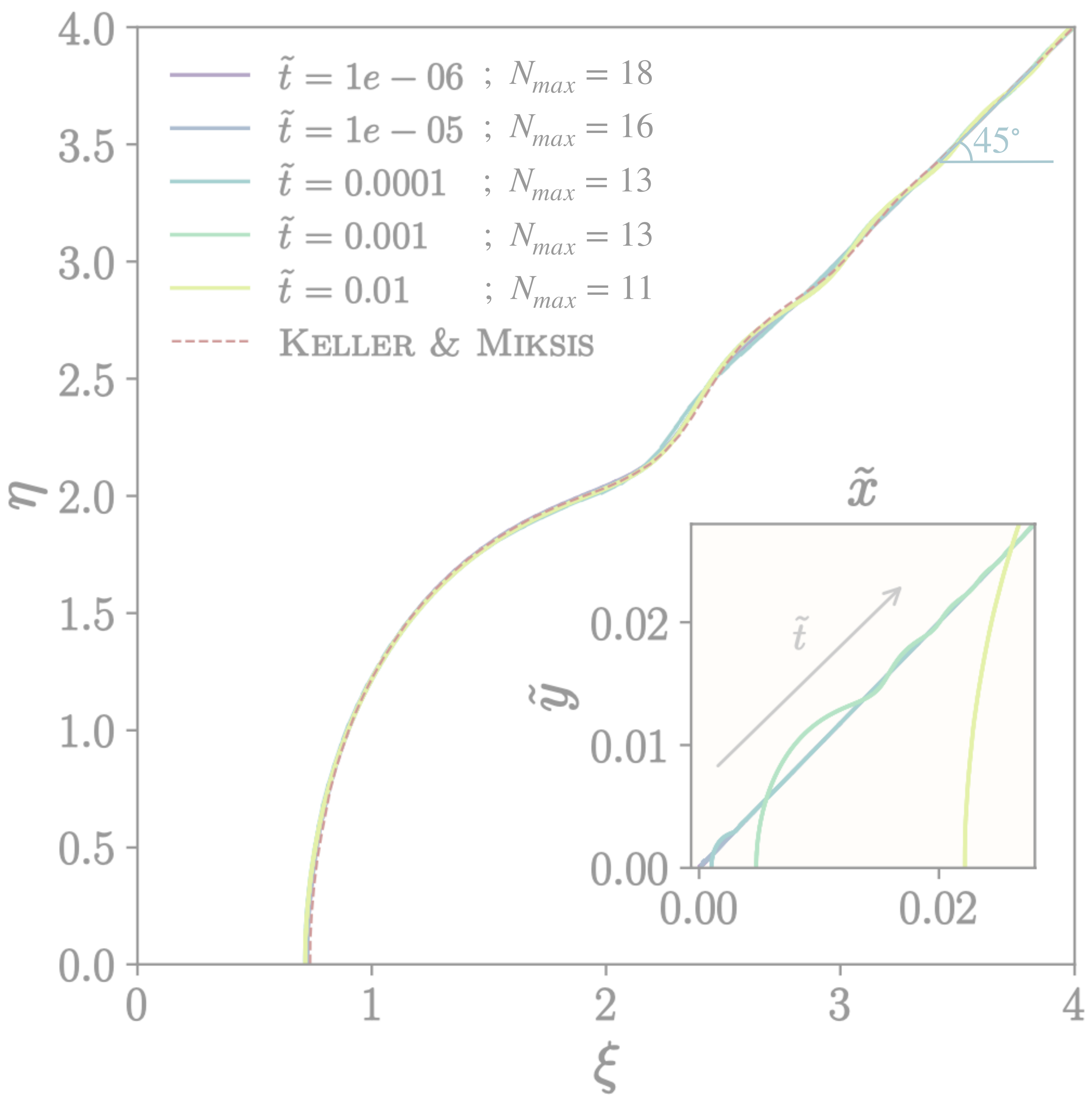


Non-dimensionalized *Euler* eqns solved

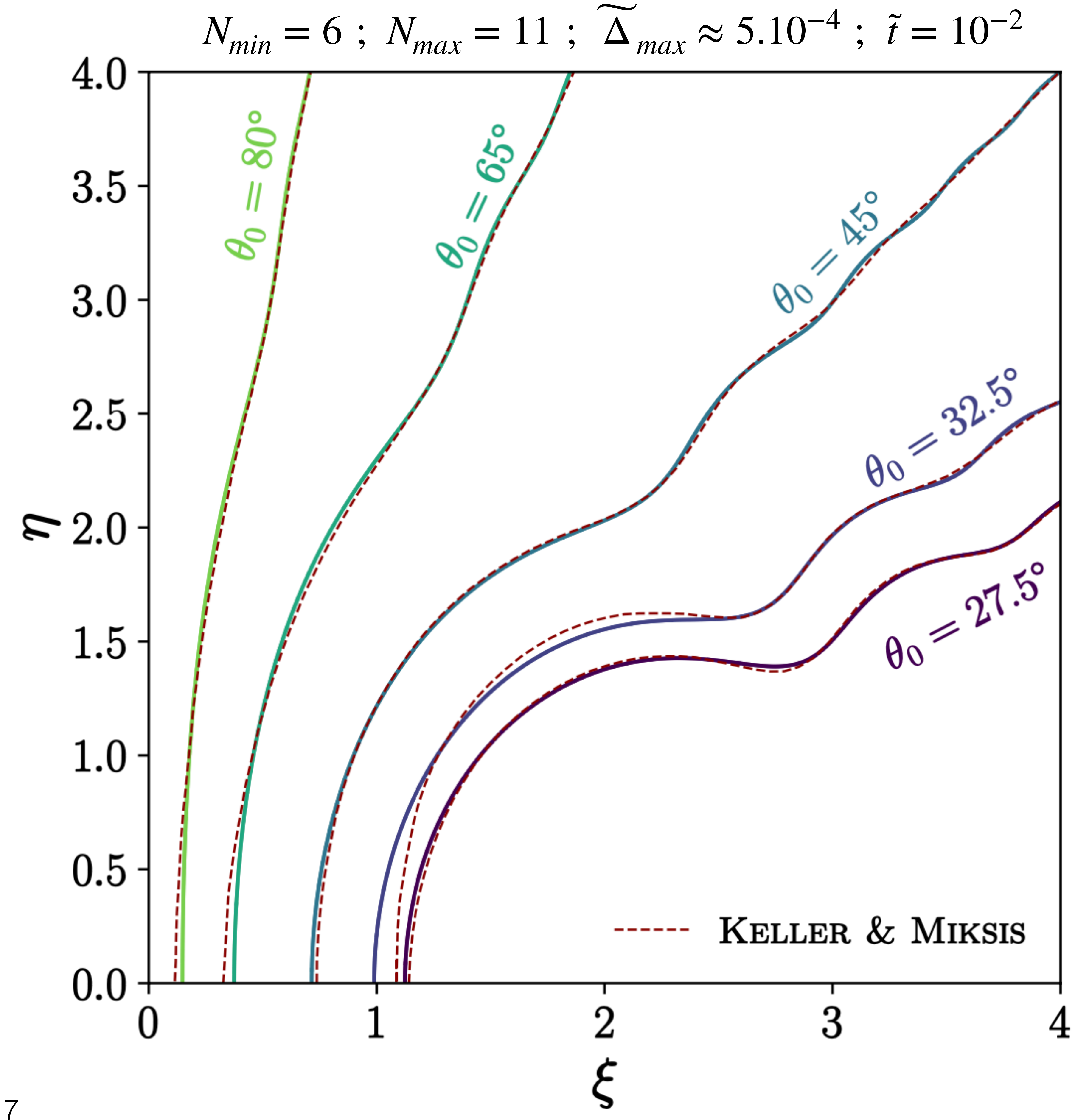
$$L_0 = 1 \quad ; \quad \rho_{liq} = 1 \quad ; \quad \rho_{gas} = 10^{-3} \quad ; \quad \sigma^{num} = 1$$

II - Keller & Miksis (1983) problem





$\tilde{t} = 1e-06$; $N_{max} = 18$
$\tilde{t} = 1e-05$; $N_{max} = 16$
$\tilde{t} = 0.0001$; $N_{max} = 13$
$\tilde{t} = 0.001$; $N_{max} = 13$
$\tilde{t} = 0.01$; $N_{max} = 11$
Keller & Miksis
45°
η
ξ
$\tilde{x}$
$\tilde{y}$
$\tilde{t}$

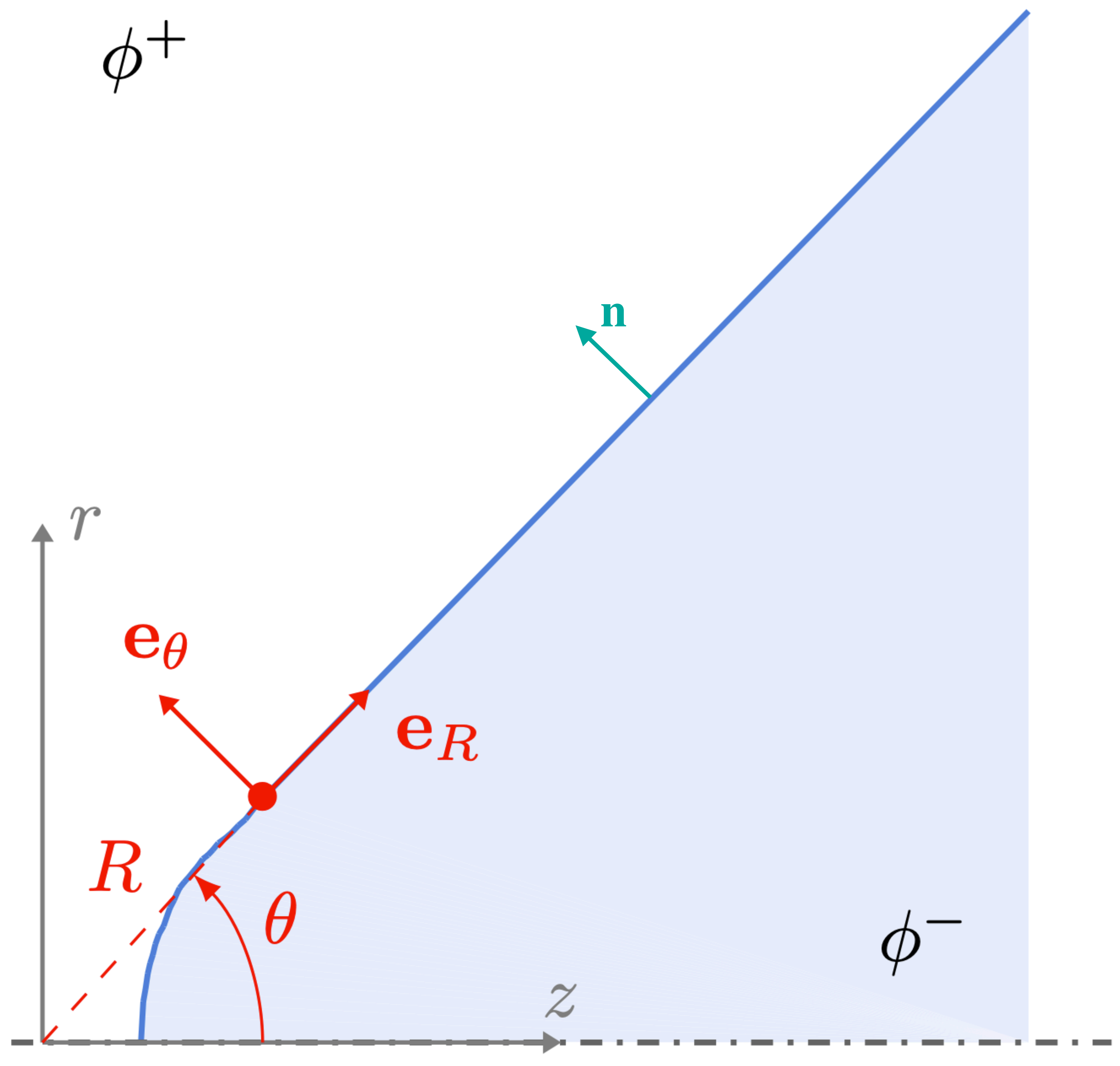


$N_{min} = 6$; $N_{max} = 11$; $\widetilde{\Delta}_{max} \approx 5.10^{-4}$; $\tilde{t} = 10^{-2}$
$\theta_0 = 80°$
$\theta_0 = 65°$
$\theta_0 = 45°$
$\theta_0 = 32.5°$
$\theta_0 = 27.5°$
Keller & Miksis
η
ξ

III - 3D-AXI Recoil under a dipolar flow

III.1 - Differences with the 2D case & Dipolar Flow

Sierou & Lister (2004): **axisymmetric** recoil of an inviscid liquid cone → *theoretical* + *numerical study (BIM)*



$$R = \sqrt{r^2 + z^2} \quad ; \quad \tan\theta = r/z$$

$$\mathcal{S}(R, \theta, t) := \theta - F(R, t)$$

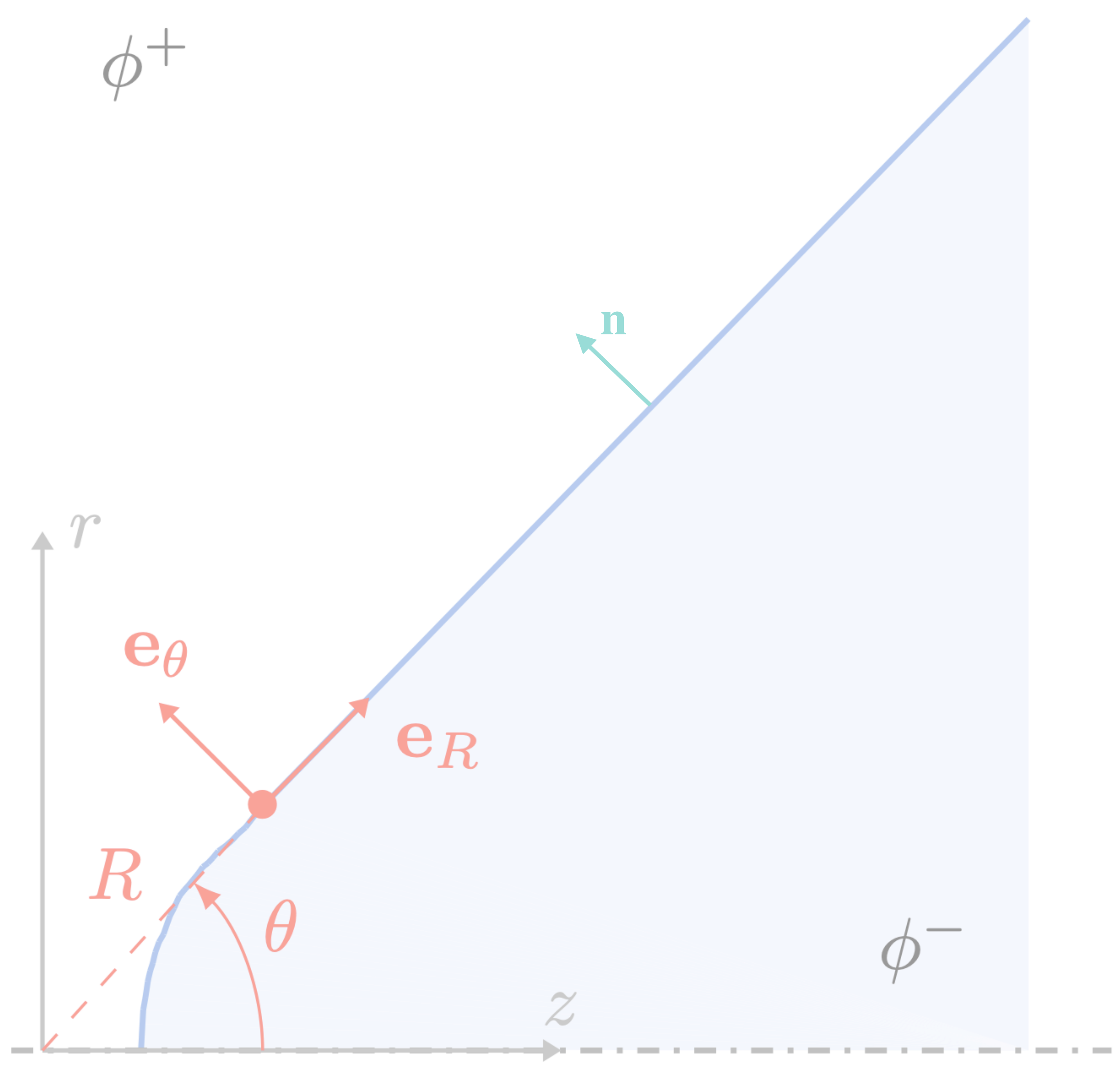
$$\mathbf{n} = \nabla\mathcal{S}/\|\nabla\mathcal{S}\|$$

New features in AXI (*cone*):

- ***inhomogeneous curvature*** ⇒ *Laplace pressure gradient* ⇒ **capillary flow** (*general movement*);
- ***geometrical spreading*** ⇒ capillary waves of *smaller amplitude* $\left(\sim R^{-5} \right)$ than in 2D $\left(\sim R^{-7/2} \right)$;

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

Sierou & Lister (2004): **axisymmetric** recoil of an inviscid liquid cone → *theoretical* + *numerical study (BIM)*



$$R = \sqrt{r^2 + z^2} \quad ; \quad \tan\theta = r/z$$

$$\mathcal{S}(R,\theta,t) := \theta - F(R,t)$$

$$\mathbf{n} = \nabla\mathcal{S}/\|\nabla\mathcal{S}\|$$

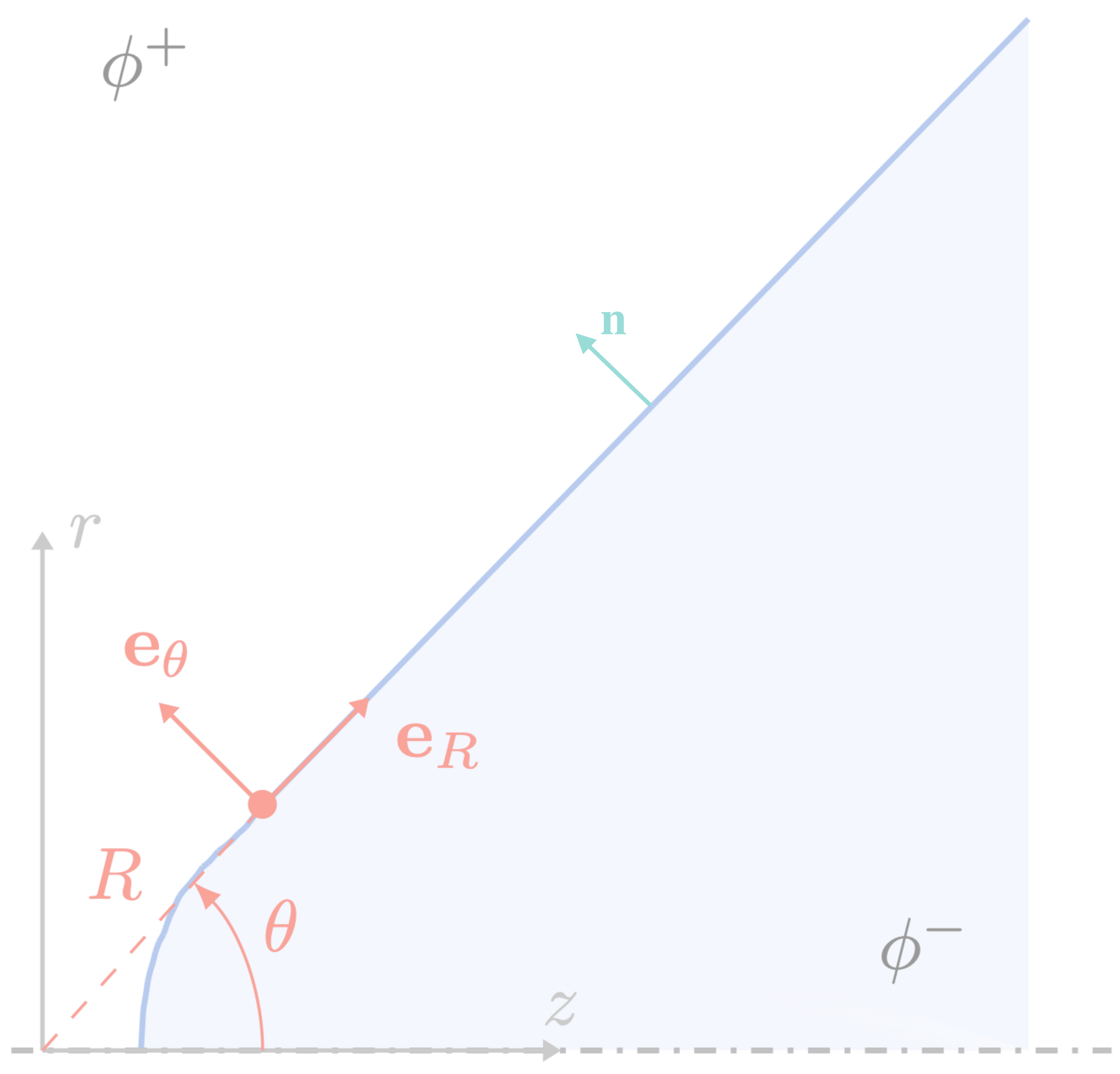
New features in AXI (*cone*):

- ***inhomogeneous curvature*** ⇒ *Laplace pressure gradient* ⇒ **capillary flow** (*general movement*);
- ***geometrical spreading*** ⇒ capillary waves of *smaller amplitude* $\left(\sim R^{-5}\right)$ than in 2D $\left(\sim R^{-7/2}\right)$;

A **far-field dipolar distribution flow** μ_d is added upon the capillary flow:

- **K&M:** waves ↔ surface vorticity distribution
  → **tangential velocity discontinuities** in inviscid flows

Sierou & Lister (2004): **axisymmetric** recoil of an inviscid liquid cone → *theoretical* + *numerical study (BIM)*



$$R = \sqrt{r^2 + z^2} \quad ; \quad \tan\theta = r/z$$

$$\mathcal{S}(R, \theta, t) := \theta - F(R, t)$$

$$\mathbf{n} = \nabla\mathcal{S}/||\nabla\mathcal{S}||$$

New features in AXI (*cone*):

- ***inhomogeneous curvature*** ⇒ *Laplace pressure gradient* ⇒ **capillary flow** (*general movement*);
- ***geometrical spreading*** ⇒ capillary waves of *smaller amplitude* $\left(\sim R^{-5}\right)$ than in 2D $\left(\sim R^{-7/2}\right)$;

A **far-field dipolar distribution flow** μ_d is added upon the capillary flow:

- **K&M:** waves ↔ surface vorticity distribution
  → **tangential velocity discontinuities** in inviscid flows

⇓

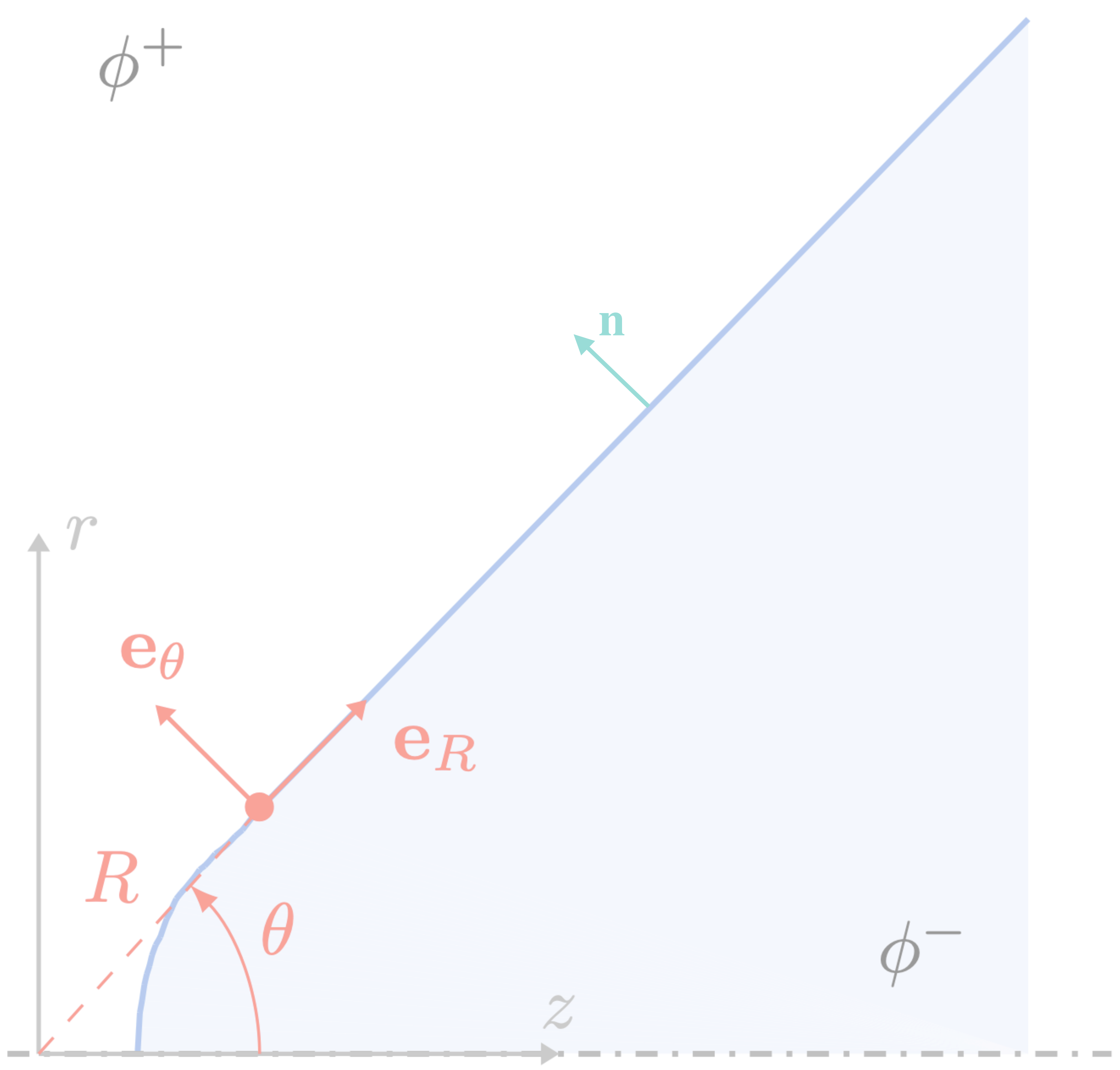
- **S&L:** add a ***variable*** *surface vorticity distribution* at far-field → *modelling complex flows*

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

III - 3D-AXI Recoil under a dipolar flow

III.1 - Differences with the 2D case & Dipolar Flow

Sierou & Lister (2004): **axisymmetric** recoil of an inviscid liquid cone → *theoretical* + *numerical study (BIM)*



$$R = \sqrt{r^2 + z^2} \quad ; \quad \tan\theta = r/z$$

$$\mathcal{S}(R,\theta,t) := \theta - F(R,t)$$

$$\mathbf{n} = \nabla\mathcal{S}/\|\nabla\mathcal{S}\|$$

New features in AXI (*cone*):

- ***inhomogeneous curvature*** ⇒ *Laplace pressure gradient* ⇒ **capillary flow** (*general movement*);
- ***geometrical spreading*** ⇒ capillary waves of *smaller amplitude* $\left(\sim R^{-5}\right)$ than in 2D $\left(\sim R^{-7/2}\right)$;

A **far-field dipolar distribution flow** μ_d is added upon the capillary flow:

- **K&M:** waves ↔ surface vorticity distribution
  → **tangential velocity discontinuities** in inviscid flows

⇓

- **S&L:** add a ***variable*** *surface vorticity distribution* at far-field → *modelling complex flows*

⇓

- Vorticity sheet → modelization → **dipolar flow** μ_d (*potential theory*) **Nie & Baker (1998)**

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

At dominant order $(R \to +\infty)$: ***capillary flow ≪ dipolar flow***

$$F(R,t) = \theta_0$$

$$\mu_d(R,t) = \sqrt{\frac{\sigma}{\rho}}\,\widetilde{\mu}_0\,R^{1/2}$$

$$\phi^-(R,\theta,t) = A_0^-\,P_{1/2}(\cos\theta)\,R^{1/2}$$

$$\phi^+(R,\theta,t) = \left(A_{0,P}^+\,P_{1/2}\cos(\theta) + A_{0,Q}^+\,Q_{1/2}\cos(\theta)\right)R^{1/2}$$

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

At dominant order ($R \rightarrow +\infty$): ***capillary flow $\ll$ dipolar flow***

$$F(R,t) = \theta_0$$

$\longrightarrow$ **Two free parameters:** $\theta_0,\ \widetilde{\mu}_0$

$$\mu_d(R,t) = \sqrt{\frac{\sigma}{\rho}}\,\widetilde{\mu}_0\,R^{1/2}$$

$$\phi^-(R,\theta,t) = A_0^-\,P_{1/2}(\cos\theta)\,R^{1/2}$$

$$\phi^+(R,\theta,t) = \left(A_{0,P}^+\,P_{1/2}\cos(\theta) + A_{0,Q}^+\,Q_{1/2}\cos(\theta)\right)R^{1/2}$$

$\widetilde{\mu}_0$ **corresponds to the *intensity* of the dipolar flow**

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

At dominant order $(R \to +\infty)$: ***capillary flow $\ll$ dipolar flow***

$$F(R,t) = \theta_0$$

$\longrightarrow$ **Two free parameters:** $\theta_0,\ \widetilde{\mu}_0$

$$\mu_d(R,t) = \sqrt{\frac{\sigma}{\rho}}\,\widetilde{\mu}_0\,R^{1/2}$$

$$\phi^-(R,\theta,t) = A_0^- P_{1/2}(\cos\theta)\,R^{1/2}$$

$$\phi^+(R,\theta,t) = \left(A_{0,P}^+ P_{1/2}\cos(\theta) + A_{0,Q}^+ Q_{1/2}\cos(\theta)\right)R^{1/2}$$

Unknown coefficients to be determined.

$\widetilde{\mu}_0$ corresponds to the *intensity* of the dipolar flow

Sierou & Lister (2004), *Self-similar recoil of inviscid drops.* Phys. Fluids **16**

At dominant order ($R \to +\infty$): *capillary flow $\ll$ dipolar flow*

$$F(R,t) = \theta_0$$

$\longrightarrow$ **Two free parameters:** $\theta_0, \ \widetilde{\mu}_0$

$$\mu_d(R,t) = \sqrt{\frac{\sigma}{\rho}}\, \widetilde{\mu}_0 \, R^{1/2}$$

$$\phi^-(R,\theta,t) = A_0^- \, P_{1/2}(\cos\theta)\, R^{1/2}$$

$$\phi^+(R,\theta,t) = \left(A_{0,P}^+ \, P_{1/2} \cos(\theta) + A_{0,Q}^+ \, Q_{1/2} \cos(\theta) \right) R^{1/2}$$

Unknown coefficients to be determined.

We can show analytically that: $A_{0,P}^+ = 0 \quad ; \quad A_0^-,\, A_{0,Q}^+ \propto f^{\pm}(\theta_0)\, \widetilde{\mu}_0$

$\widetilde{\mu}_0$ corresponds to the *intensity* of the dipolar flow

Far-field velocity depends on $\widetilde{\mu}_0$

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

At dominant order ($R \to +\infty$): ***capillary flow ≪ dipolar flow***

$F(R,t) = \theta_0$

$\mu_d(R,t) = \sqrt{\frac{\sigma}{\rho}}\,\widetilde{\mu}_0\,R^{1/2}$

$\longrightarrow$ **Two free parameters:** $\theta_0,\ \widetilde{\mu}_0$

$\phi^-(R,\theta,t) = A_0^- P_{1/2}(\cos\theta)\,R^{1/2}$

$\phi^+(R,\theta,t) = \left(A_{0,P}^+ P_{1/2}\cos(\theta) + A_{0,Q}^+ Q_{1/2}\cos(\theta)\right) R^{1/2}$

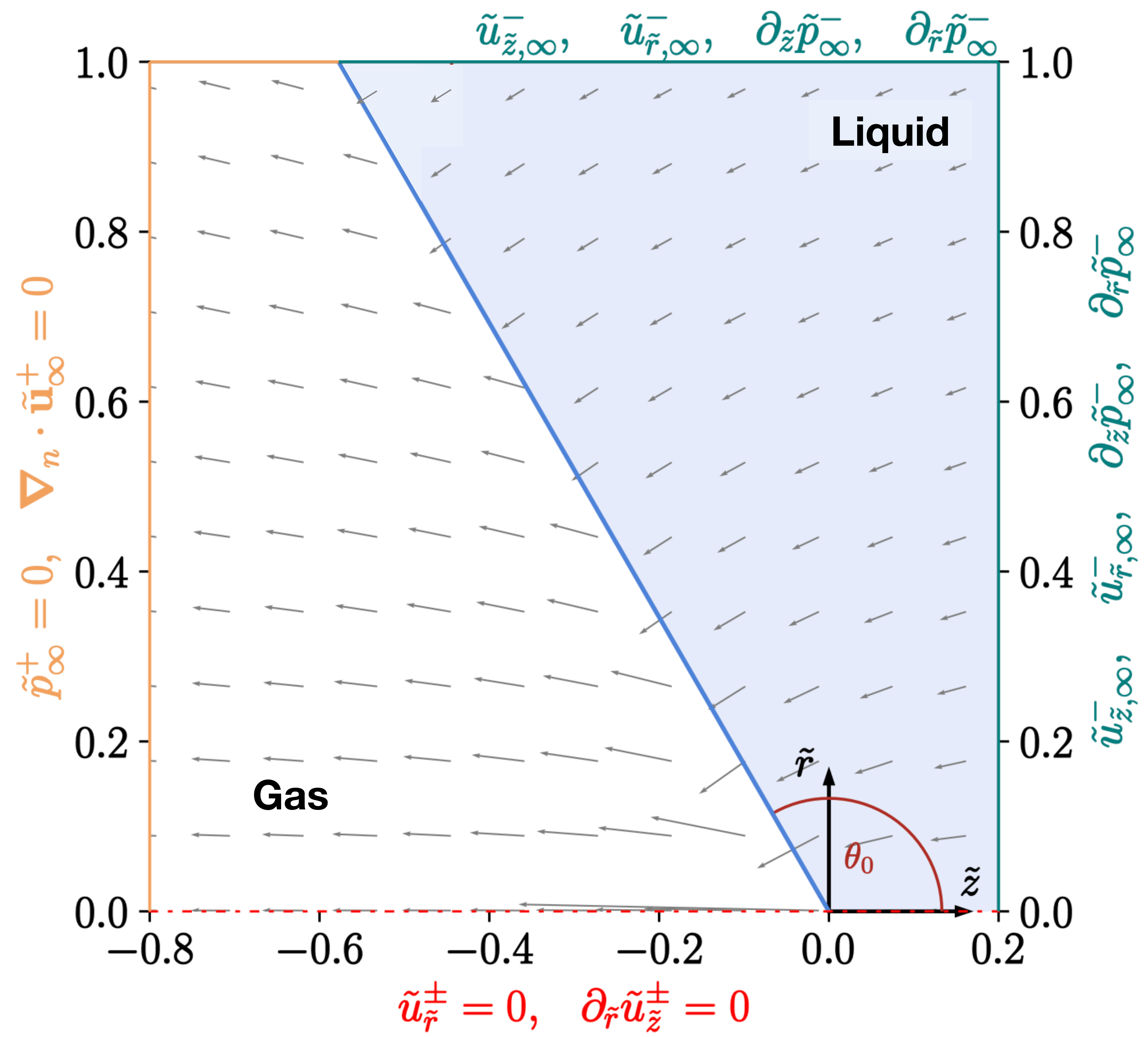
Unknown coefficients to be determined.

We can show analytically that: $A_{0,P}^+ = 0 \quad ; \quad A_0^-,\ A_{0,Q}^+ \propto f^{\pm}(\theta_0)\ \widetilde{\mu}_0$

$\widetilde{\mu}_0$ corresponds to the *intensity* of the dipolar flow

Far-field velocity depends on $\widetilde{\mu}_0$

$\theta_0 > 90^\circ \qquad \widetilde{\mu}_0 \in [-10\,;10]$



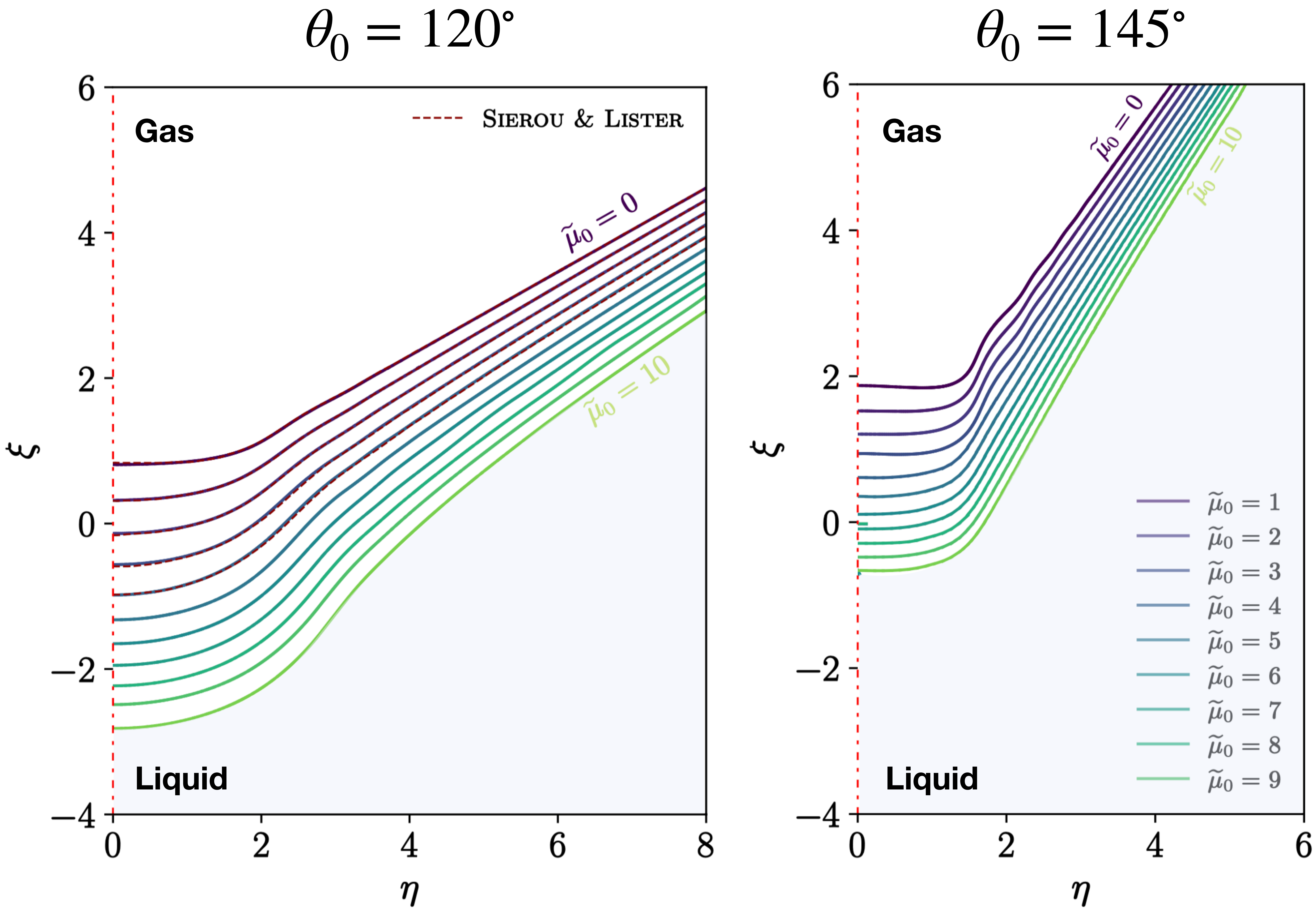
$L_0 = 1 \quad ; \quad \rho_{liq} = 1 \quad ; \quad \rho_{gas} = 10^{-3} \quad ; \quad \sigma^{num} = 1$

Sierou & Lister (2004), *Self-similar recoil of inviscid drops.* Phys. Fluids **16**

III - 3D-AXI Recoil under a dipolar flow

Extended Results of S&L

$(\widetilde{\mu}_0 > 0)$



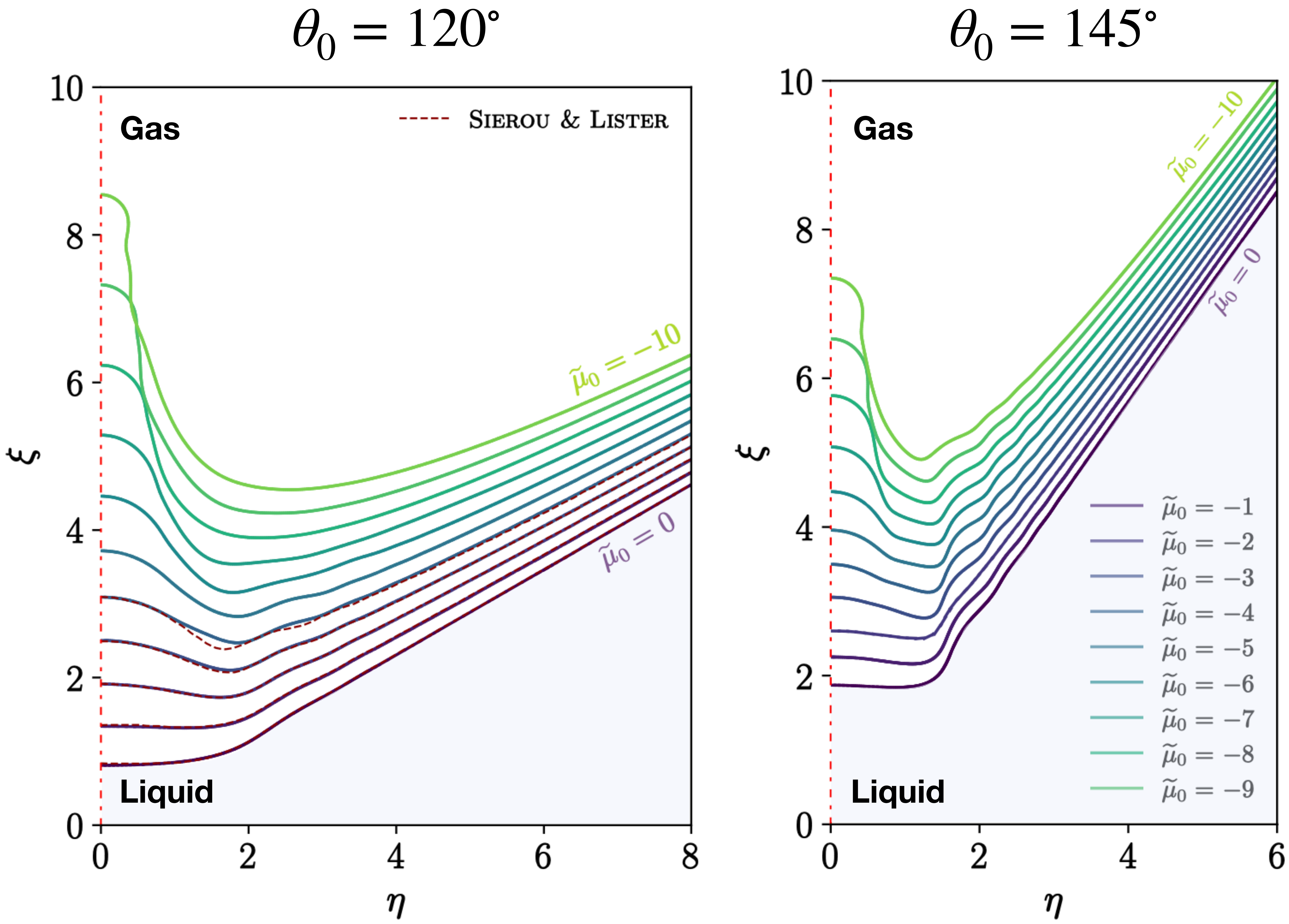
- Self-similar solutions indexed by $(\theta_0, \widetilde{\mu}_0)$
- Capillary waves ↗ when θ_0 ↗ , $\widetilde{\mu}_0$ ↘
- $\widetilde{\mu}_0 = 0,\ \theta_0 > 90°$: *capillary flow* **moves forward** the liquid
- $\widetilde{\mu}_0 > 0$: **counterbalances** the *capillary flow*

Sierou & Lister (2004), *Self-similar recoil of inviscid drops.* Phys. Fluids **16**

III - 3D-AXI Recoil under a dipolar flow

Jets as Extended Results of S&L

($\widetilde{\mu}_0 < 0$)



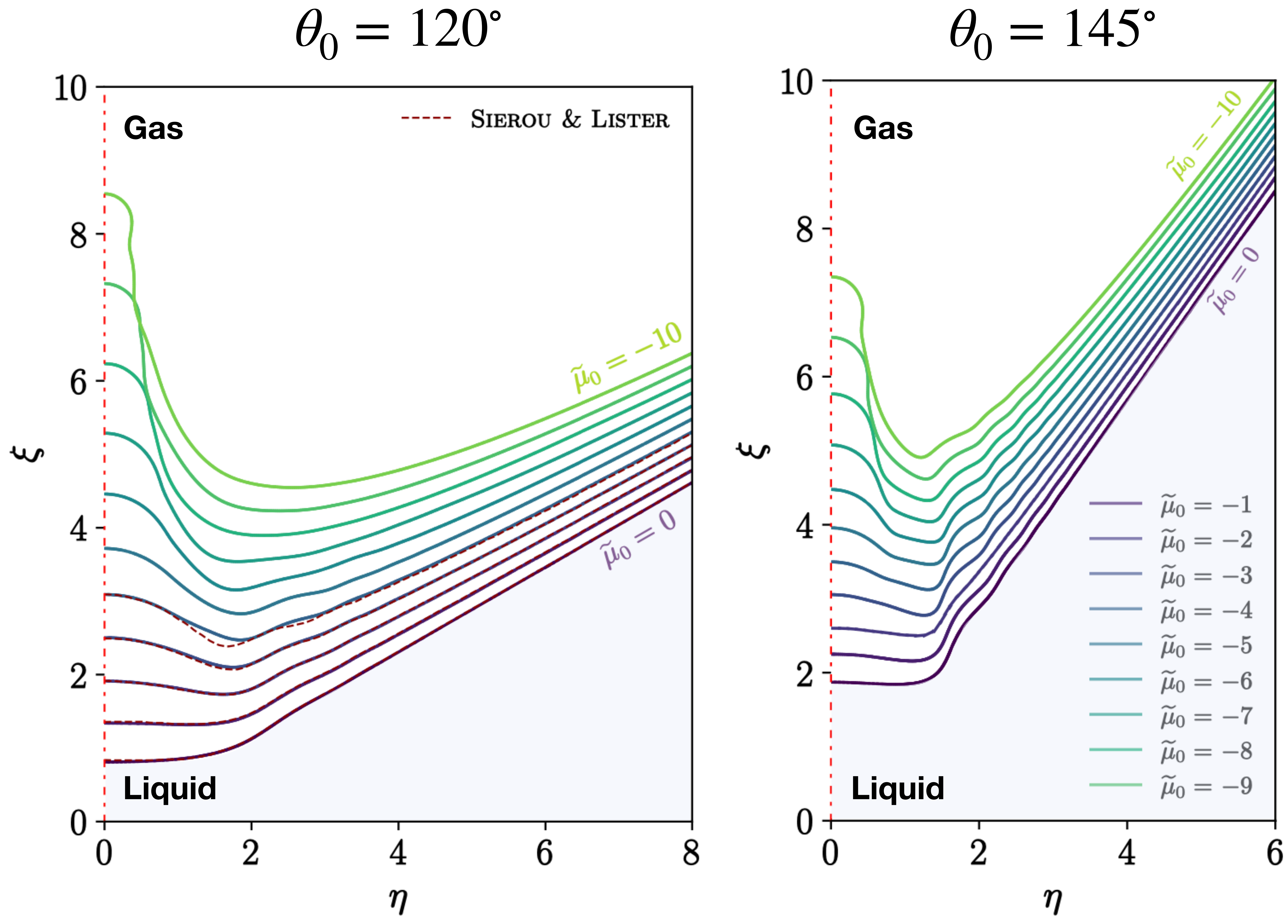
- $\widetilde{\mu}_0 < 0$: **strengthens** the *capillary flow*
- *S&L* results confirmed and extended
- **Self-similar jets profiles** unravelled for *high* $|\widetilde{\mu}_0|$

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

III - 3D-AXI Recoil under a dipolar flow

Jets as Extended Results of S&L

($\widetilde{\mu}_0 < 0$)



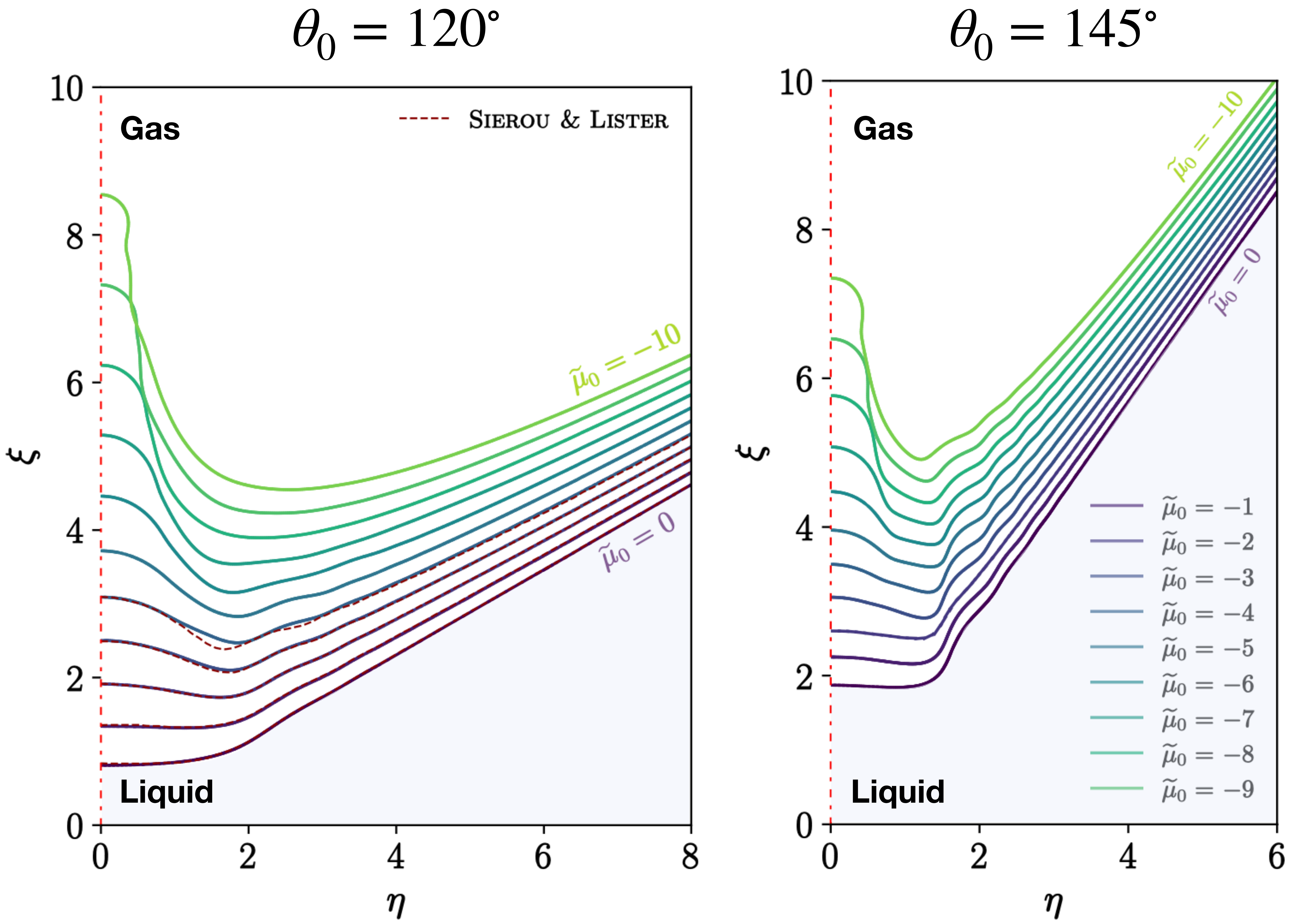
- $\widetilde{\mu}_0 < 0$: **strengthens** the *capillary flow*
- *S&L* results confirmed and extended
- **Self-similar jets profiles** unravelled for *high* $|\widetilde{\mu}_0|$

Bartolo & Josserand (2006)
Brasz *et al.* (2018)
Lai *et al.* (2018)

Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**

III - 3D-AXI Recoil under a dipolar flow

Jets as Extended Results of S&L ($\widetilde{\mu}_0 < 0$)

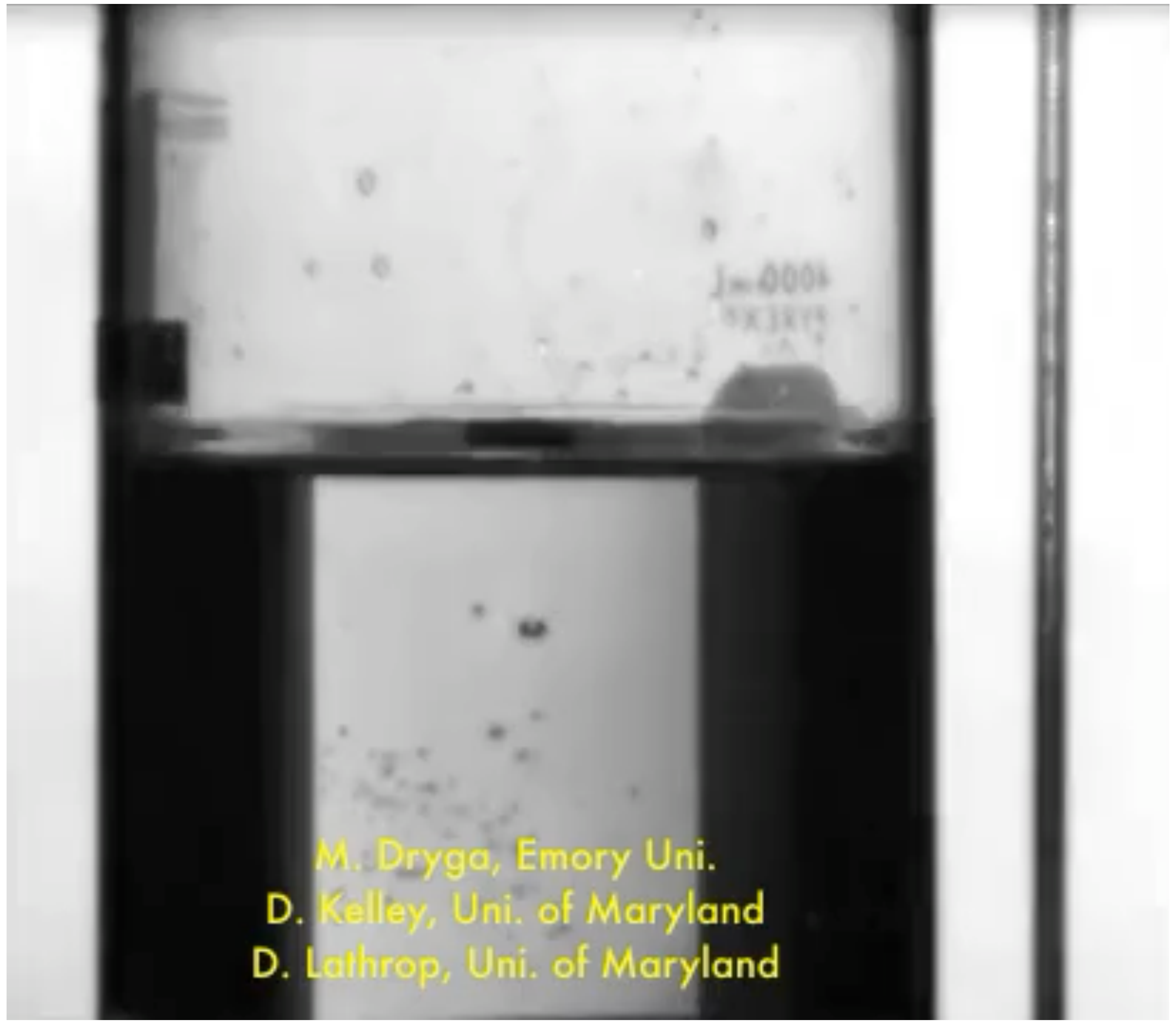


- $\widetilde{\mu}_0 < 0$: **strengthens** the *capillary flow*
- *S&L* results confirmed and extended
- **Self-similar jets profiles** unravelled for *high* $|\widetilde{\mu}_0|$

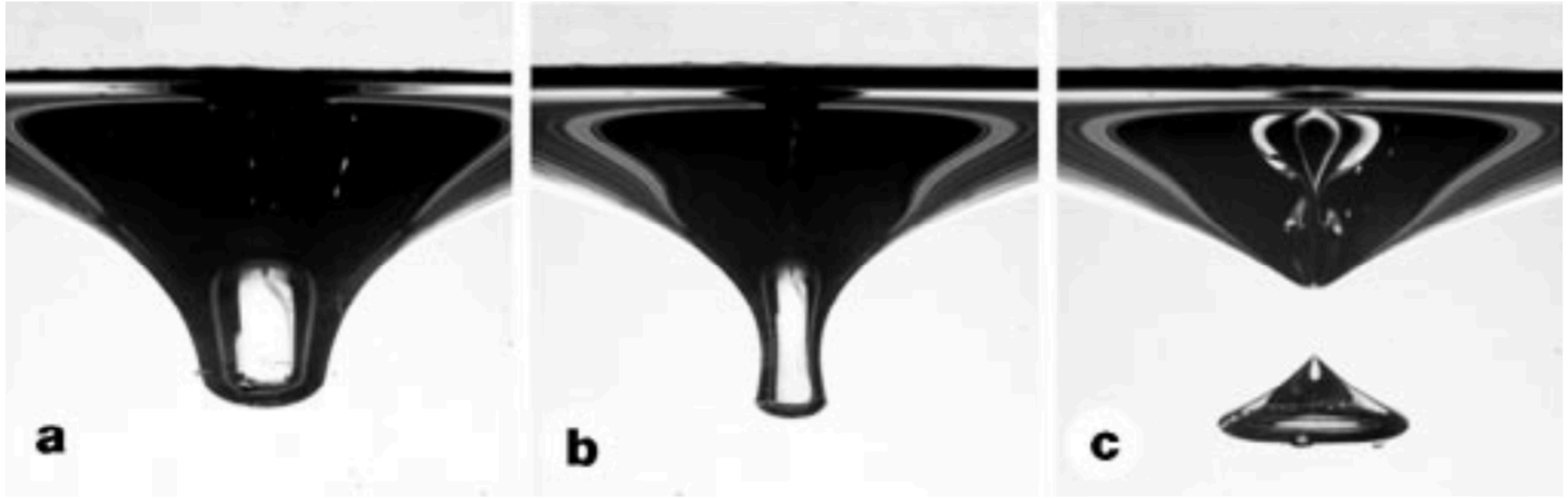
Bartolo & Josserand (2006)
Brasz *et al.* (2018)
Lai *et al.* (2018)

No crossing of the singularity,
which has yet to be addressed!

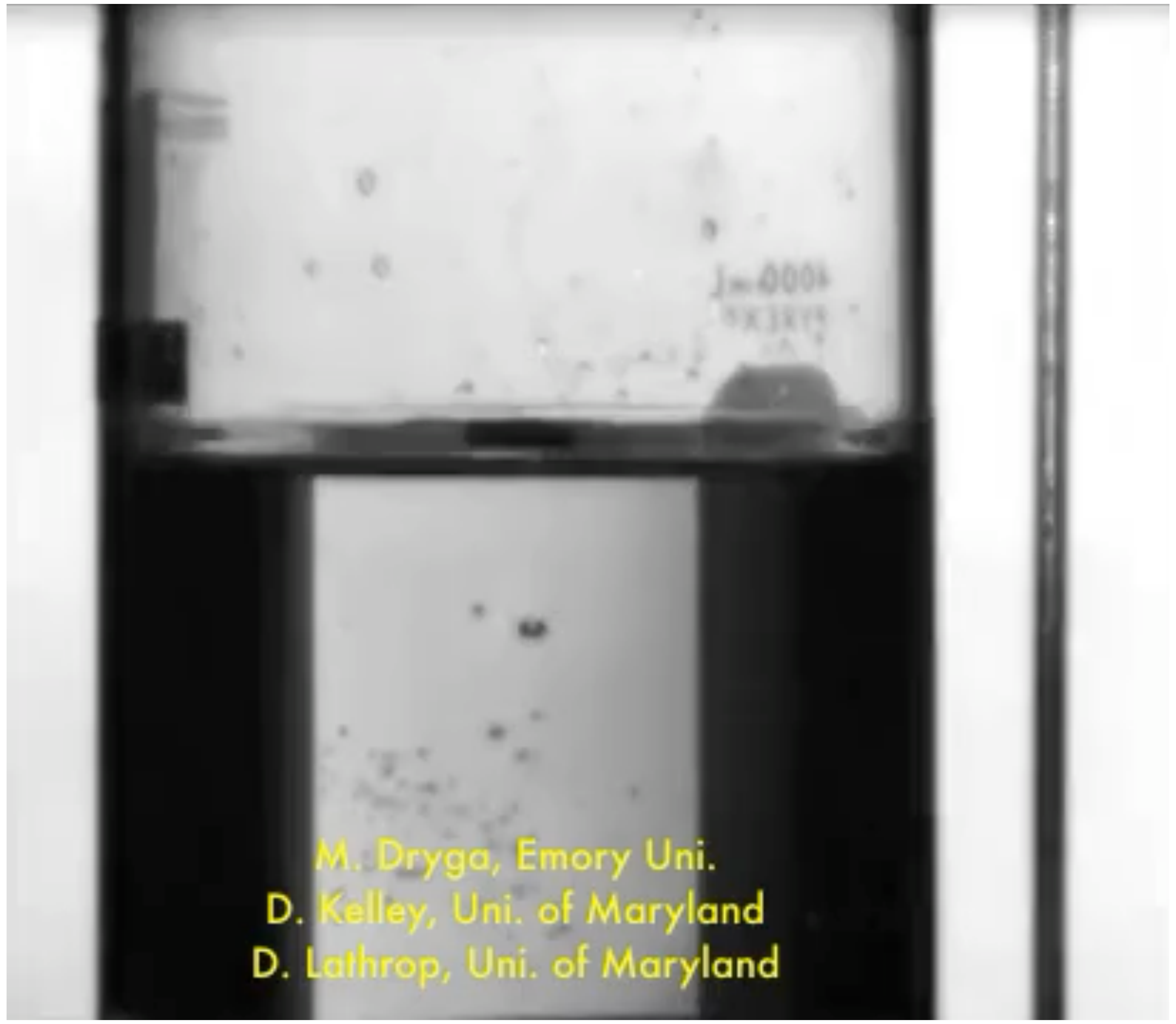
Sierou & Lister (2004), *Self-similar recoil of inviscid drops*. Phys. Fluids **16**



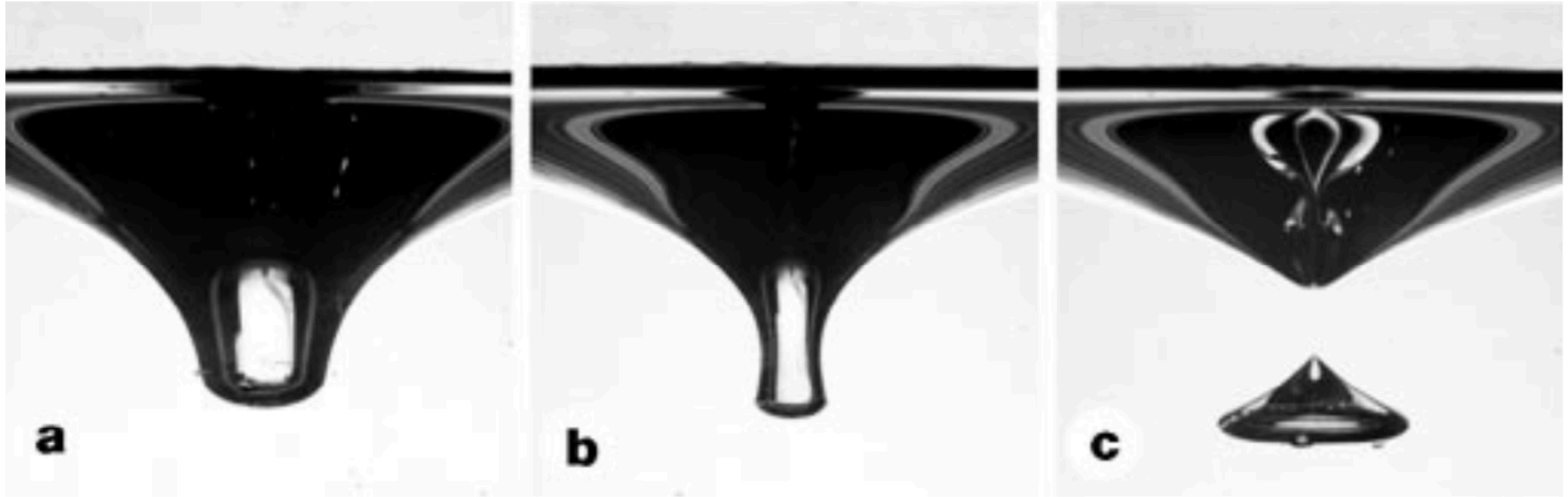
Zeff *et al.* **(2000)**. *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



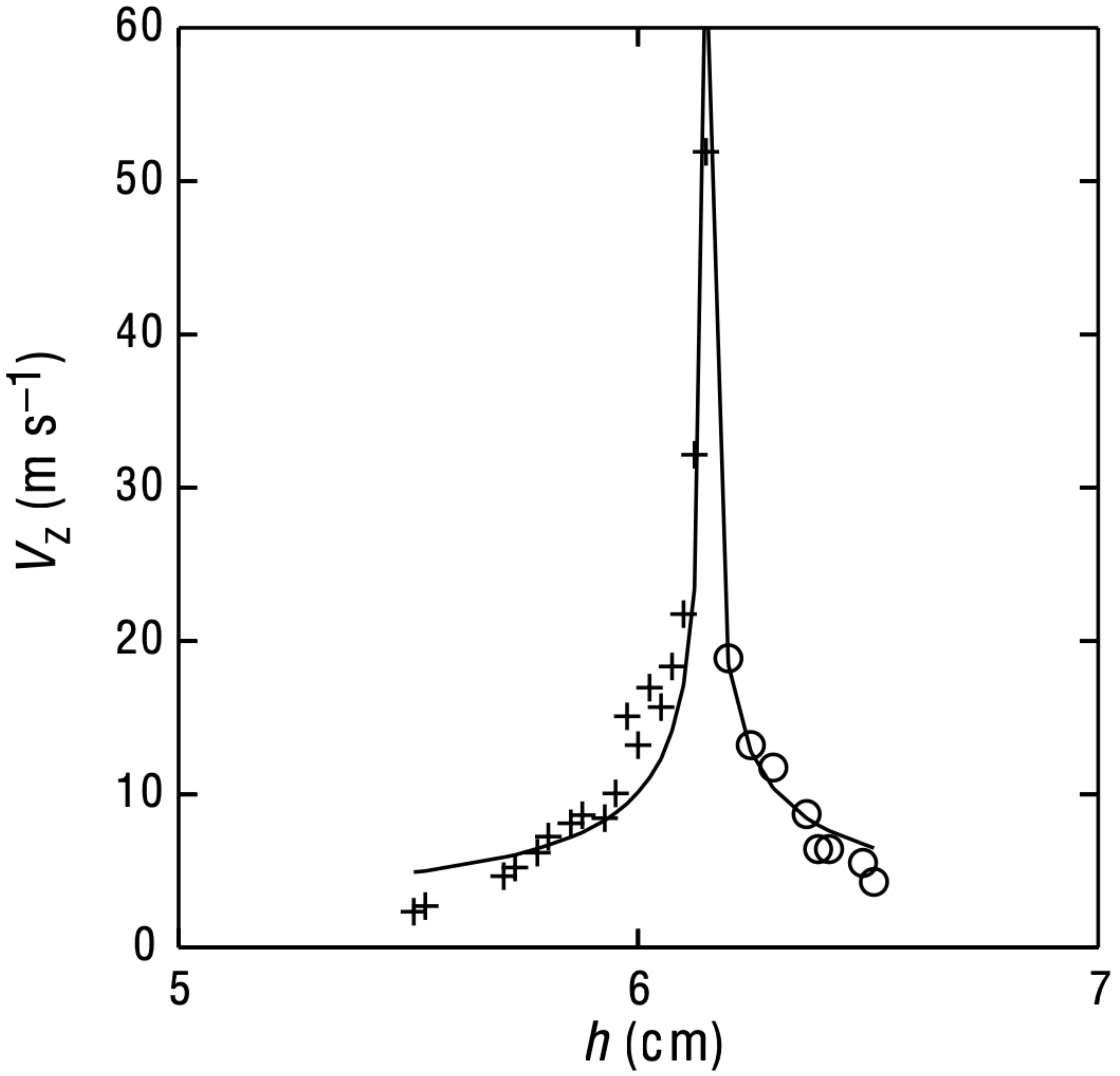
- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**



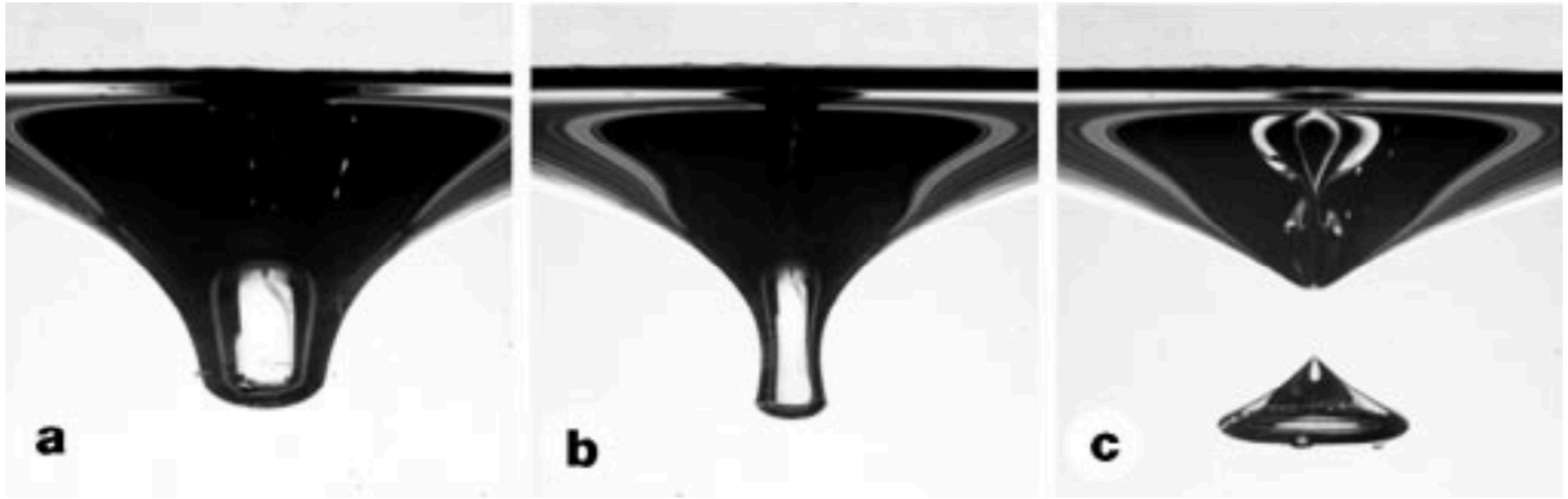
Zeff et al. (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**

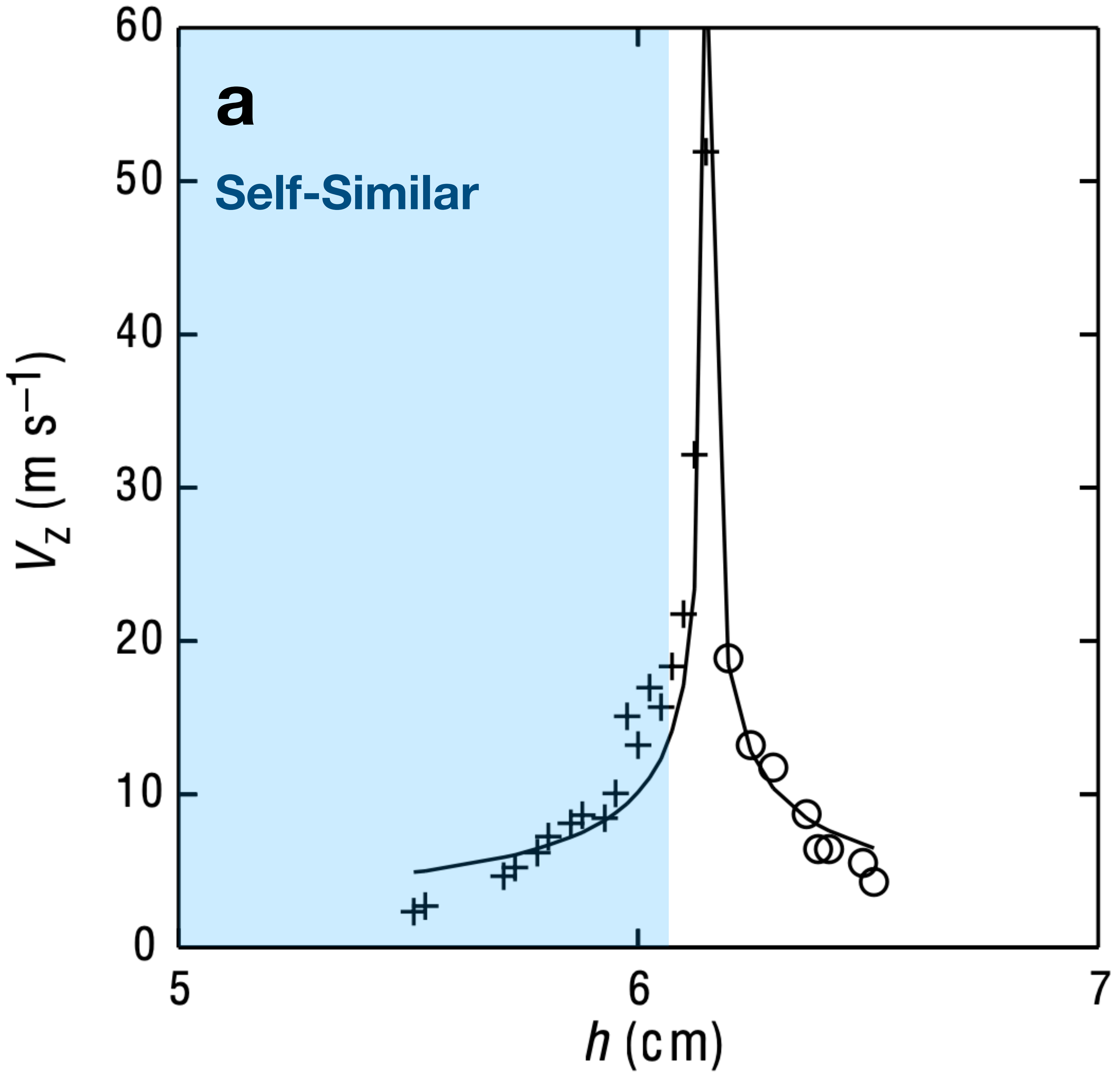


Zeff *et al.* (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**

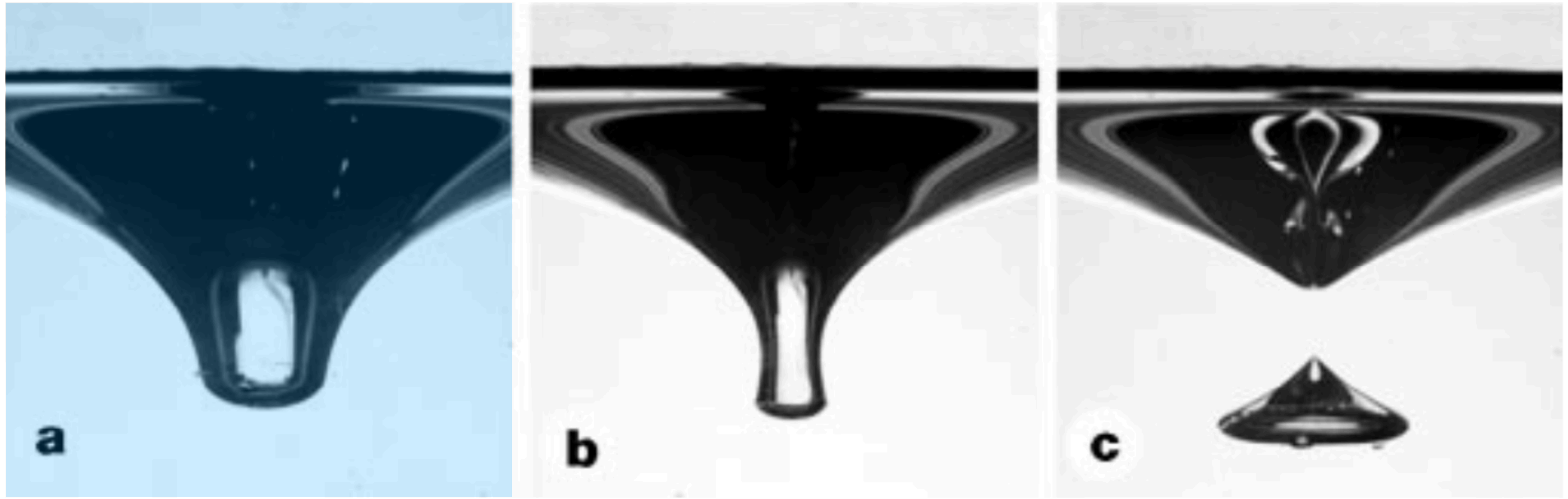


- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**

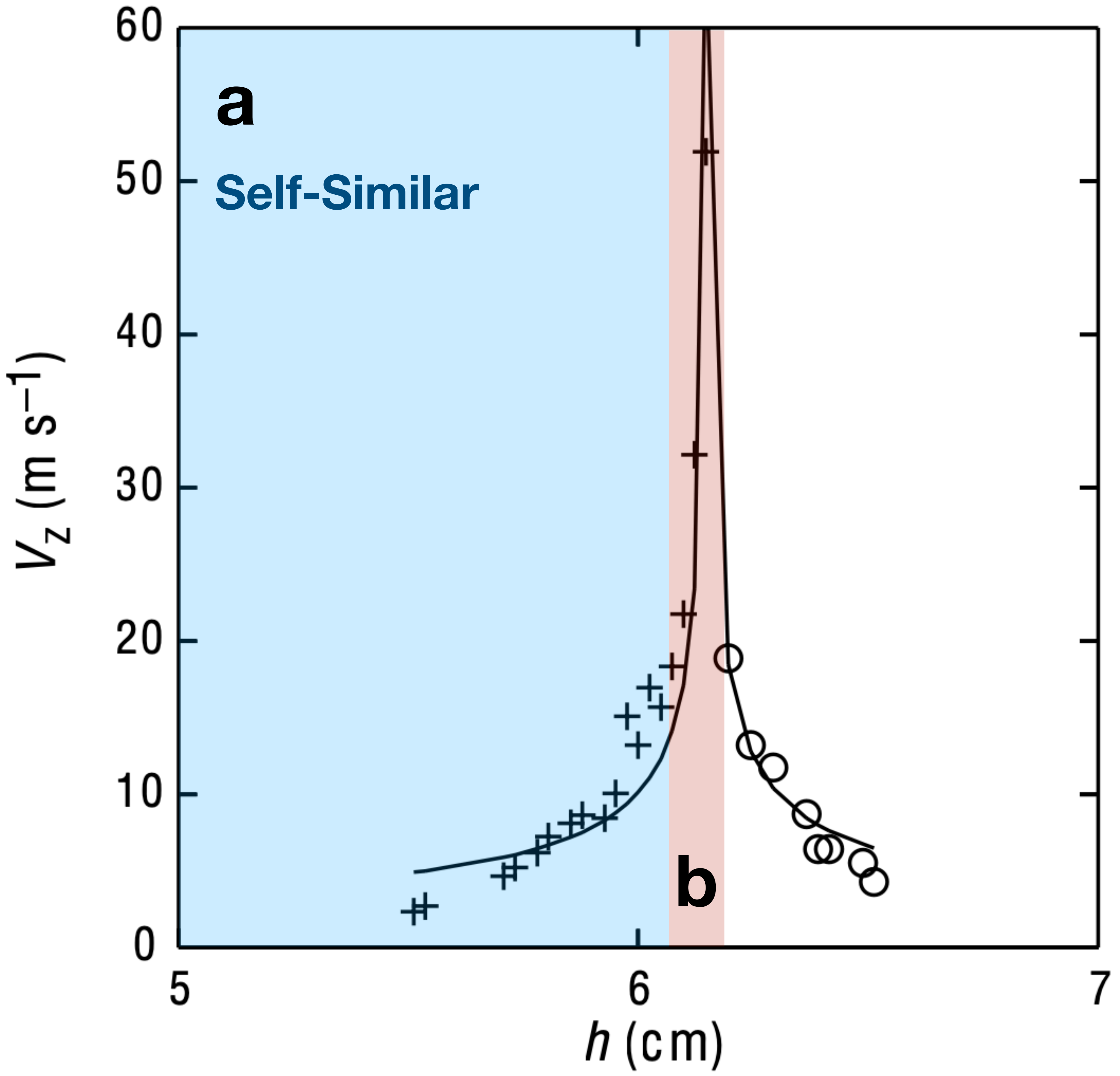
IV - Conical Collapsing Cavities



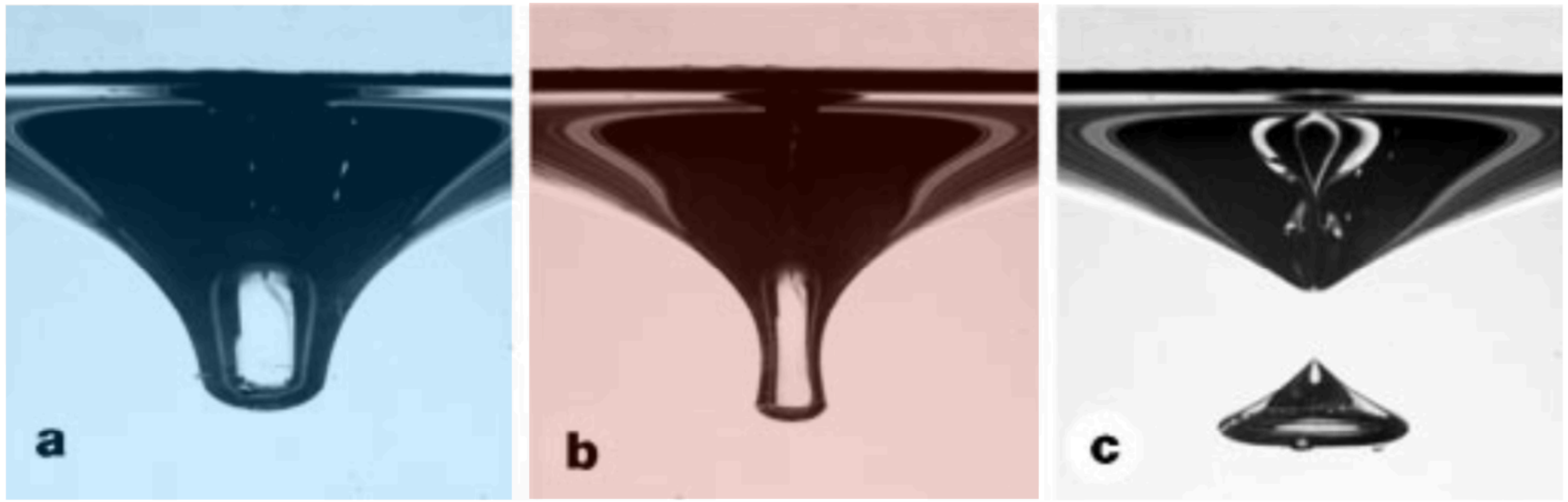
Zeff *et al.* (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



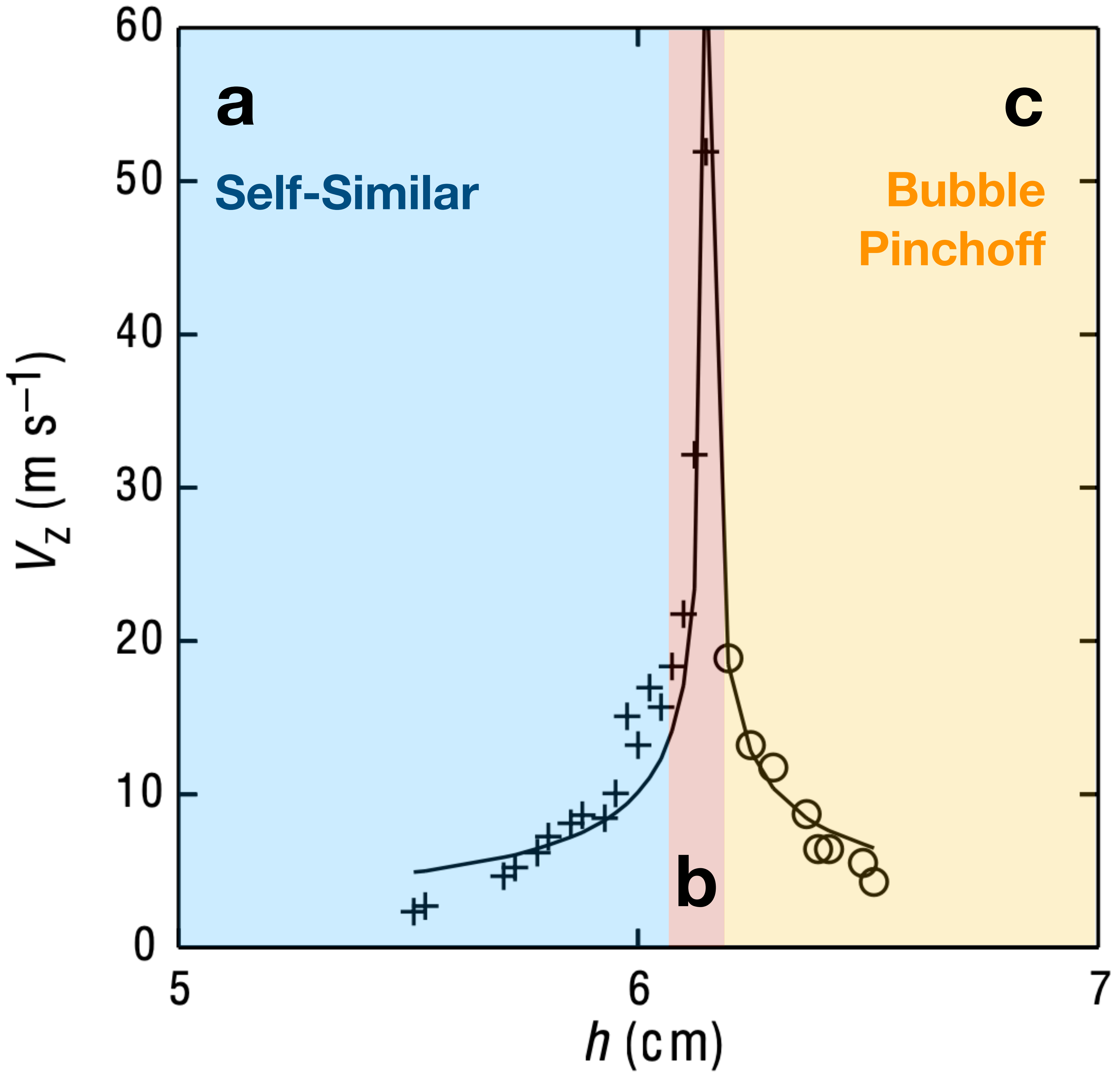
- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**



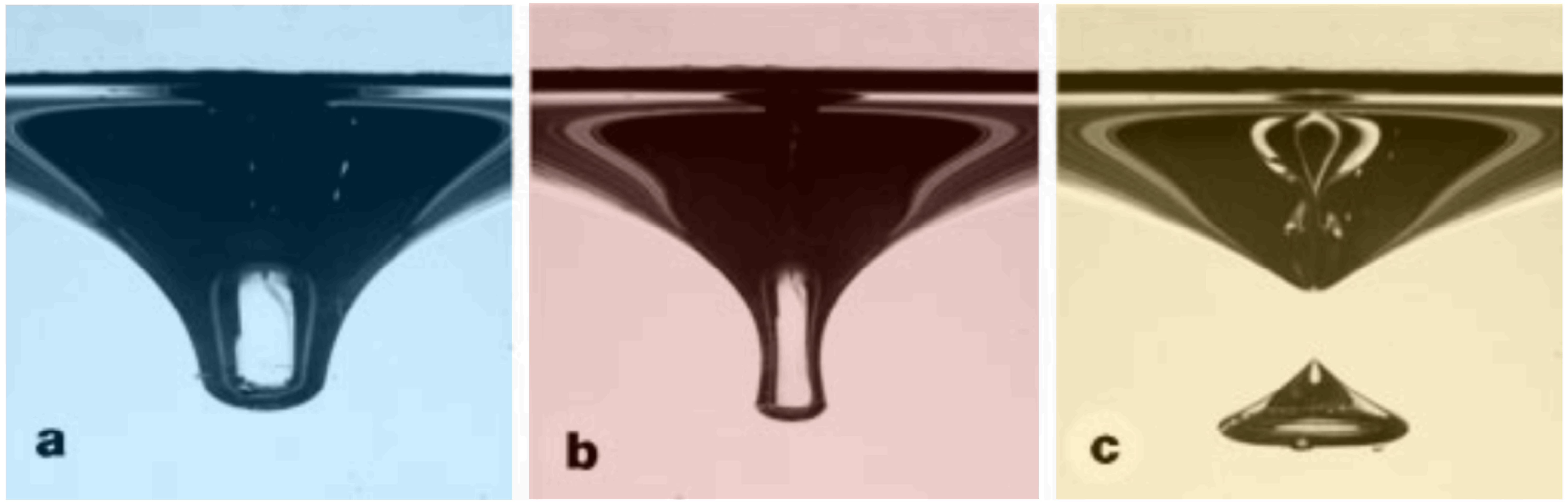
Zeff *et al.* (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



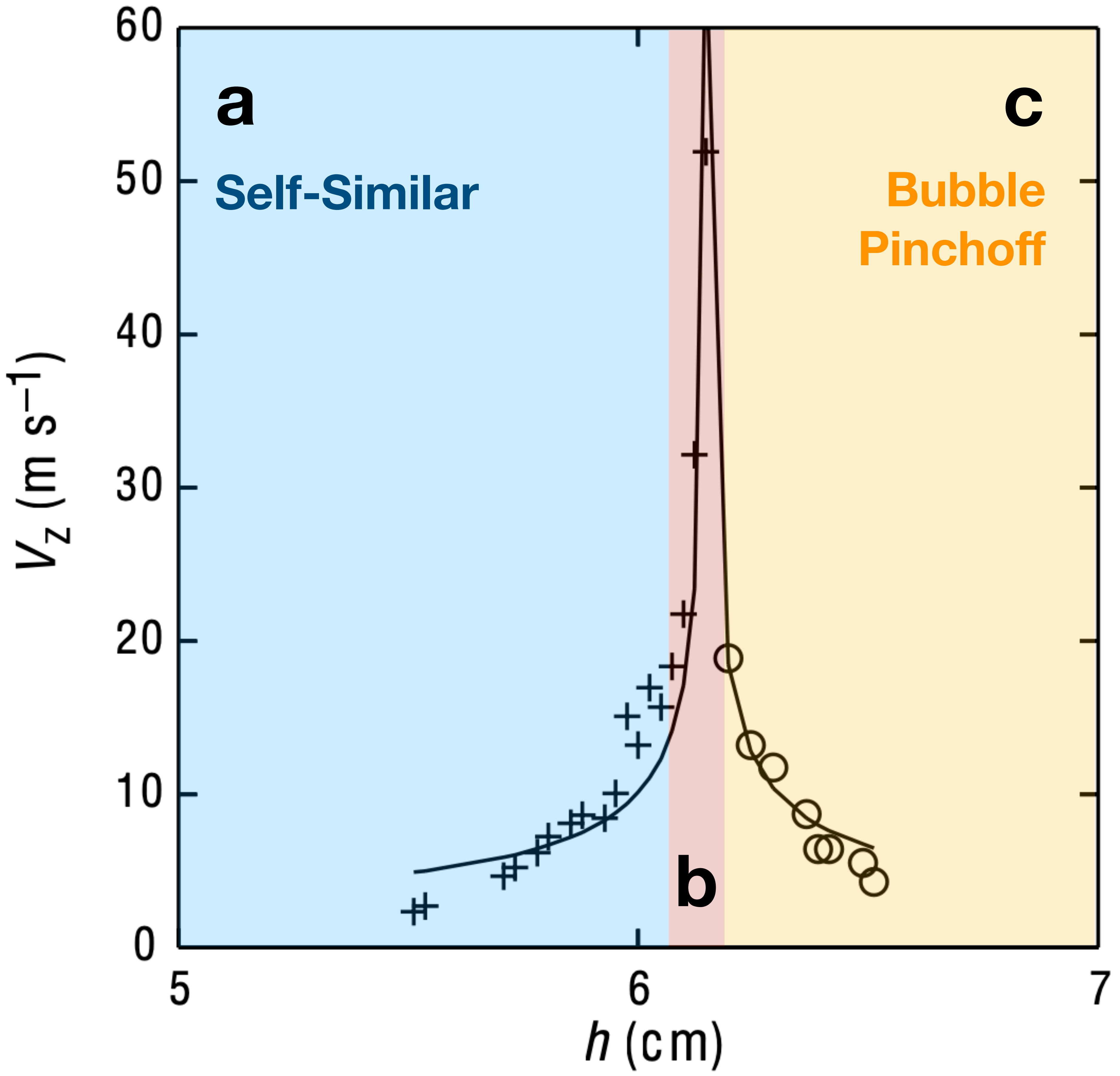
- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**



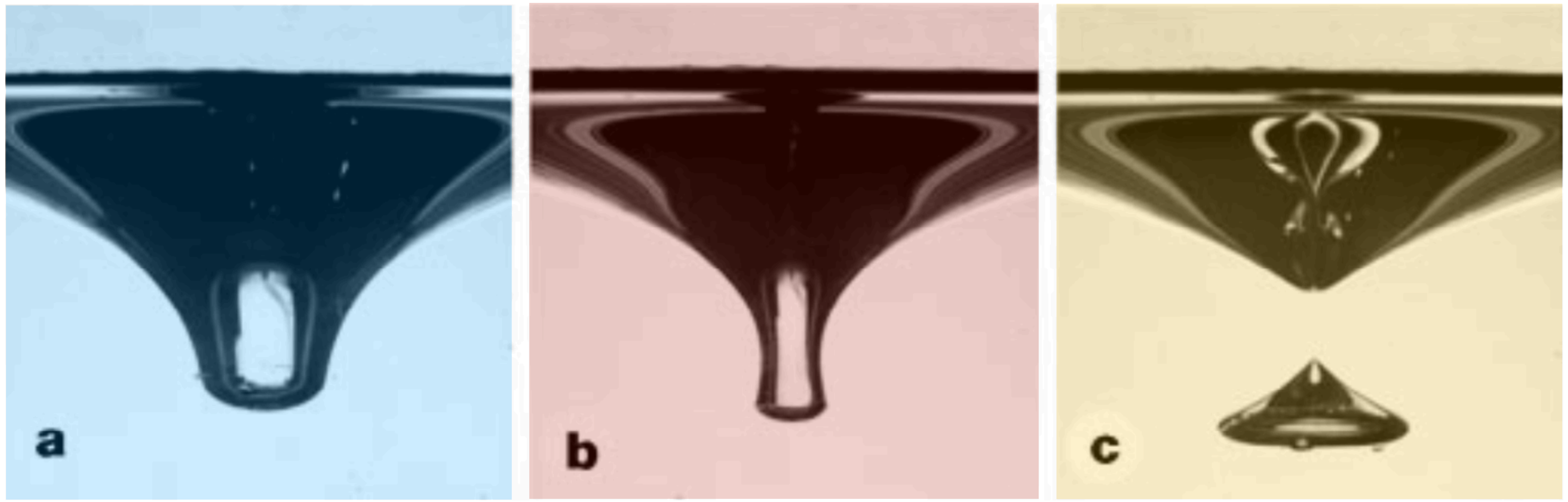
Zeff *et al.* (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**



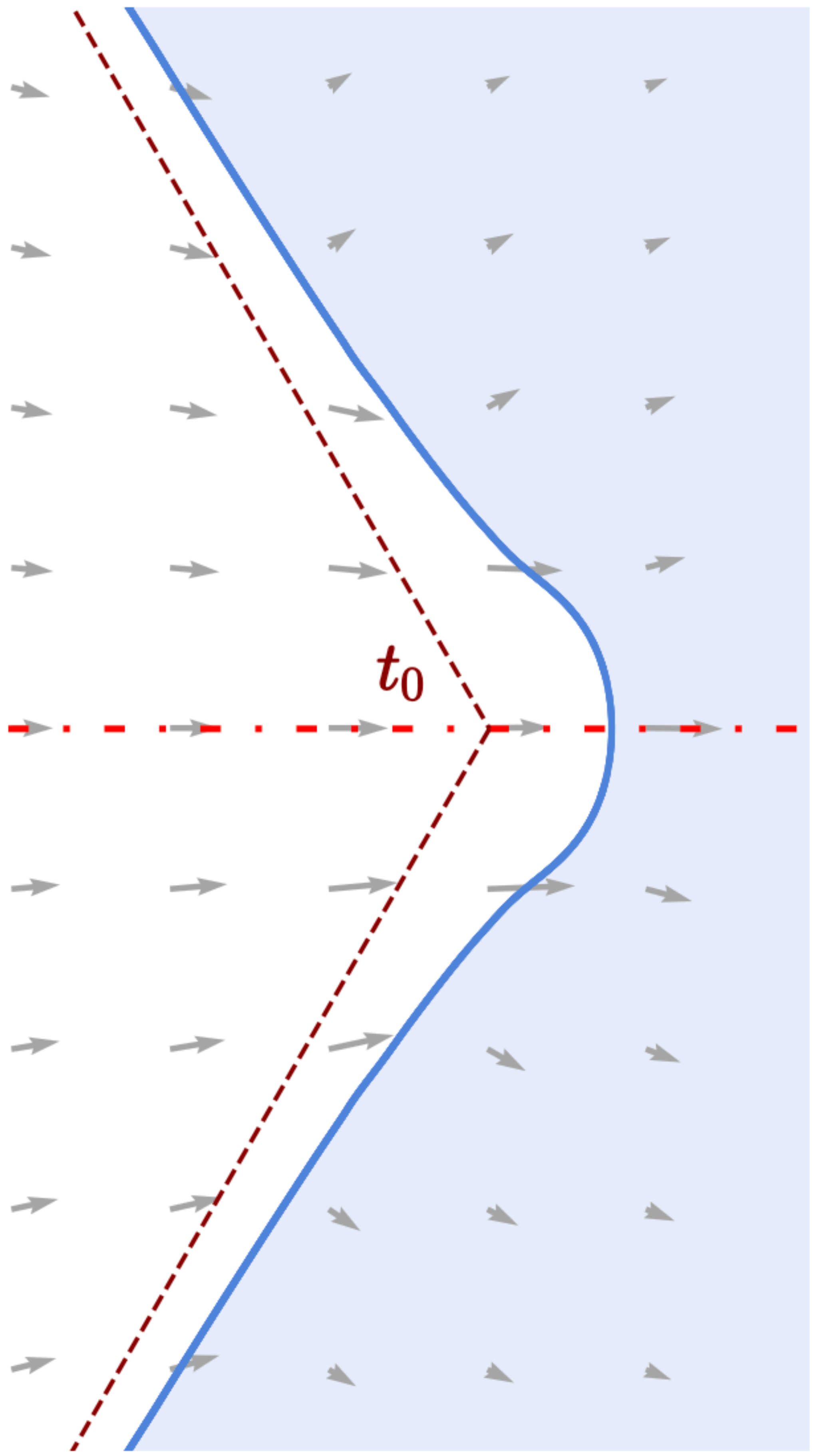
Zeff *et al.* (2000). *Singularity dynamics in curvature collapse and jet eruption on fluid surface.* Nature **403**



- **Topological diversity of cavities**
- **When the flow viscosity increases: *blunt tip* of the cone**
- **Jet velocity does not diverge**

Zeff's "ultraviolet cutoff": *viscosity as a regularization mechanism?*

IV - Conical Collapsing Cavities



$t \nearrow \; (> t_0), \quad \widetilde{\mu}_0 > 0$

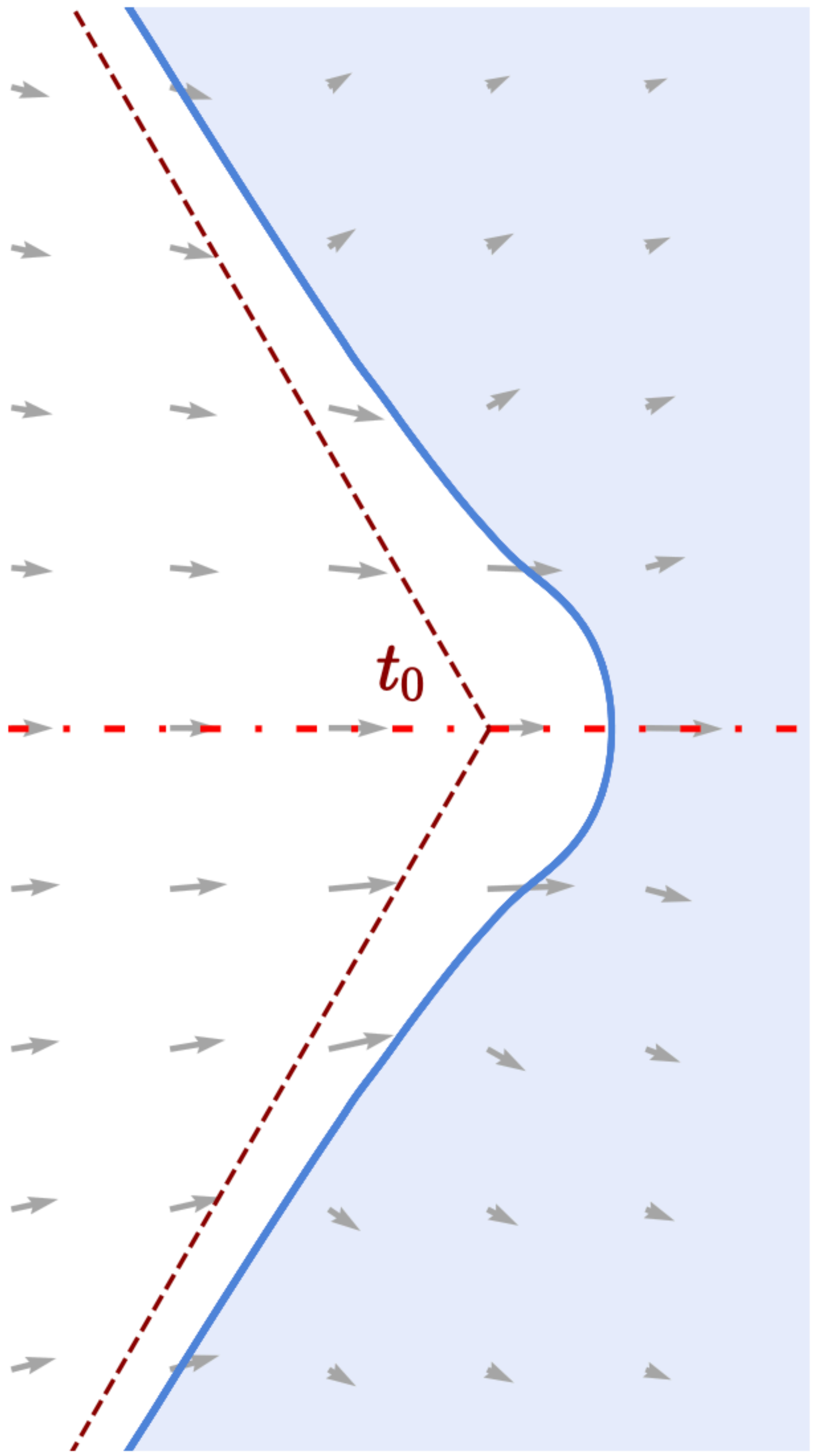
Recoil of a singular finite-time cone

Sierou & Lister (2004)

With the change of variables:

$$(t - t_0) \rightarrow (t_0 - t)$$

$$\Rightarrow \mathbf{u} \rightarrow -\mathbf{u}, \quad \tilde{\mu}_0 \rightarrow -\tilde{\mu}_0$$



$t \nearrow \ (> t_0), \quad \tilde{\mu}_0 > 0$

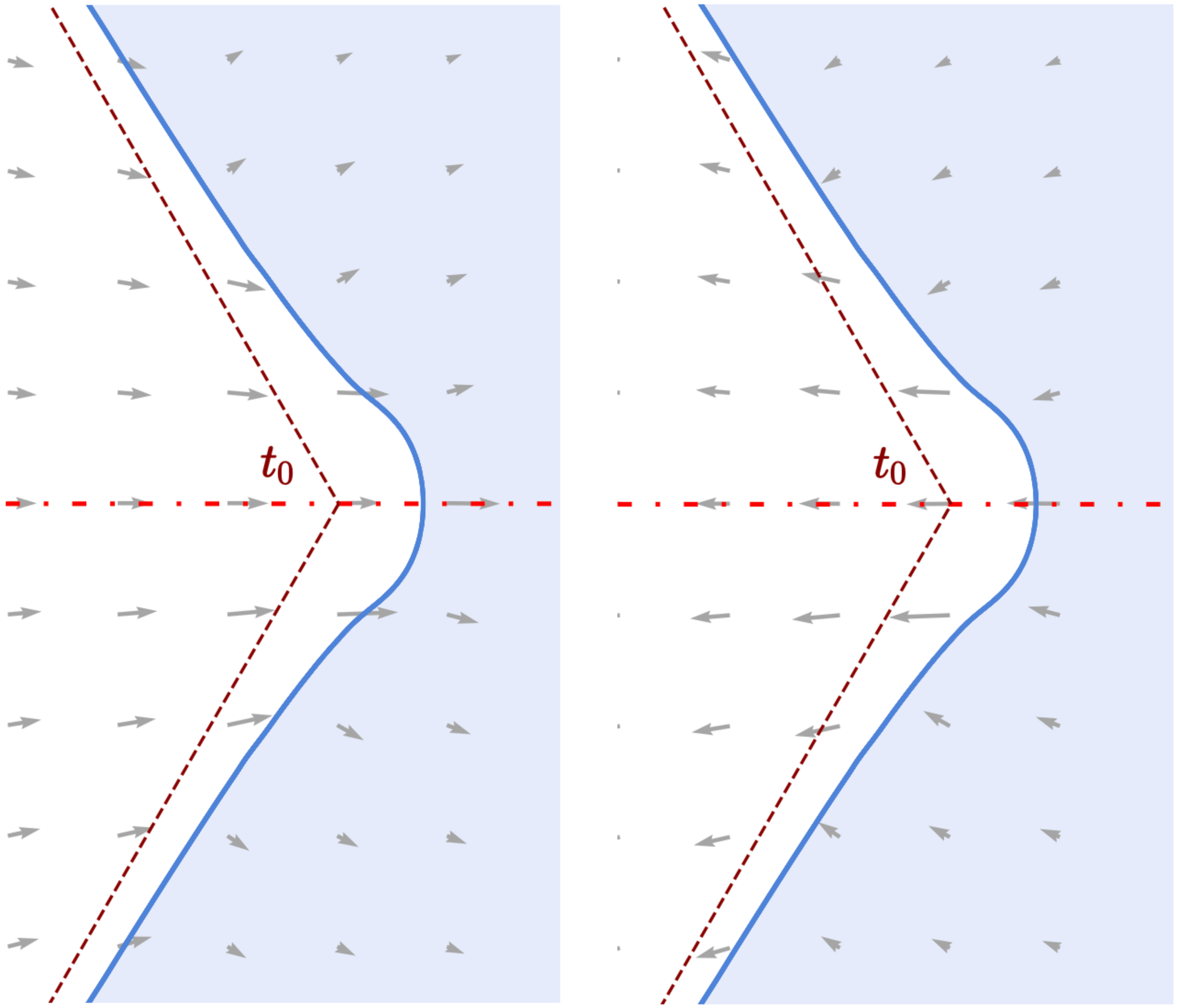
Recoil of a singular finite-time cone

Sierou & Lister (2004)

With the change of variables:

$$(t - t_0) \rightarrow (t_0 - t)$$

$$\Rightarrow \mathbf{u} \rightarrow -\mathbf{u}, \quad \tilde{\mu}_0 \rightarrow -\tilde{\mu}_0$$



$t \nearrow \ (> t_0), \quad \tilde{\mu}_0 > 0$

Recoil of a singular finite-time cone

$t < t_0, \quad \tilde{\mu}_0 < 0$

Time reversal: *cavity collapse singular at finite-time*

IV - Conical Collapsing Cavities

Sierou & Lister (2004)

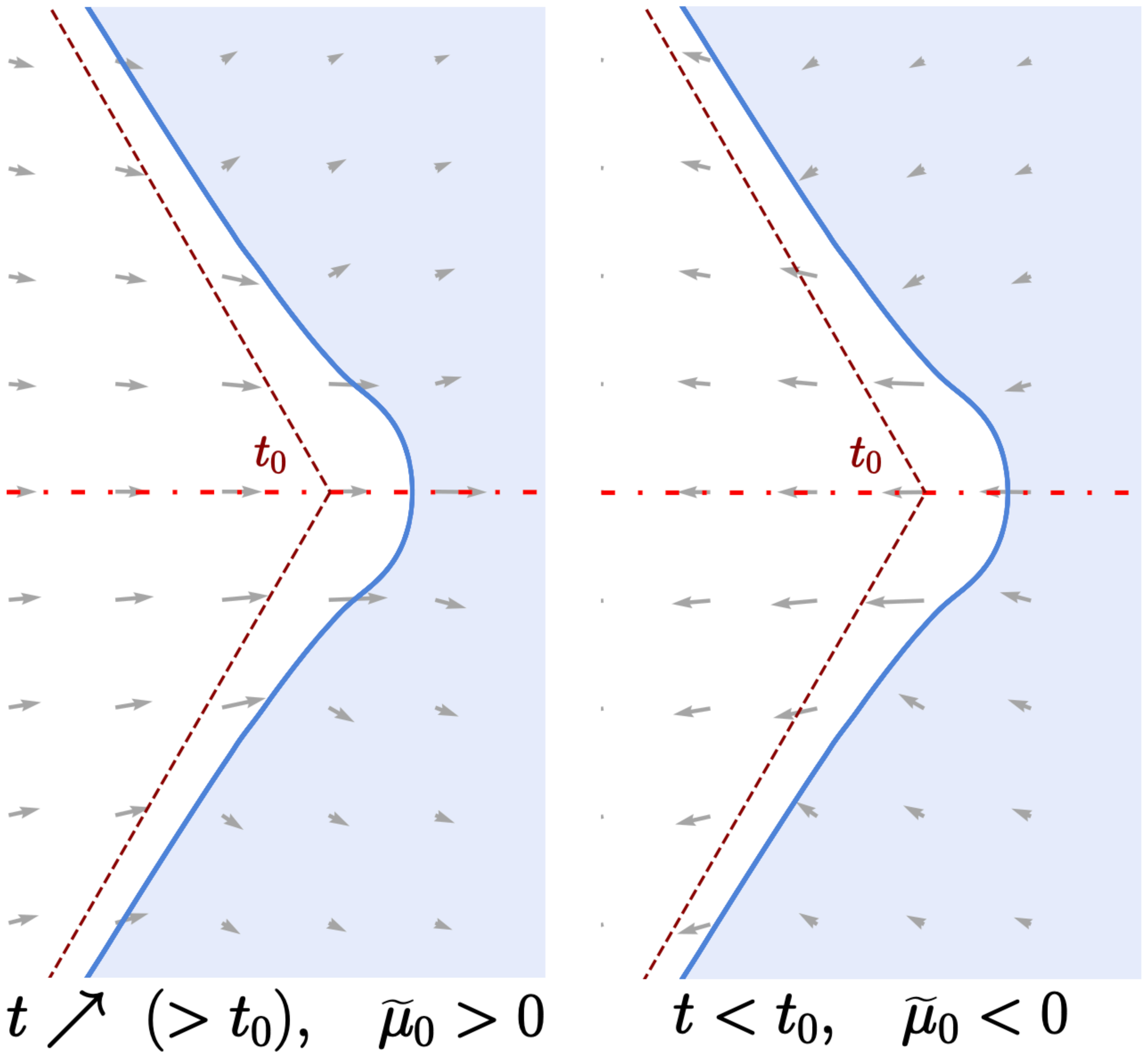
With the change of variables:

$$(t - t_0) \to (t_0 - t)$$

$$\Rightarrow \mathbf{u} \to -\mathbf{u}, \quad \tilde{\mu}_0 \to -\tilde{\mu}_0$$

Cavity collapses of bursting bubbles are the ***time reversal*** of recoiling cones with $\theta_0 > 90°$

Dipolar flow ↔ *Draining flow*



Recoil of a singular finite-time cone

Time reversal: *cavity collapse singular at finite-time*

IV - Conical Collapsing Cavities

Sierou & Lister (2004)

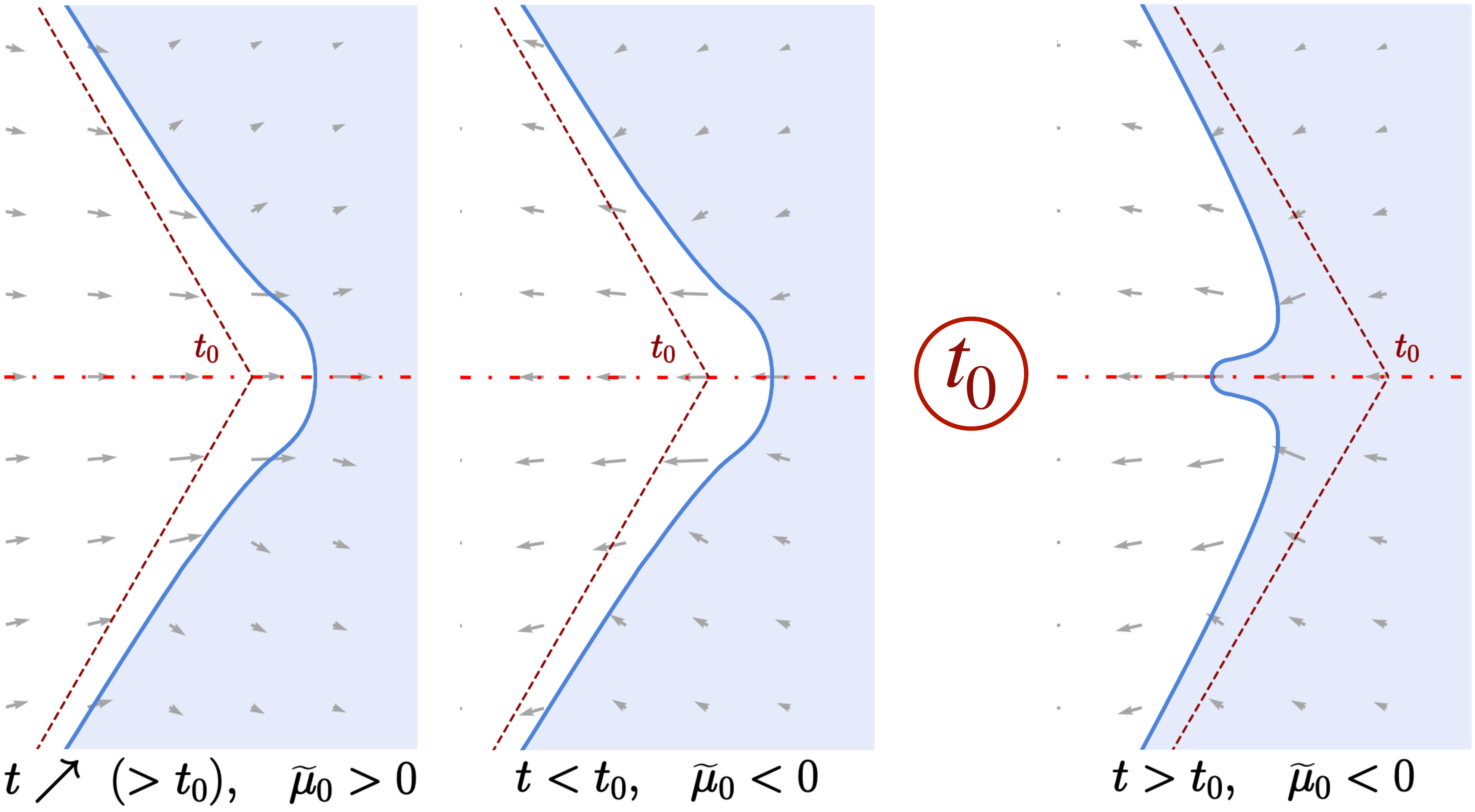
With the change of variables:

$$(t - t_0) \to (t_0 - t)$$

$$\Rightarrow \mathbf{u} \to -\mathbf{u}, \quad \tilde{\mu}_0 \to -\tilde{\mu}_0$$

Cavity collapses of bursting bubbles are the ***time reversal*** of recoiling cones with $\theta_0 > 90°$

Dipolar flow ↔ *Draining flow*



Recoil of a singular finite-time cone

***Time reversal:** cavity collapse singular at finite-time*

*Post-singular collapse: **jet?***

Sierou & Lister (2004)

With the change of variables:

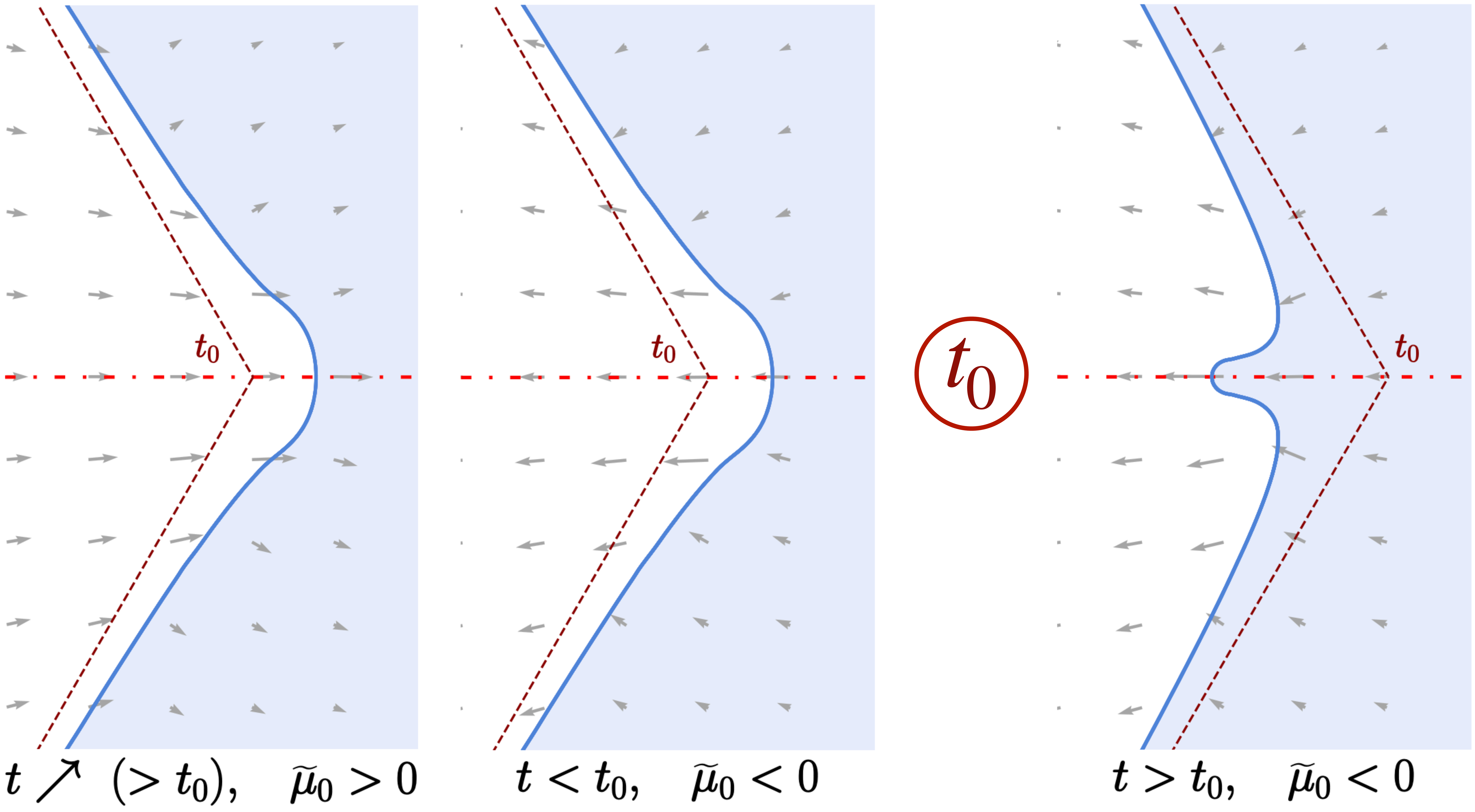
$$(t - t_0) \rightarrow (t_0 - t)$$

$$\Rightarrow \mathbf{u} \rightarrow -\mathbf{u}, \quad \widetilde{\mu}_0 \rightarrow -\widetilde{\mu}_0$$

Cavity collapses of bursting bubbles are the ***time reversal*** of recoiling cones with $\theta_0 > 90°$

Dipolar flow ↔ *Draining flow*

Test at: $|\widetilde{\mu}_0| = 50$

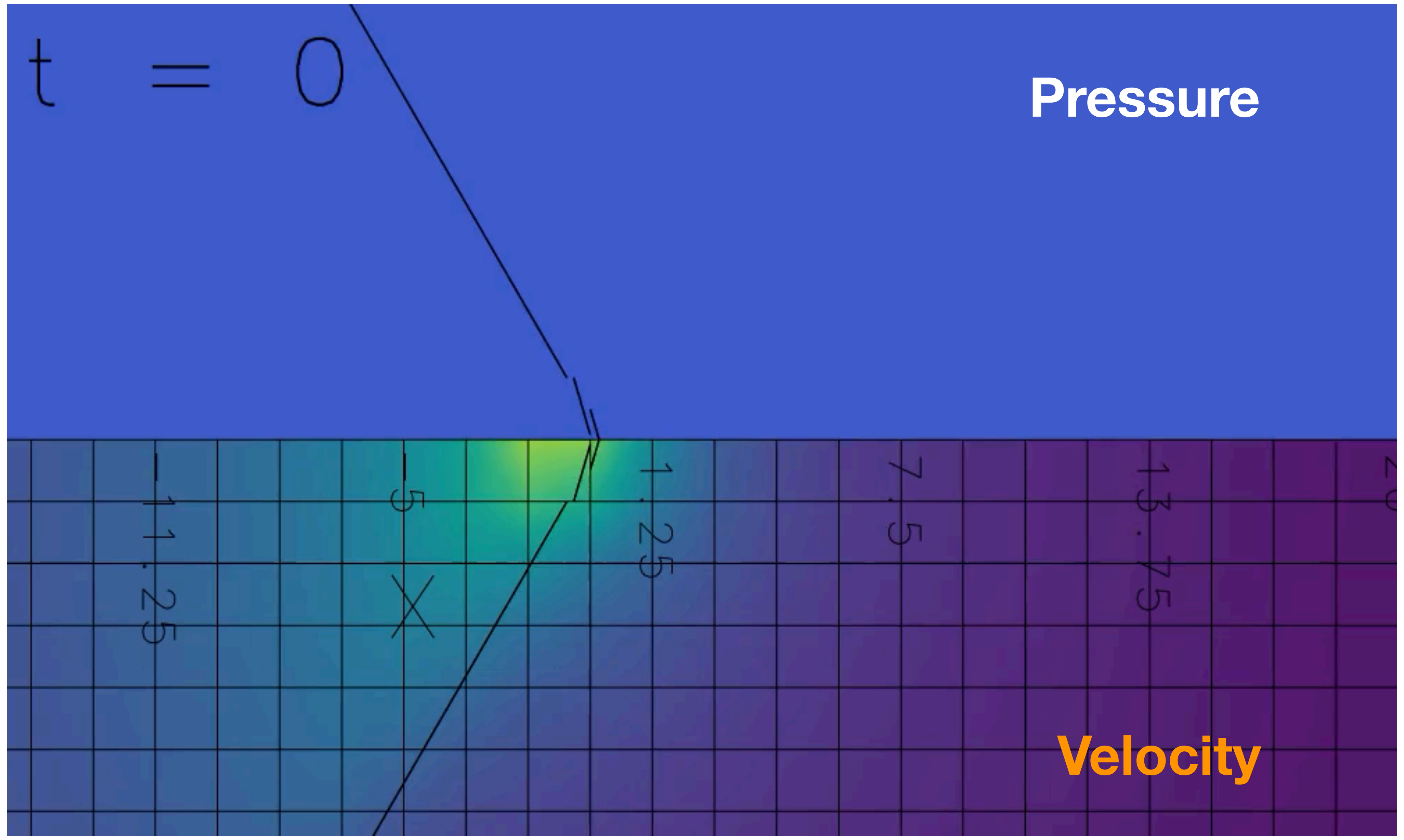


Recoil of a singular finite-time cone

Time reversal: *cavity collapse singular at finite-time*

Post-singular collapse: ***jet?***

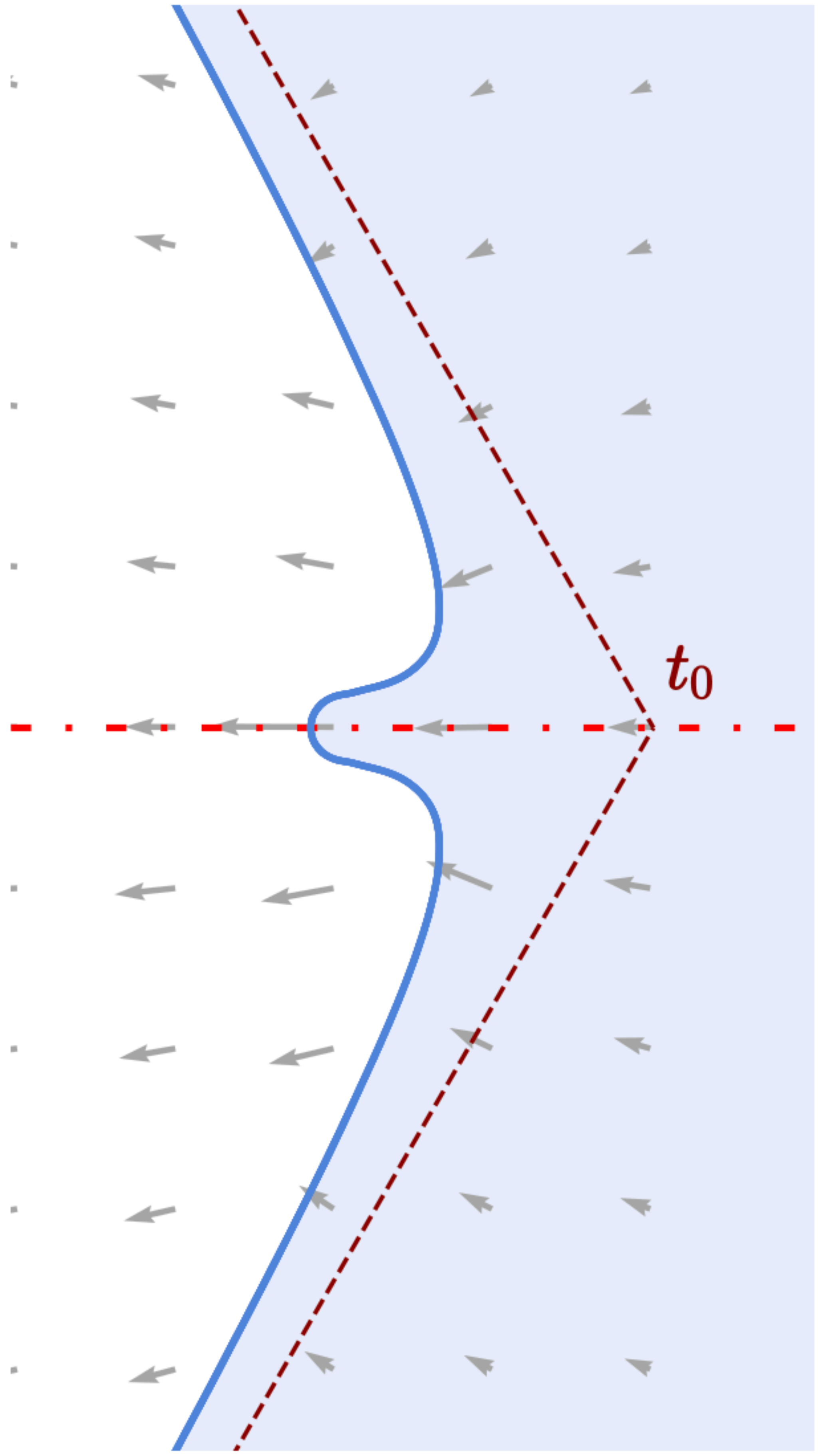
Principle of the simulation



$$t \nearrow (>t_0),\quad \widetilde{\mu}_0 > 0$$

Recoil of a singular finite-time cone

t_0



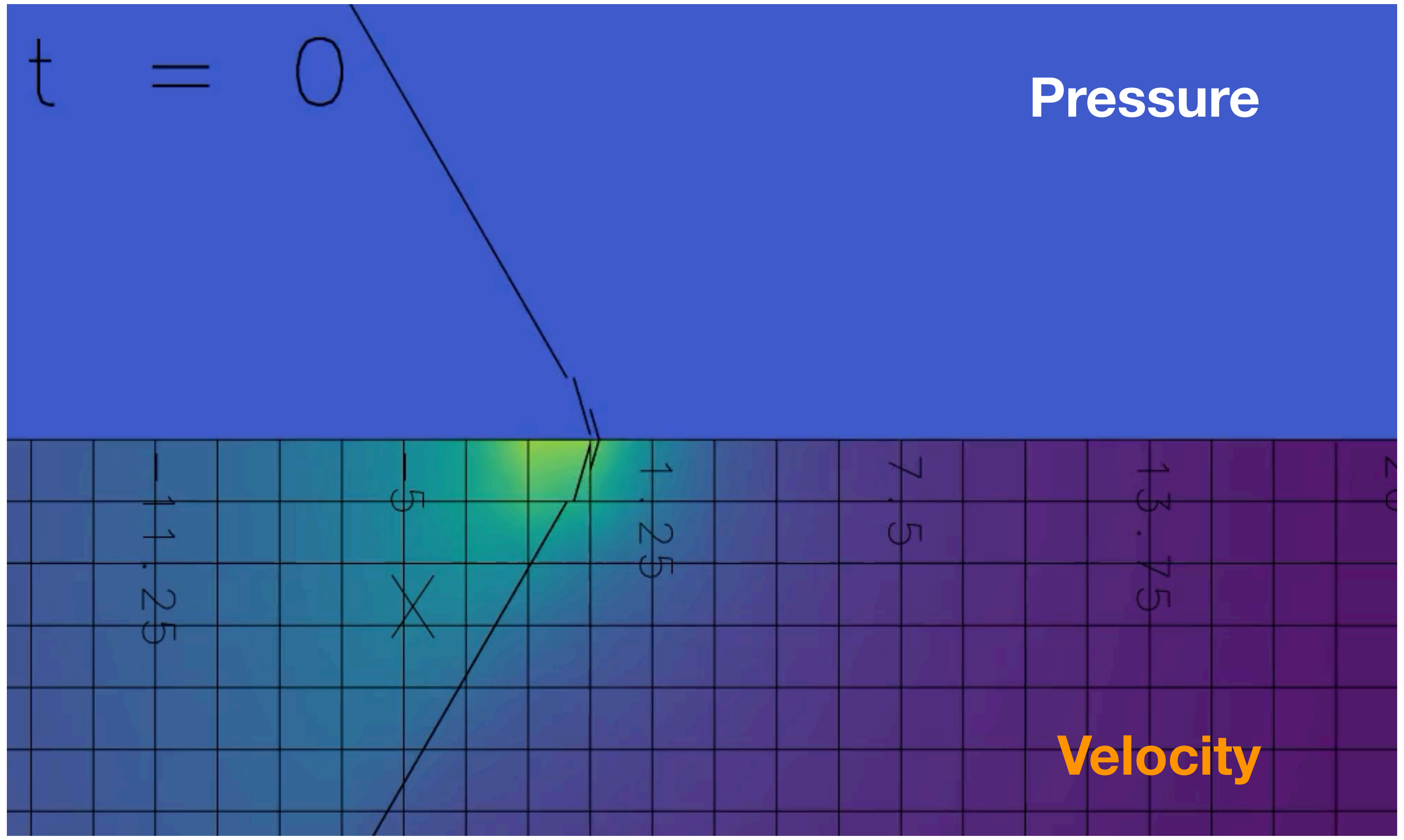
$$t > t_0, \quad \widetilde{\mu}_0 < 0$$

Post-singular collapse: ***jet?***

IV - Conical Collapsing Cavities

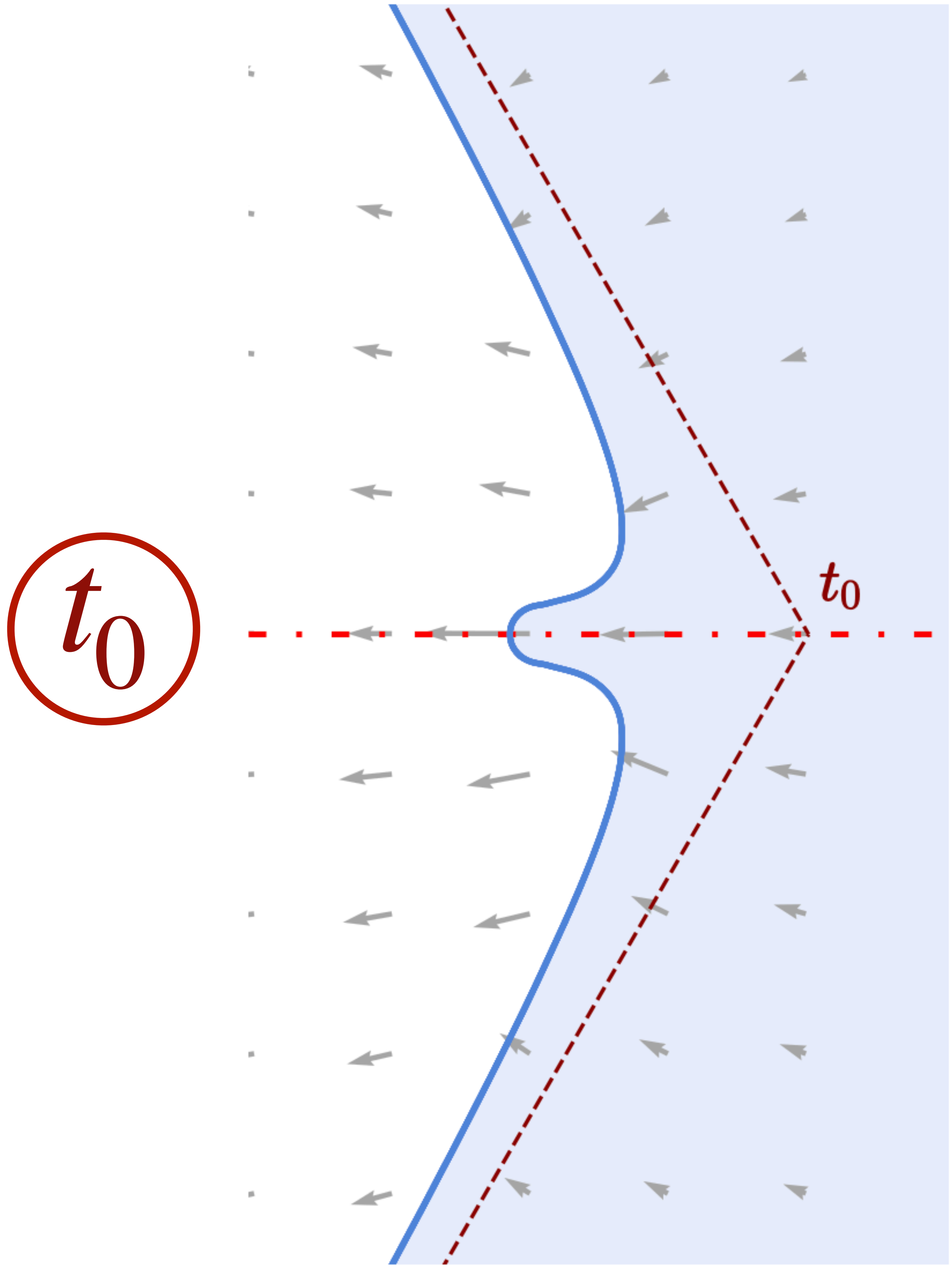
IV.3 - Inviscid Sim. don't cross singularity

Principle of the simulation



$t \nearrow (> t_0), \quad \widetilde{\mu}_0 > 0$

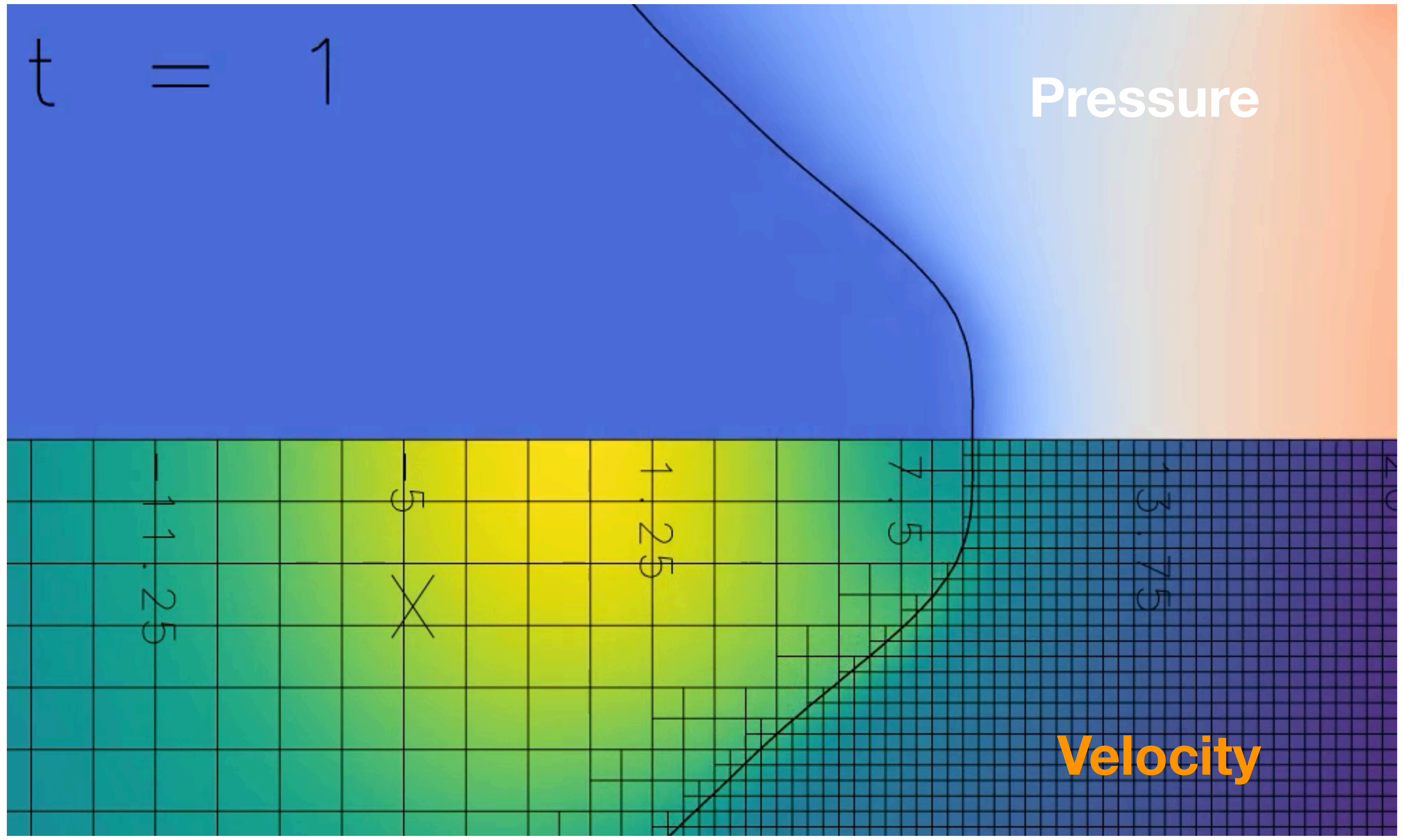
Recoil of a singular finite-time cone



$t > t_0, \quad \widetilde{\mu}_0 < 0$

*Post-singular collapse: **jet?***

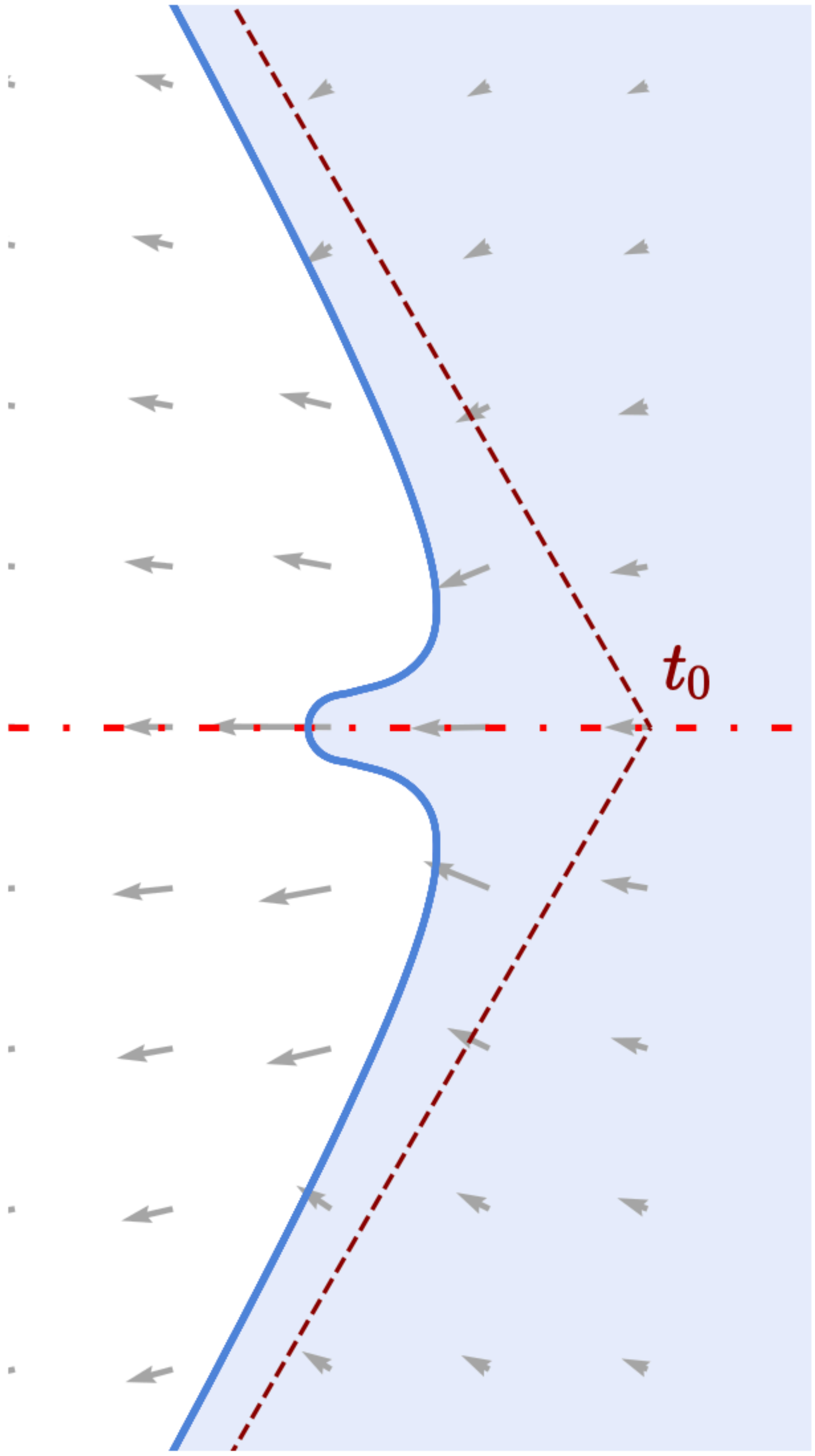
Principle of the simulation



$t < t_0, \quad \widetilde{\mu}_0 < 0$

Time reversal: *cavity collapse*

t_0

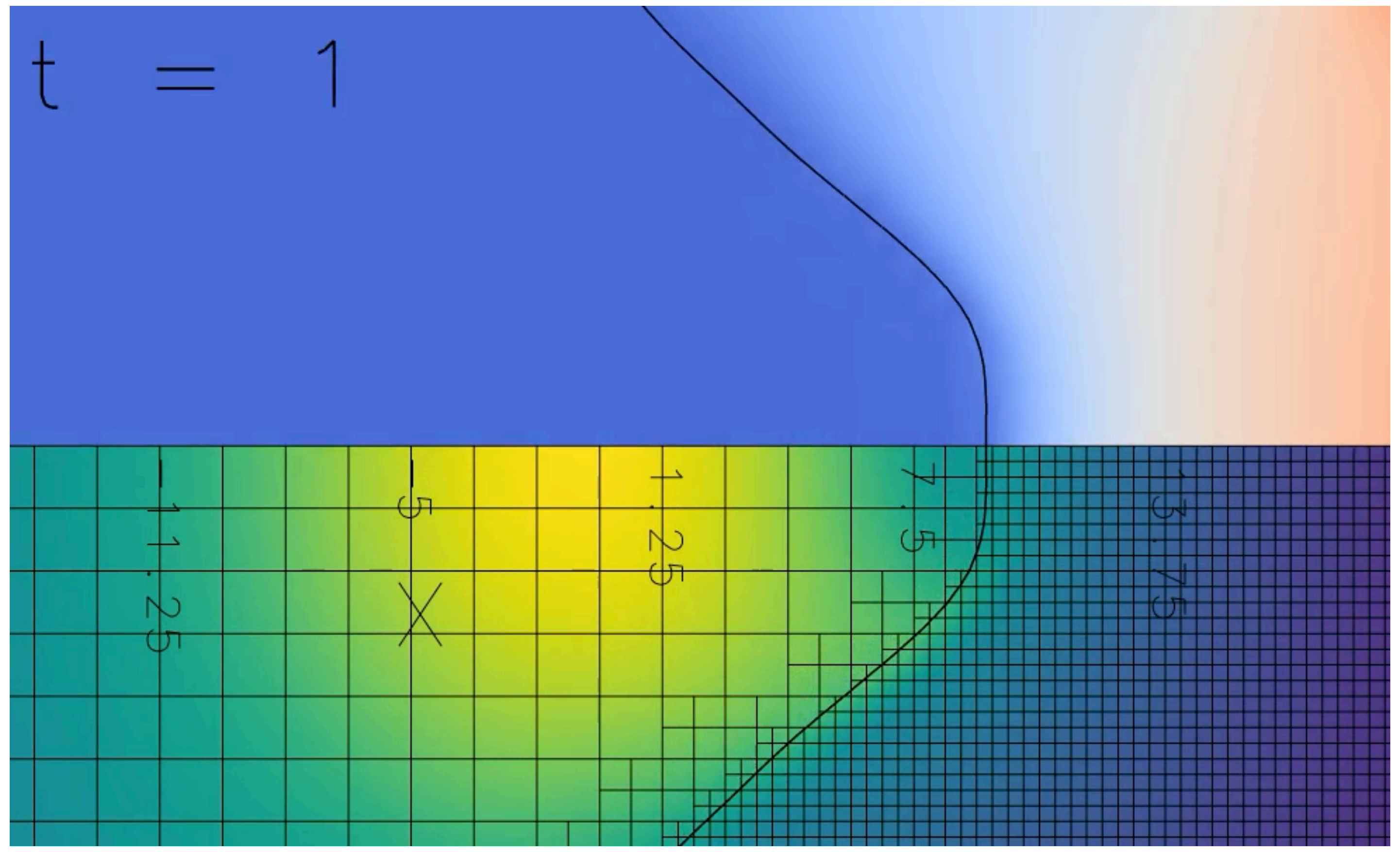


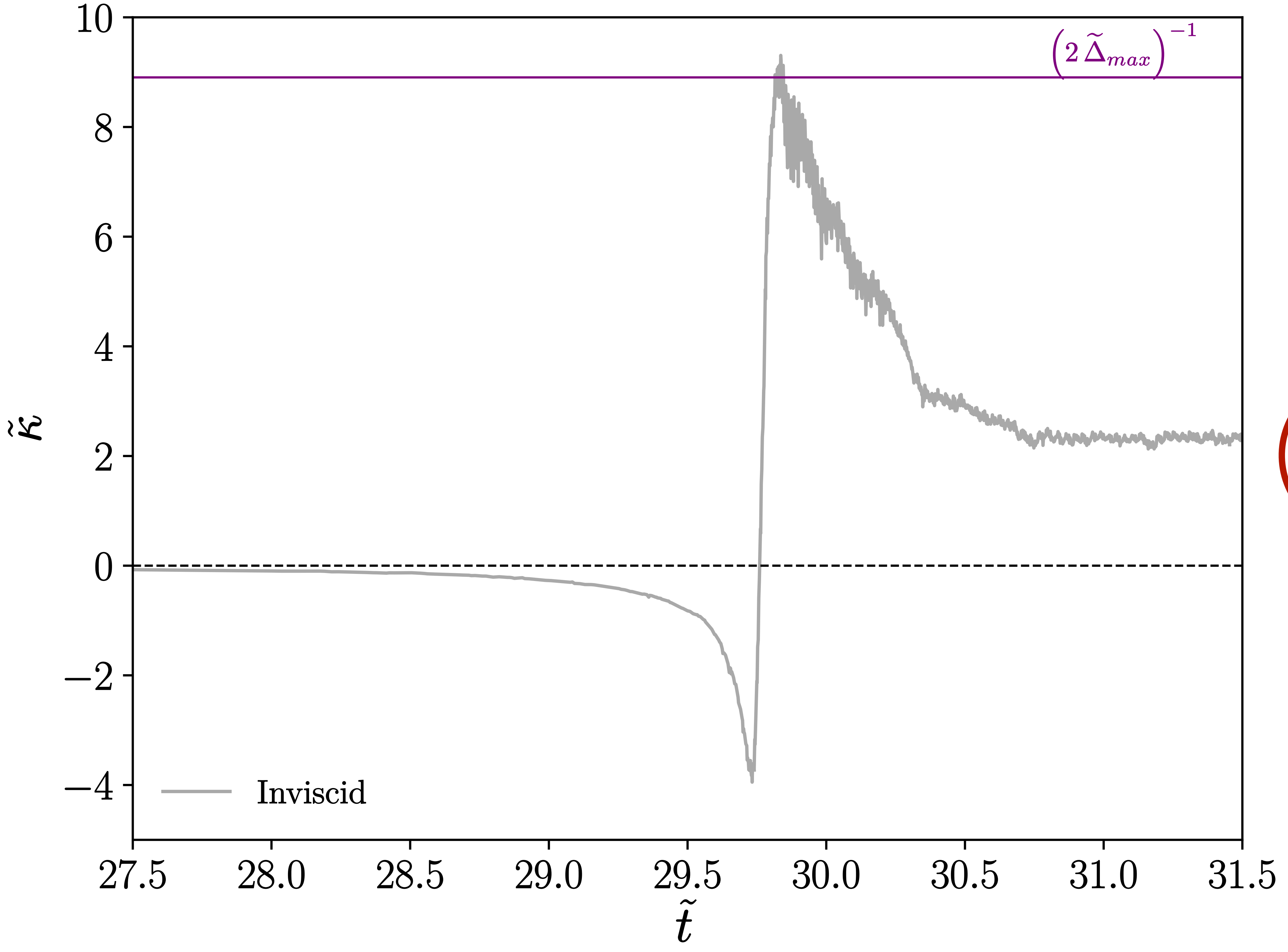
$t > t_0, \quad \widetilde{\mu}_0 < 0$

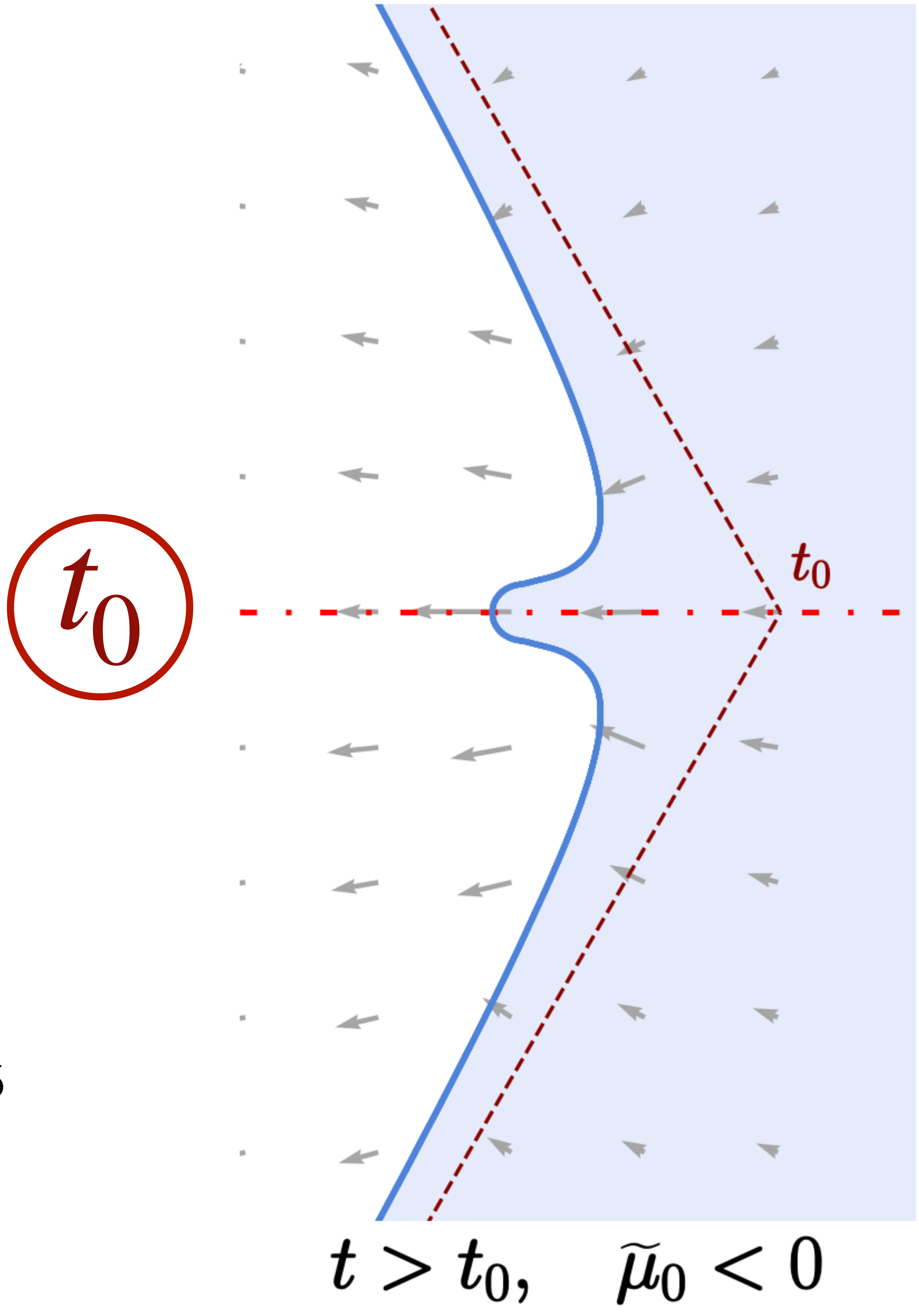
Post-singular collapse: ***jet?***

IV - Conical Collapsing Cavities

IV.3 - Inviscid Sim. don't cross singularity

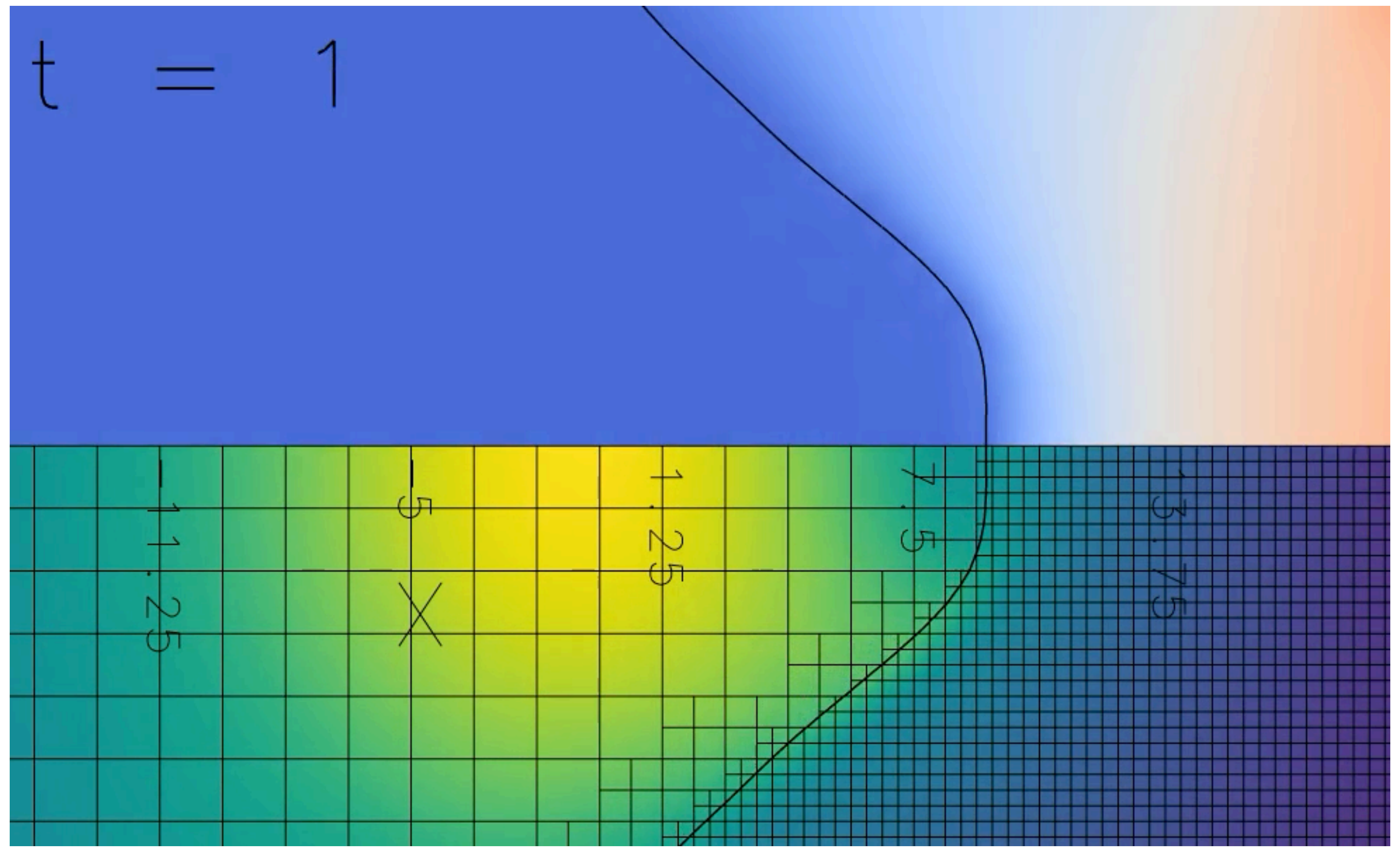


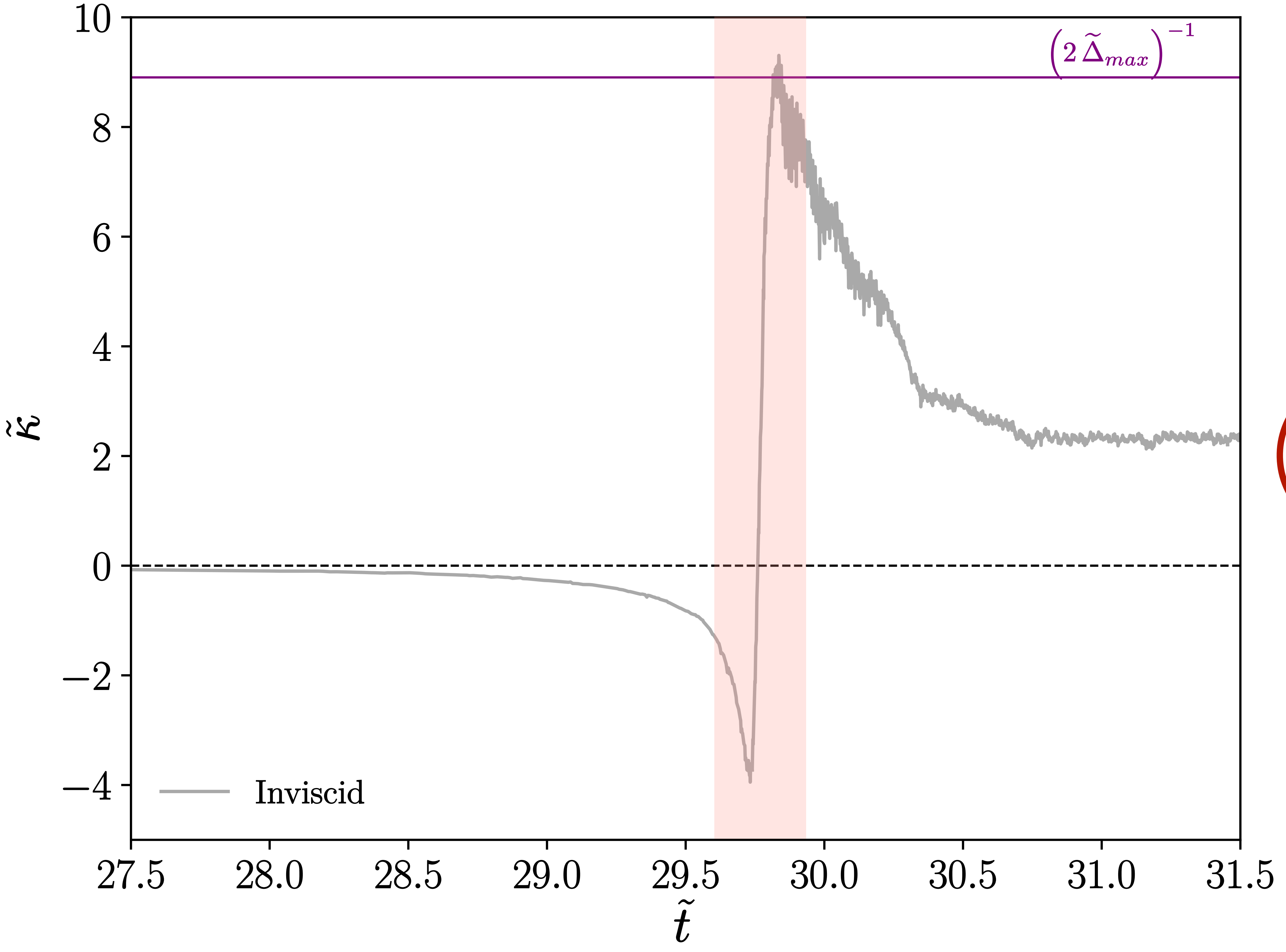


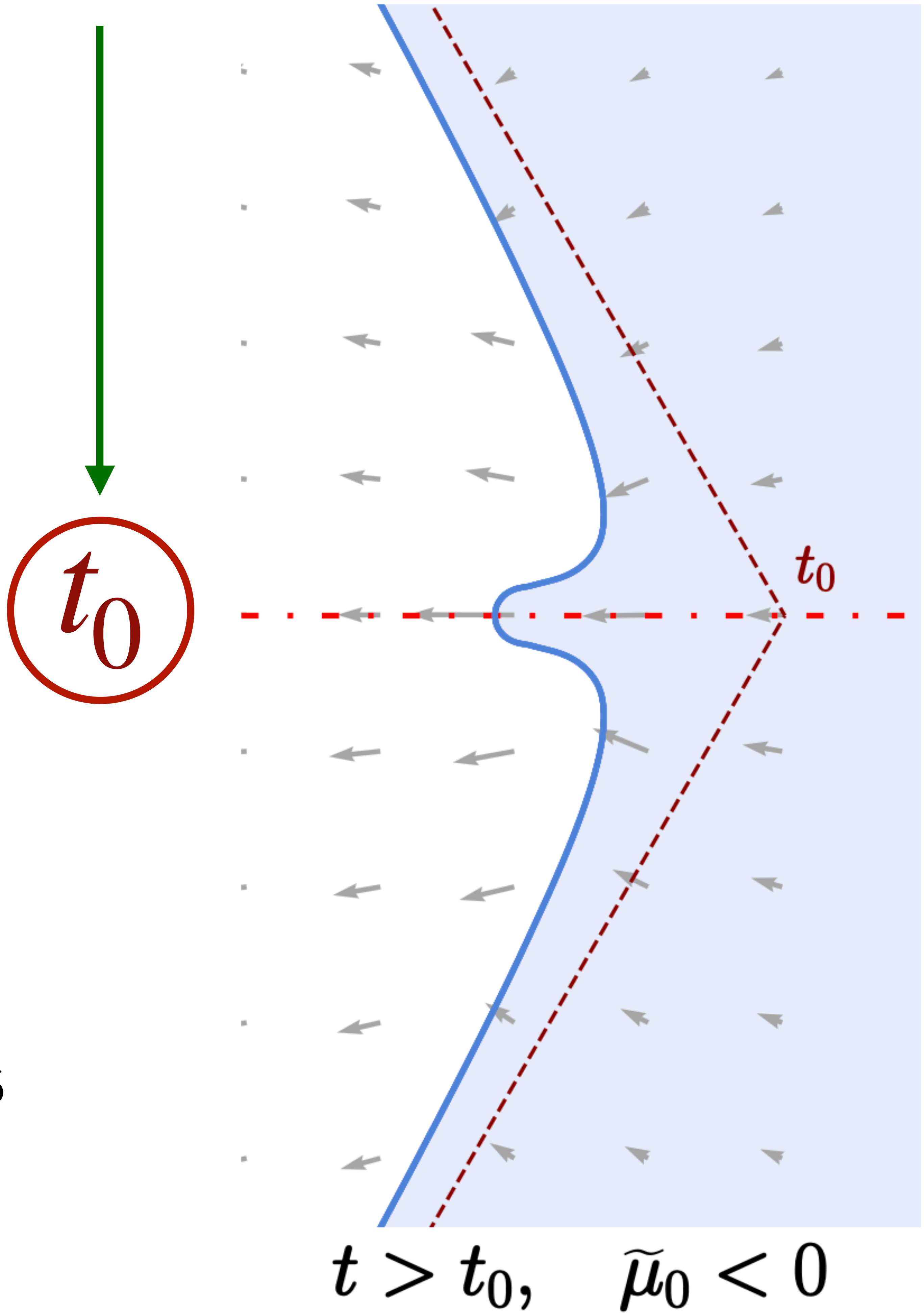


*Post-singular collapse: **jet?***

Viscosity for passing through the singularity?

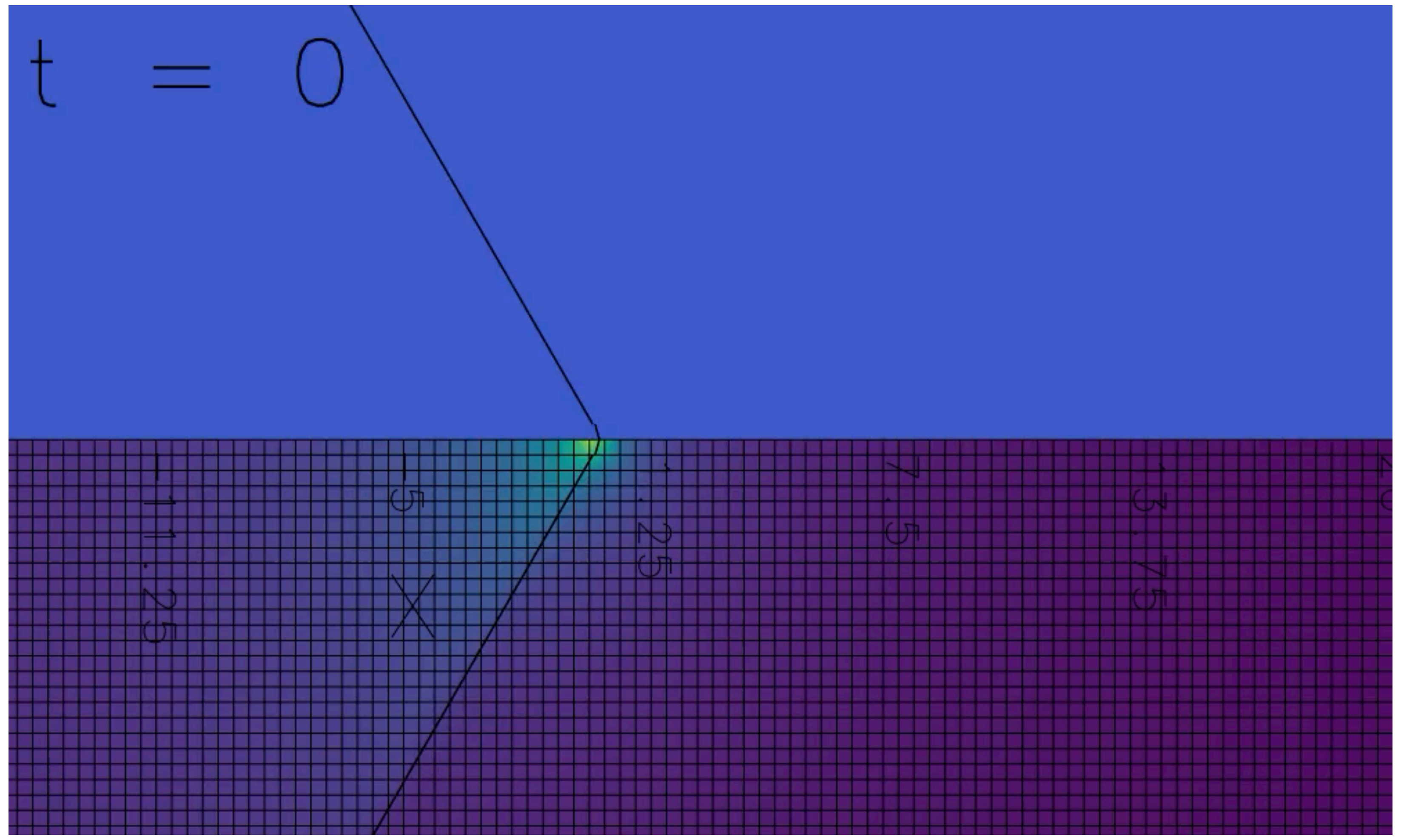


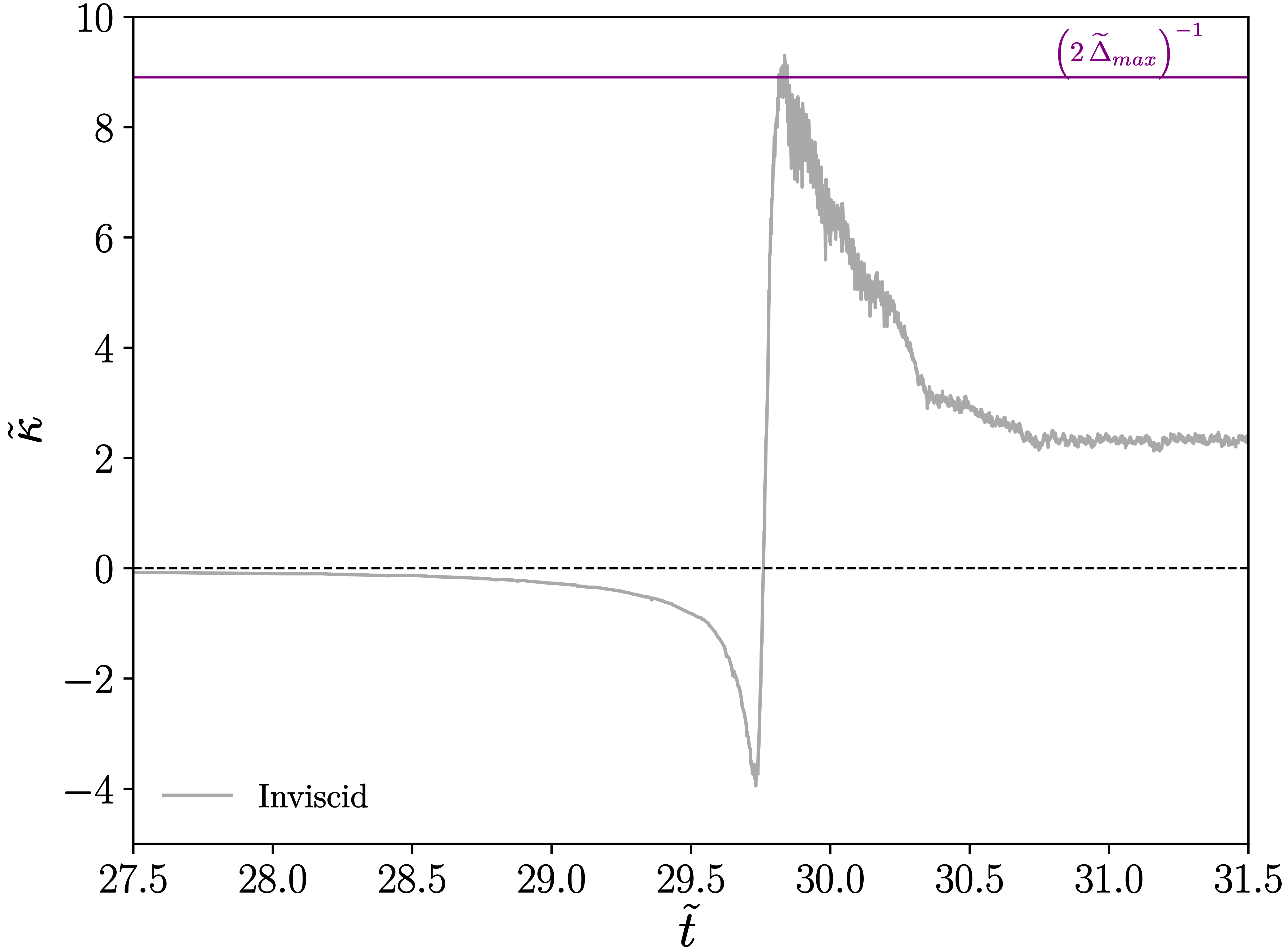




*Post-singular collapse: **jet?***

Goal: to catch a *transitory regime* towards *viscous effects* close to t_0



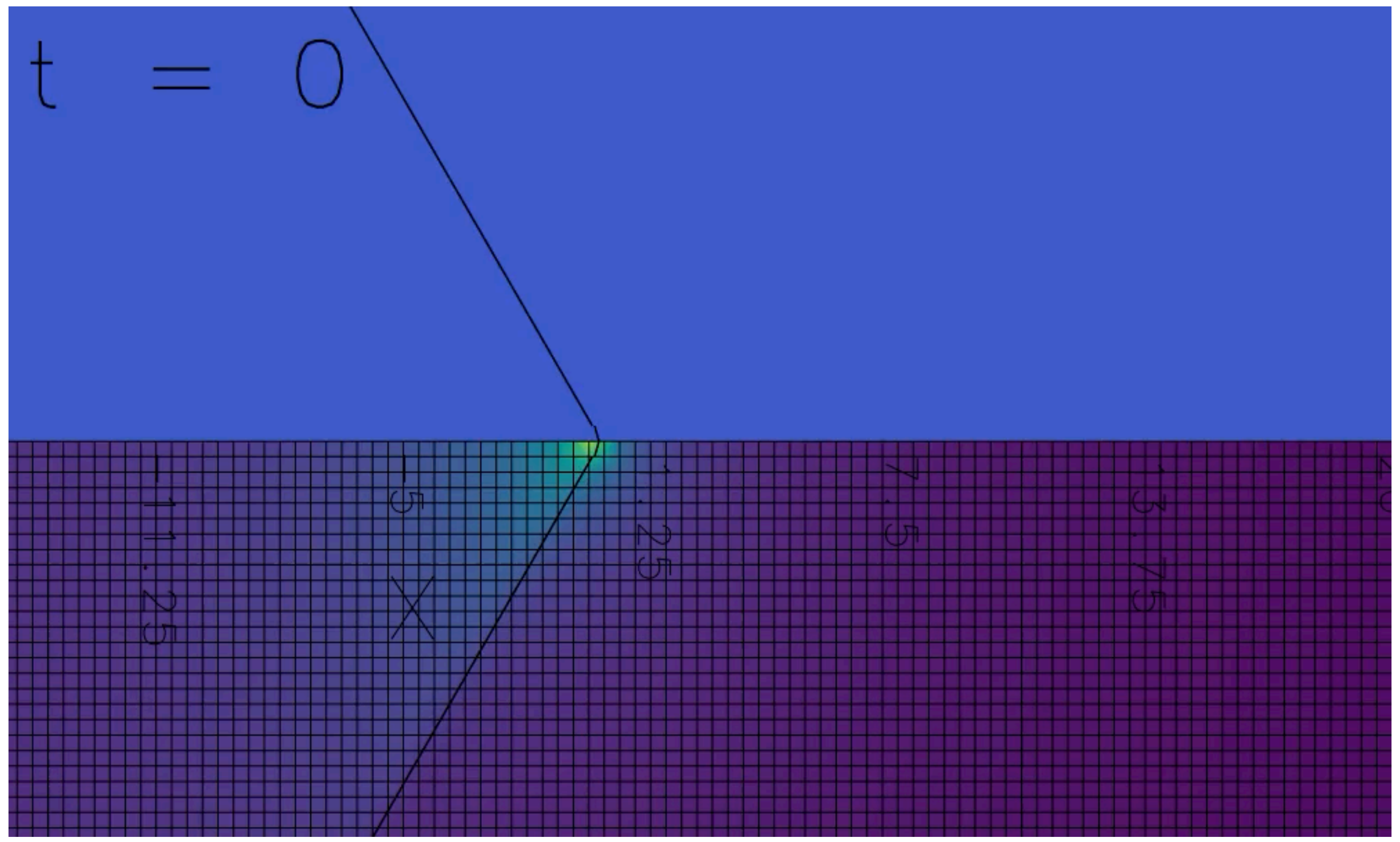


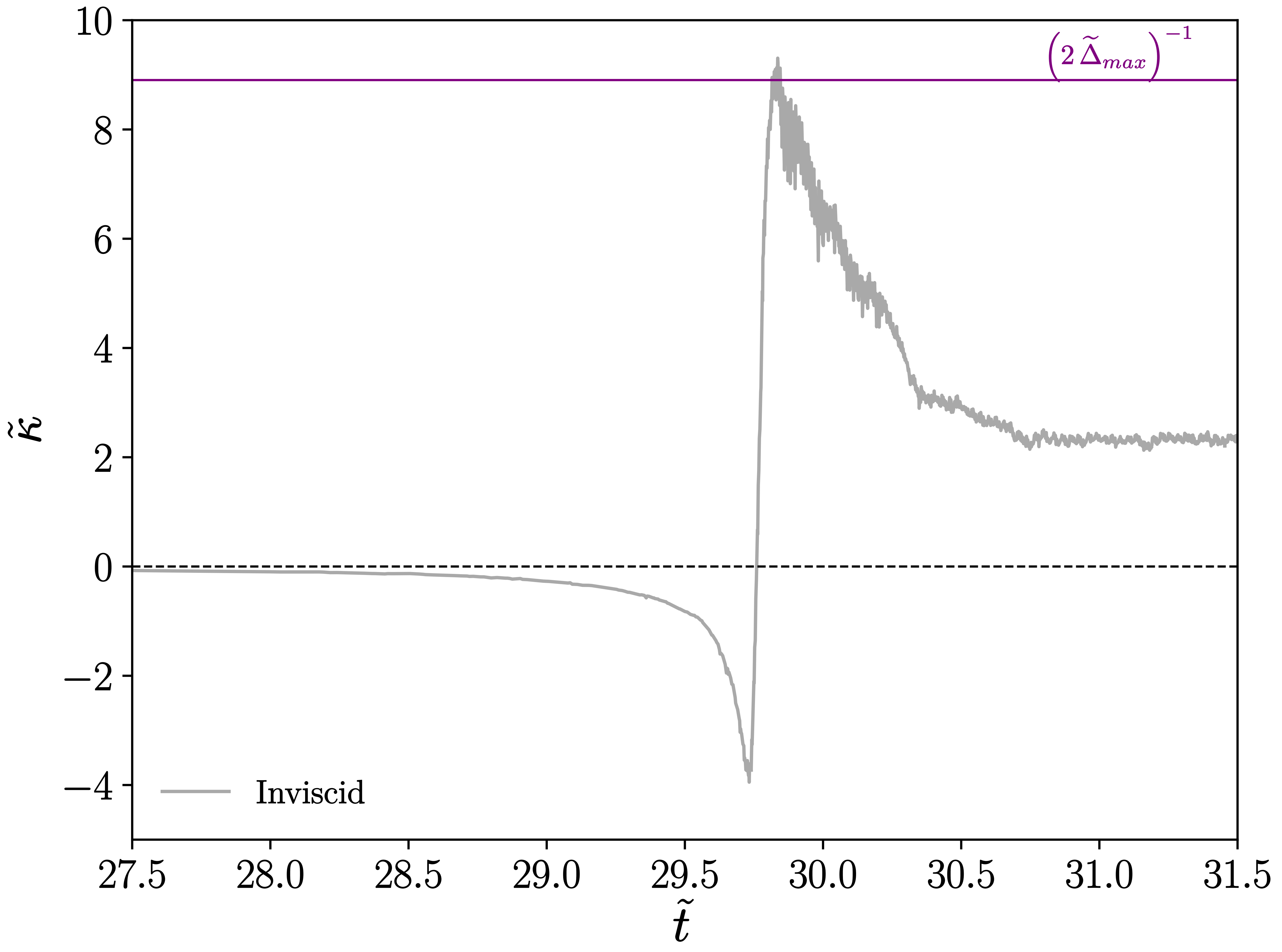
<u>Goal:</u> to catch a *transitory regime* towards *viscous effects* close to t_0

1. Problem non-dimensionalized with viscous scales:

$$\ell_\mu = \frac{{\mu_l}^2}{\rho_l \sigma}$$ ~ water: **10 nm** / oil: **100 µm**

$$t_\mu = \frac{{\mu_l}^3}{\rho_l \sigma^2}$$ ~ water: **100 ps** / oil: **100 µs**





<u>Goal:</u> to catch a *transitory regime* towards *viscous effects* close to t_0

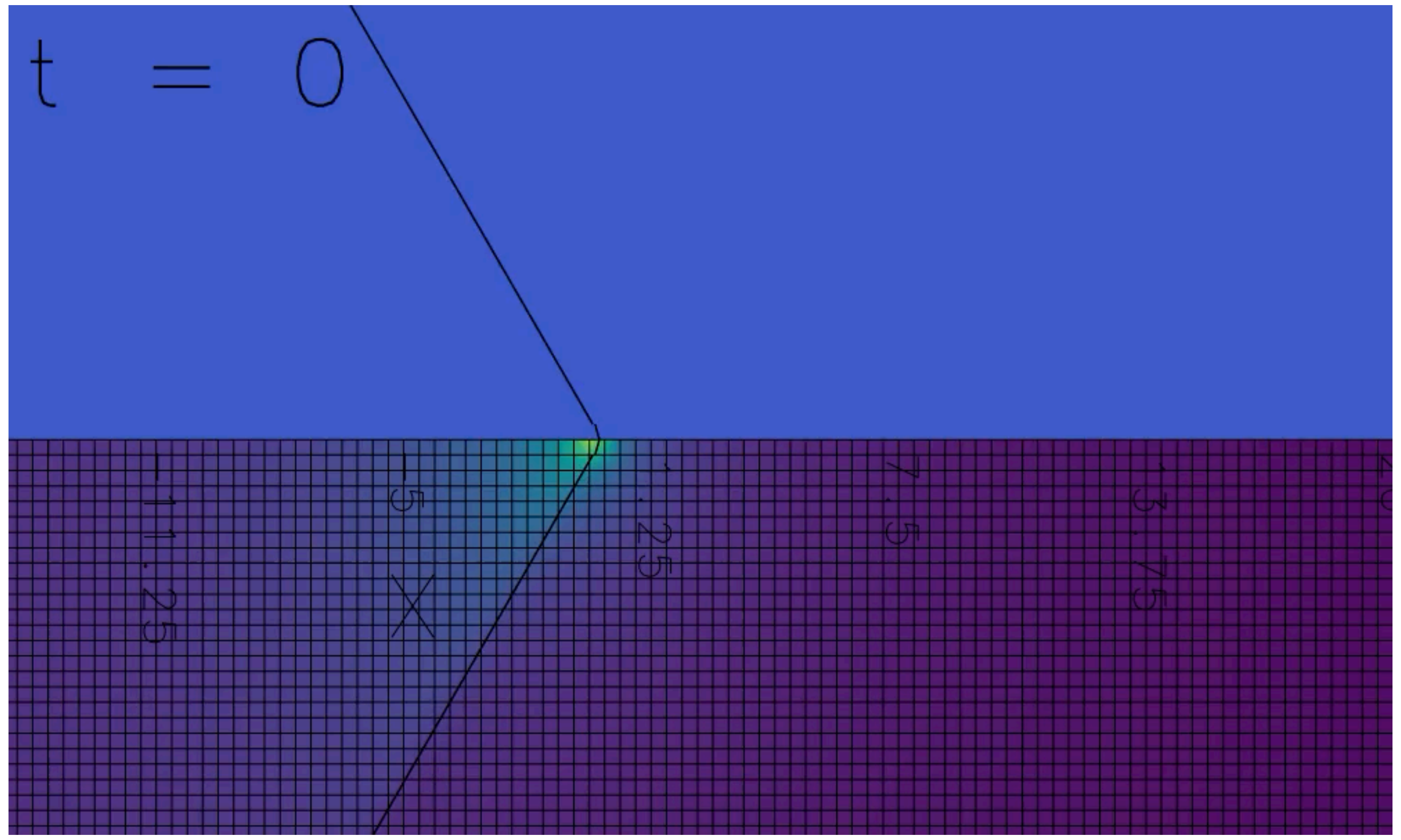
1. Problem non-dimensionalized with viscous scales:

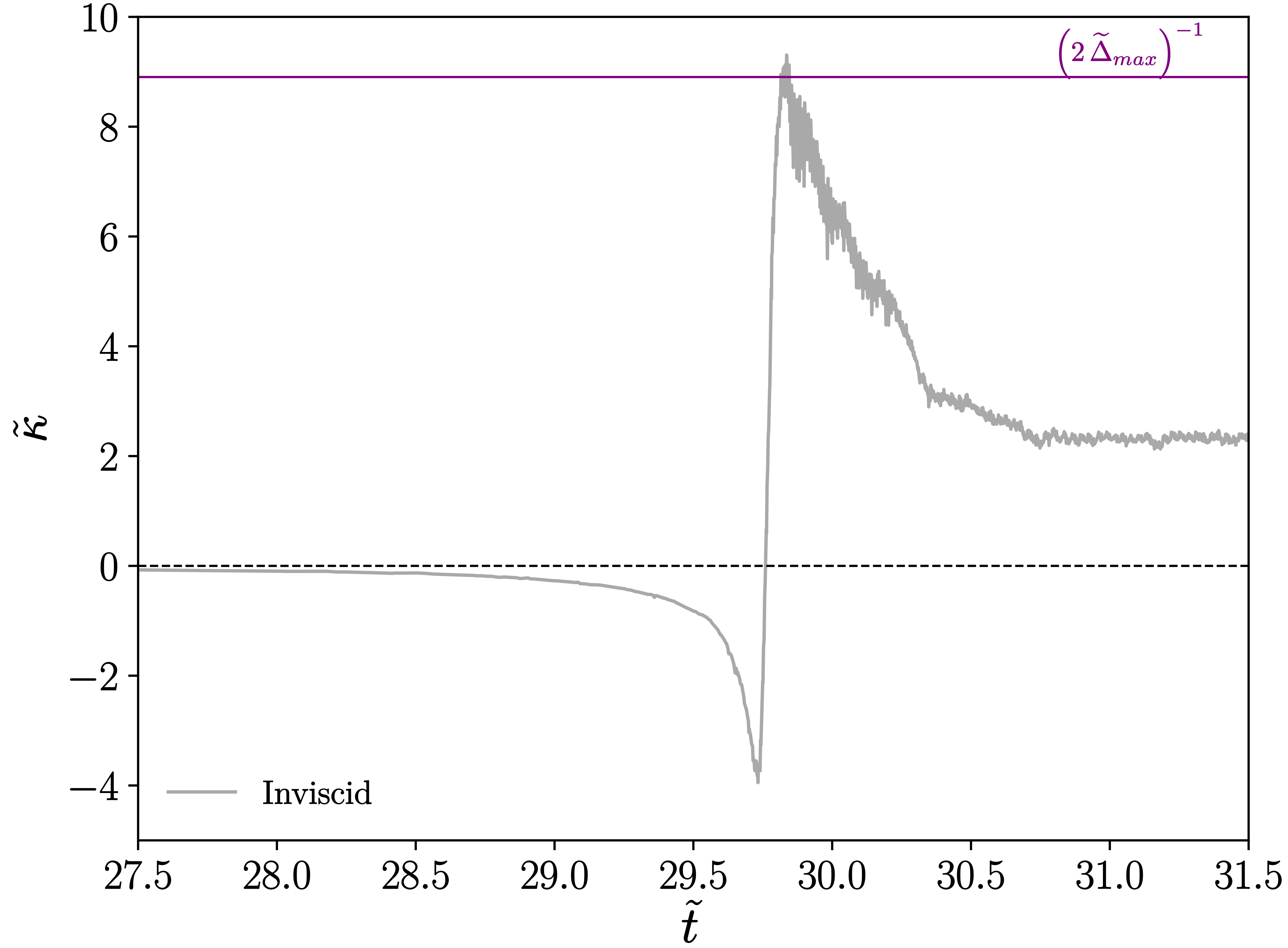
$$\ell_\mu = \frac{{\mu_l}^2}{\rho_l \sigma}$$ ~ water: **10 nm** / oil: **100 µm**

$$t_\mu = \frac{{\mu_l}^3}{\rho_l \sigma^2}$$ ~ water: **100 ps** / oil: **100 µs**

2. Take a size domain $\gg L = \ell_\mu$ $\rightarrow L = 230\,\ell_\mu$
[*to start in the cap. reg.*]

3. Take a grid resolution $\ll \ell_\mu$ (≈ 20 pts) $\rightarrow \Delta \approx 0.05\,\ell_\mu$





Goal: to catch a *transitory regime* towards *viscous effects* close to t_0

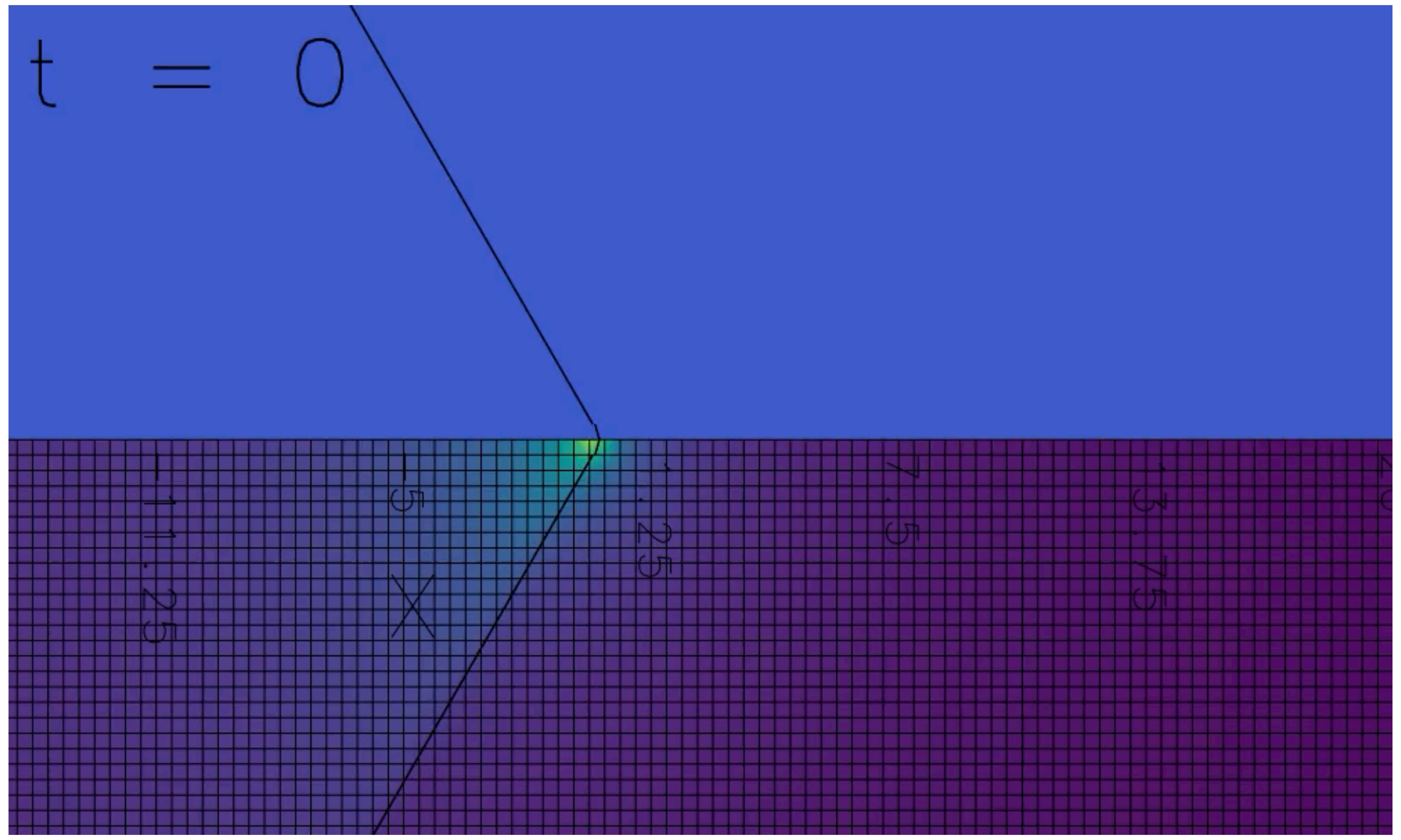
1. Problem non-dimensionalized with viscous scales:

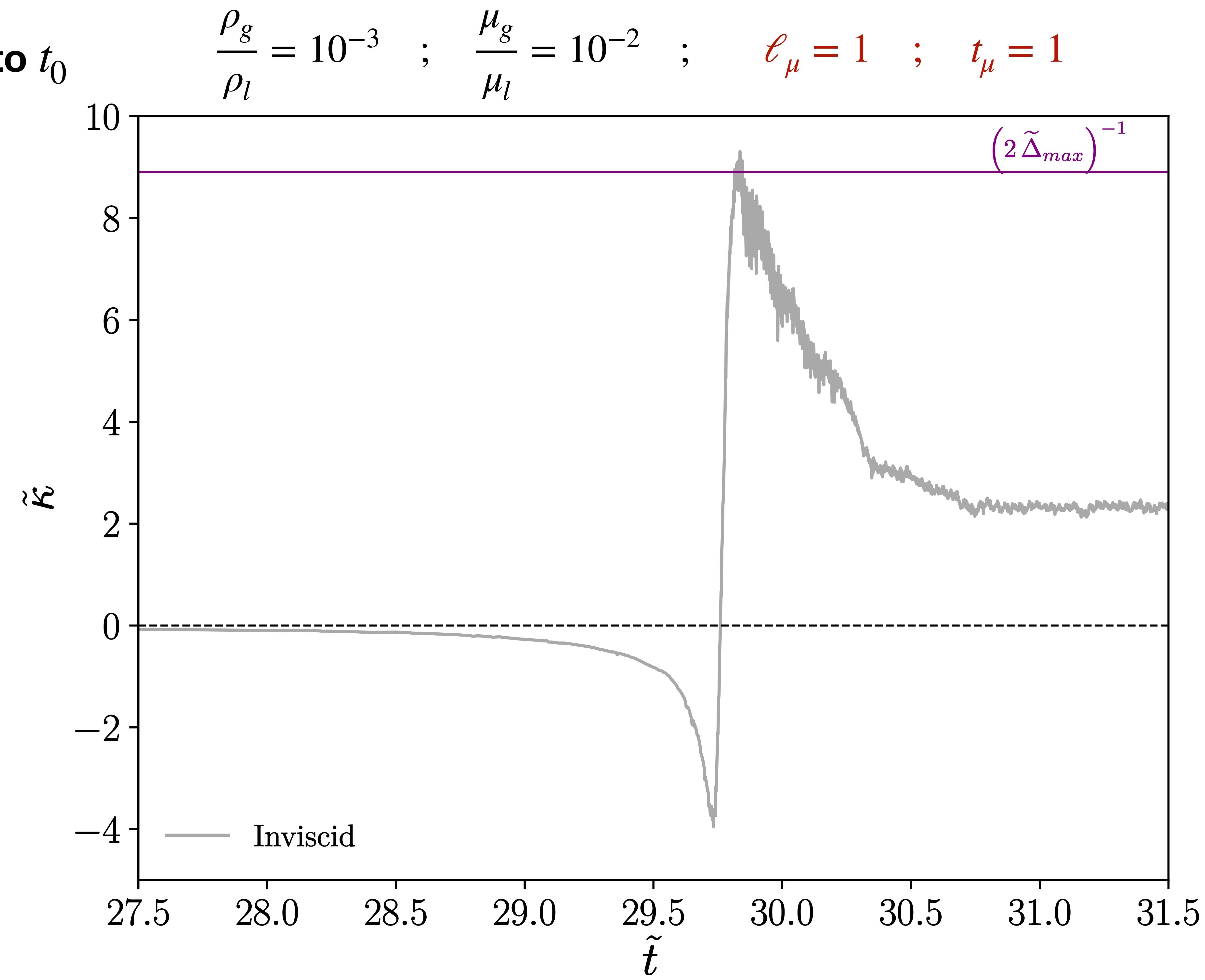
$$\ell_\mu = \frac{\mu_l^2}{\rho_l \sigma}$$ ~ water: **10 nm** / oil: **100 µm**

$$t_\mu = \frac{\mu_l^3}{\rho_l \sigma^2}$$ ~ water: **100 ps** / oil: **100 µs**

2. Take a size domain $\gg L = \ell_\mu$ $\rightarrow L = 230\,\ell_\mu$
[*to start in the cap. reg.*]

3. Take a grid resolution $\ll \ell_\mu$ (≈ 20 pts) $\rightarrow \Delta \approx 0.05\,\ell_\mu$





<u>Goal:</u> to catch a *transitory regime* towards *viscous effects* close to t_0

1. Problem non-dimensionalized with viscous scales:

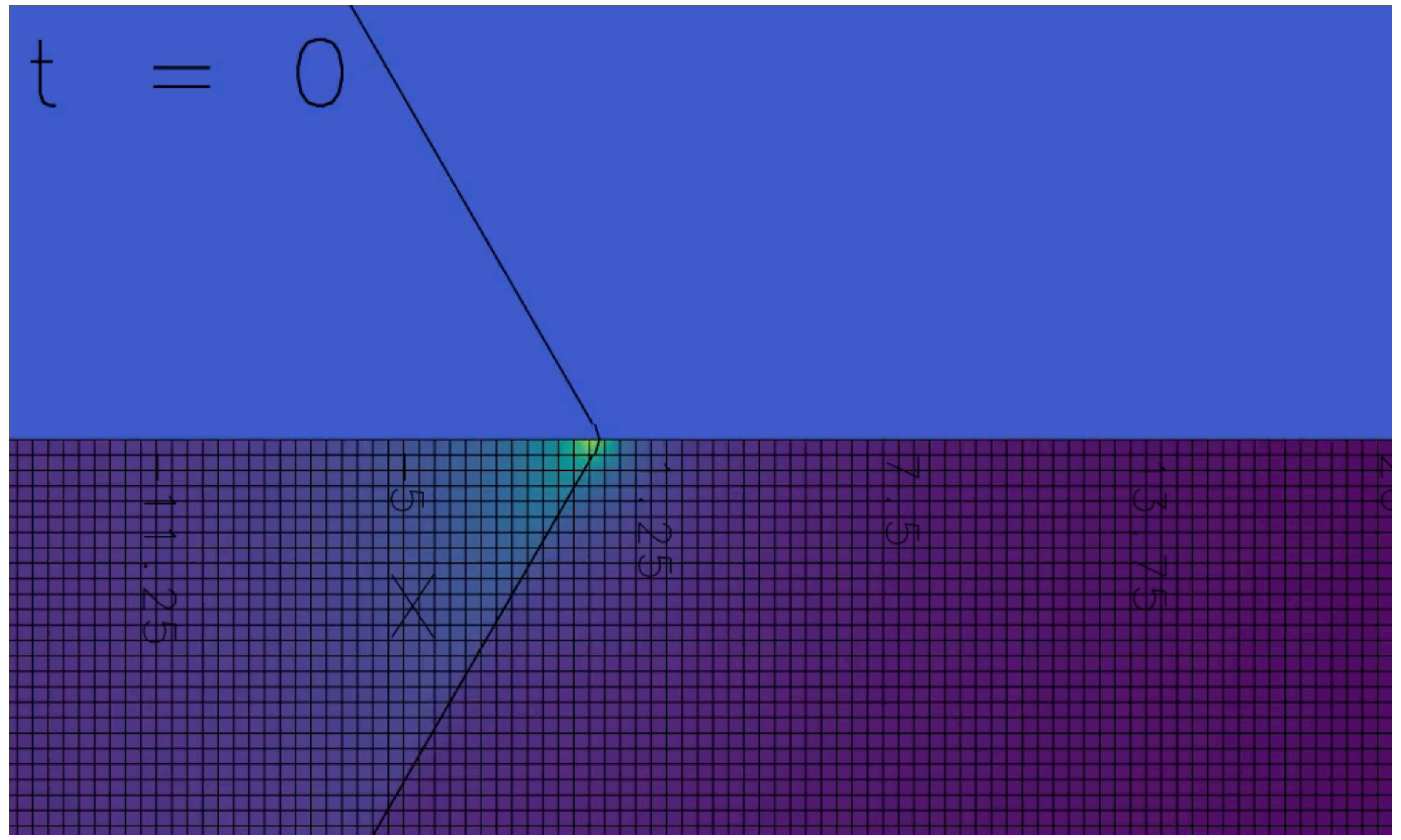
$$\ell_\mu = \frac{{\mu_l}^2}{\rho_l \sigma}$$ ~ water: **10 nm** / oil: **100 µm**

$$t_\mu = \frac{{\mu_l}^3}{\rho_l \sigma^2}$$ ~ water: **100 ps** / oil: **100 µs**

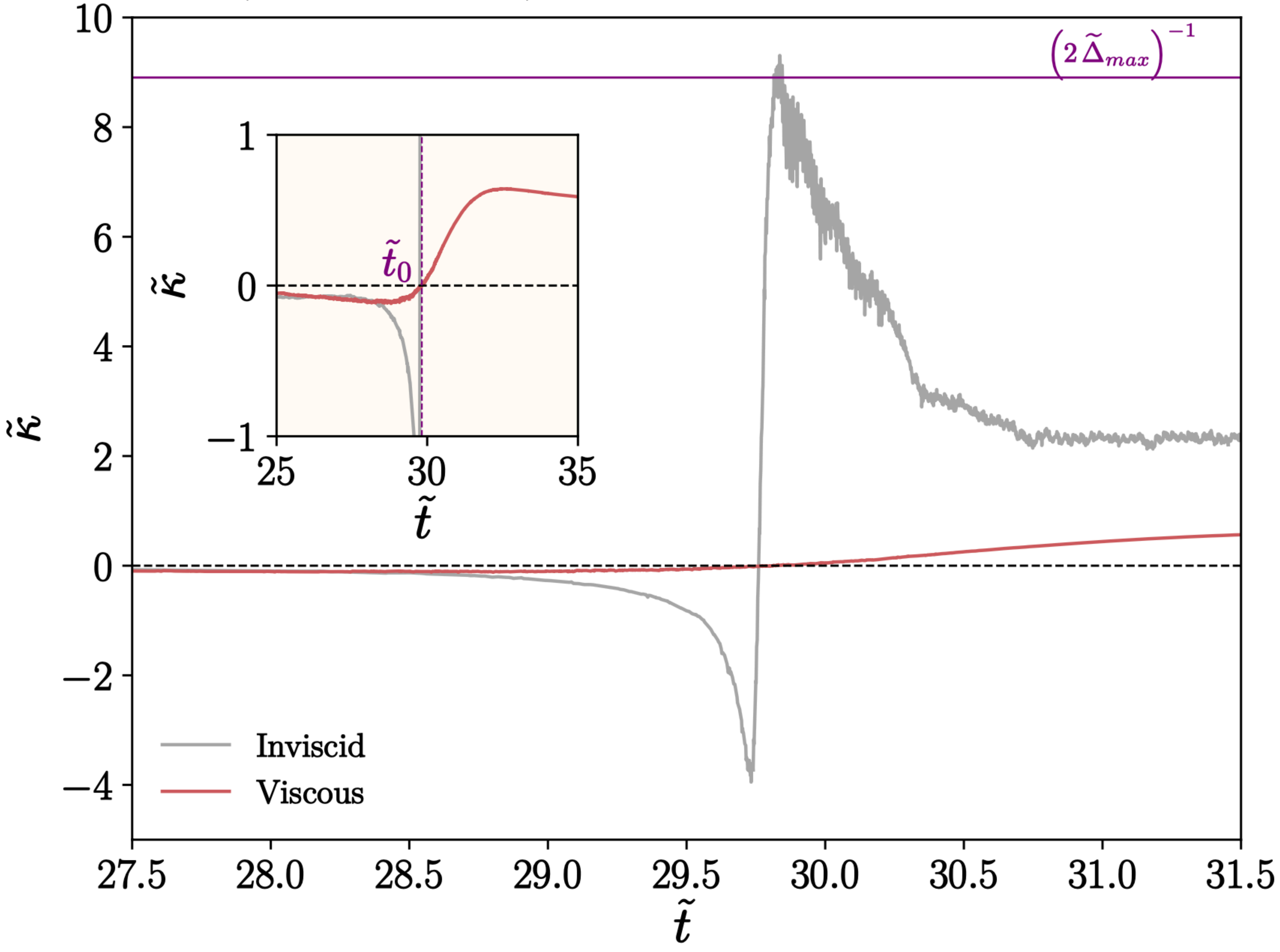
2. Take a size domain $\gg L = \ell_\mu$ $\rightarrow L = 230\,\ell_\mu$

[*to start in the cap. reg.*]

3. Take a grid resolution $\ll \ell_\mu$ (≈ 20 pts) $\rightarrow \Delta \approx 0.05\,\ell_\mu$



$$\frac{\rho_g}{\rho_l} = 10^{-3} \quad ; \quad \frac{\mu_g}{\mu_l} = 10^{-2} \quad ; \quad \ell_\mu = 1 \quad ; \quad t_\mu = 1$$



<u>Goal:</u> to catch a *transitory regime* towards *viscous effects* close to t_0

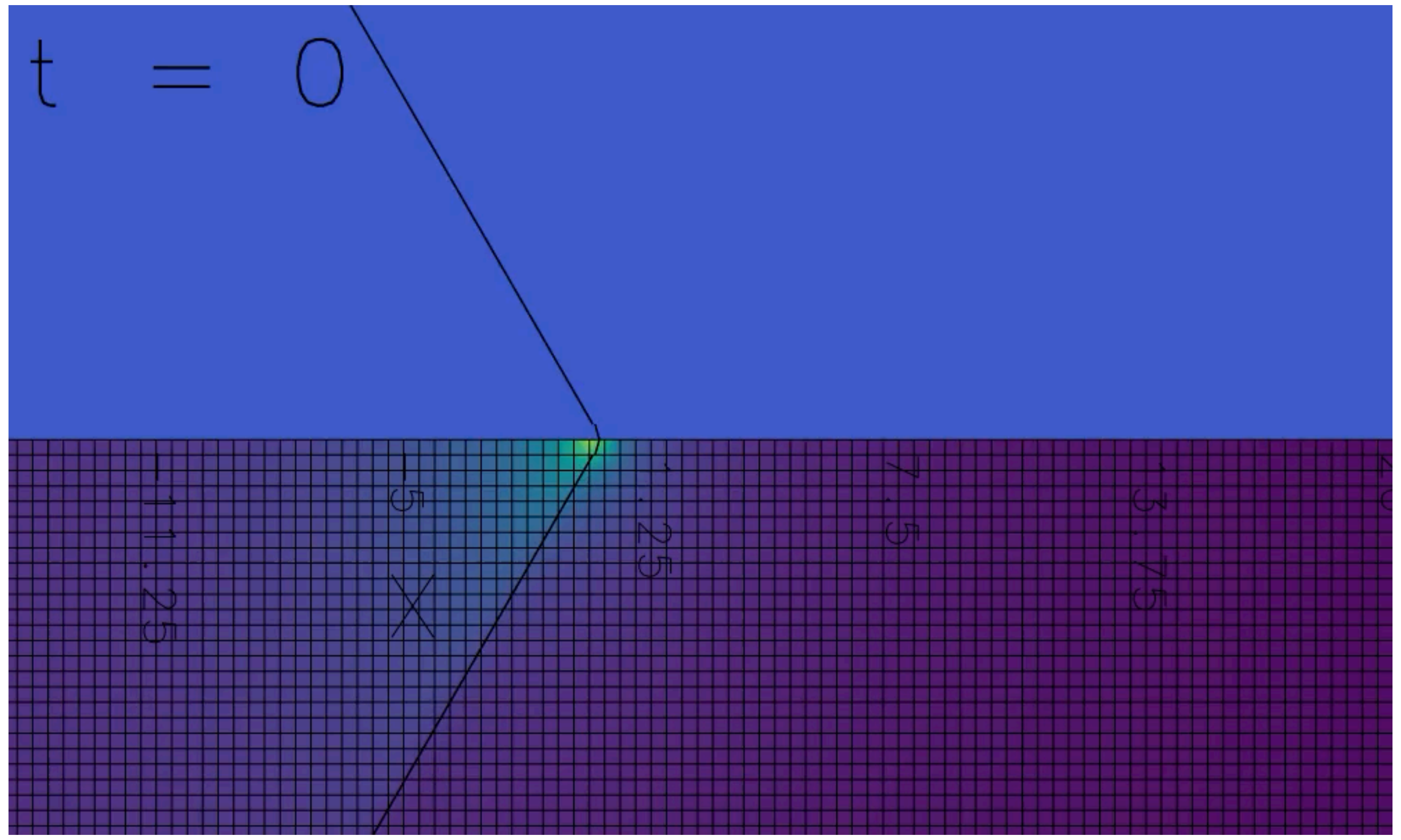
1. Problem non-dimensionalized with viscous scales:

$$\ell_\mu = \frac{{\mu_l}^2}{\rho_l \sigma}$$ ~ water: **10 nm** / oil: **100 µm**

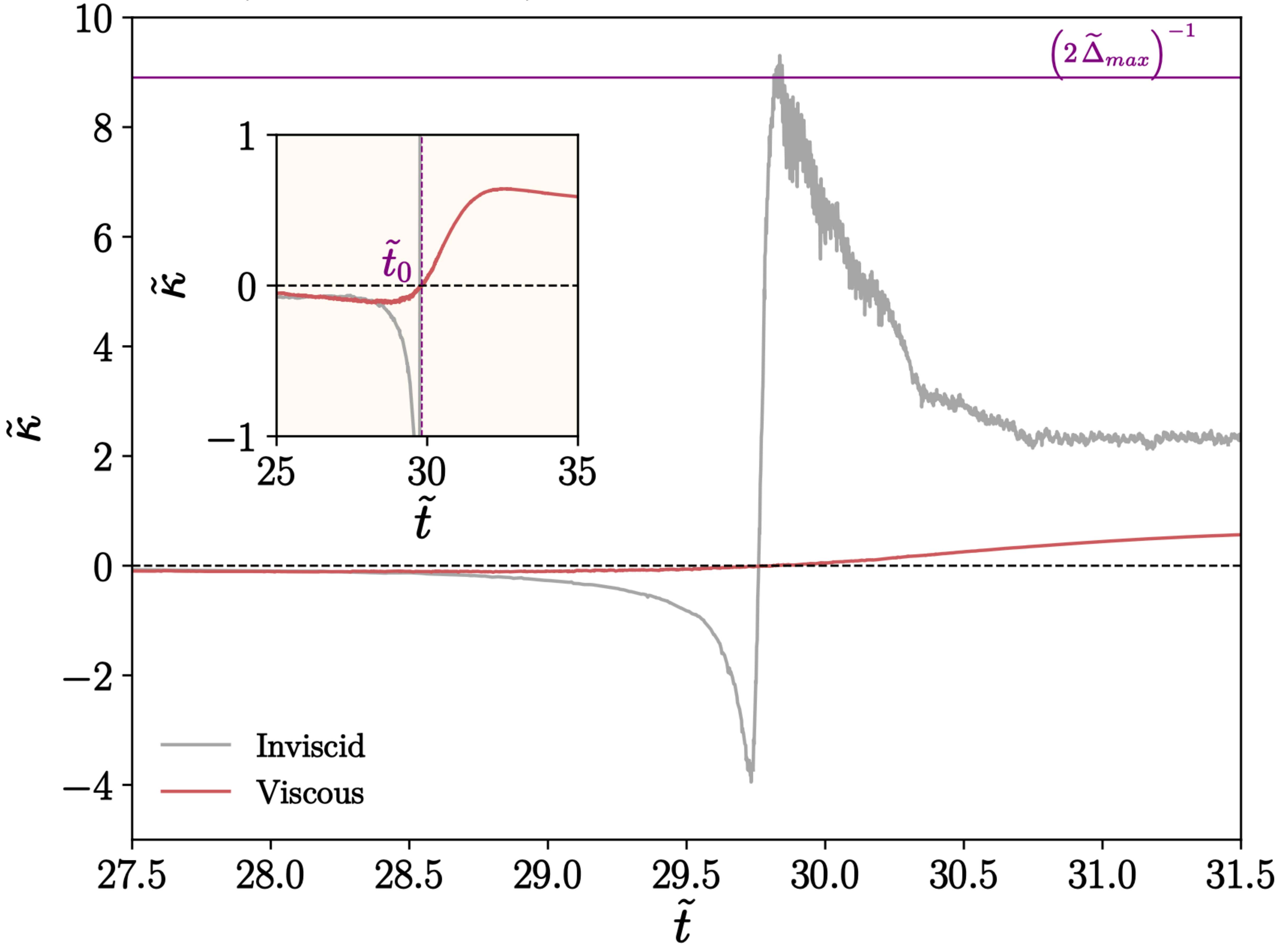
$$t_\mu = \frac{{\mu_l}^3}{\rho_l \sigma^2}$$ ~ water: **100 ps** / oil: **100 µs**

2. Take a size domain $\gg L = \ell_\mu$ $\rightarrow L = 230\, \ell_\mu$
[*to start in the cap. reg.*]

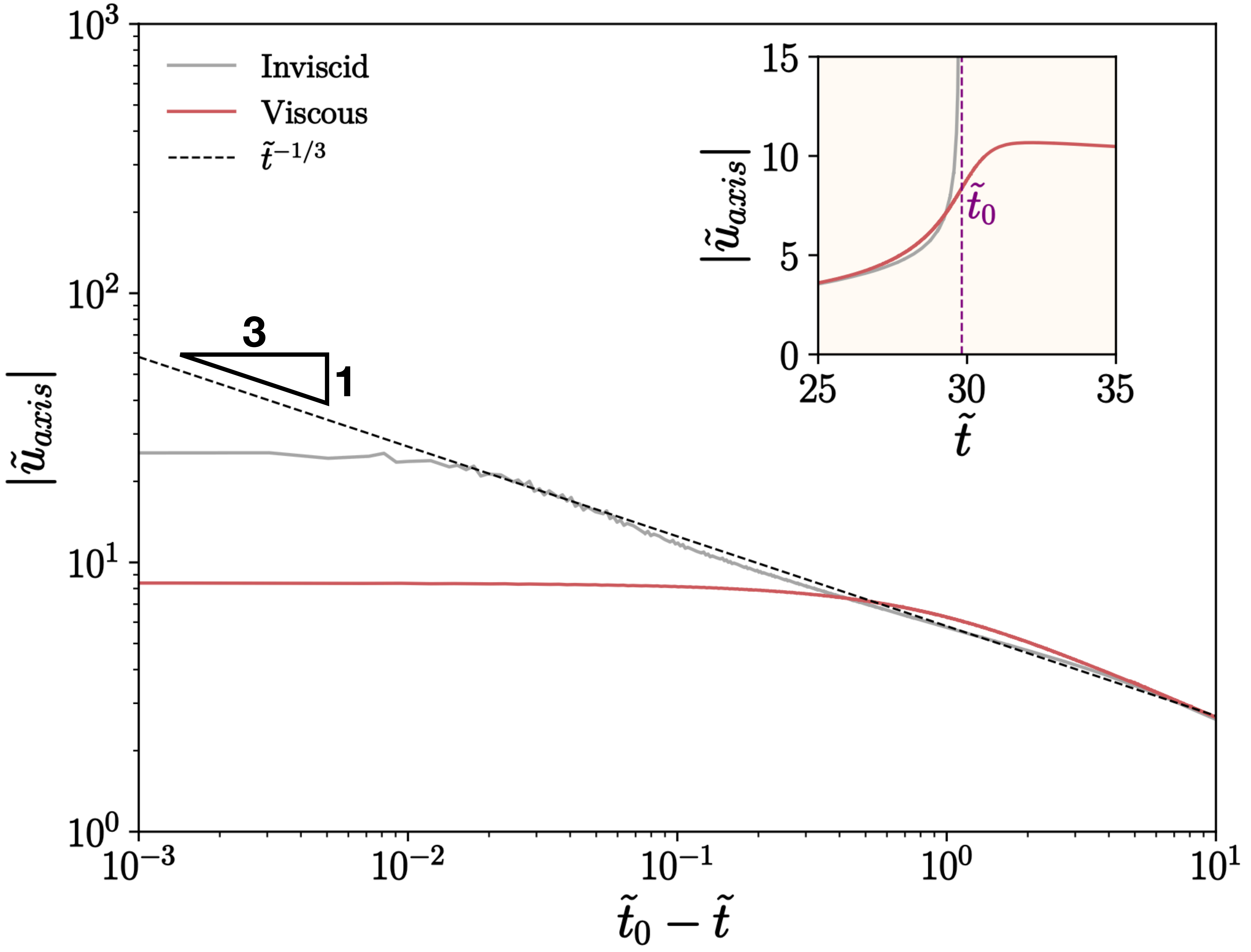
3. Take a grid resolution $\ll \ell_\mu$ ($\approx$ 20 pts) $\rightarrow \Delta \approx 0.05\, \ell_\mu$

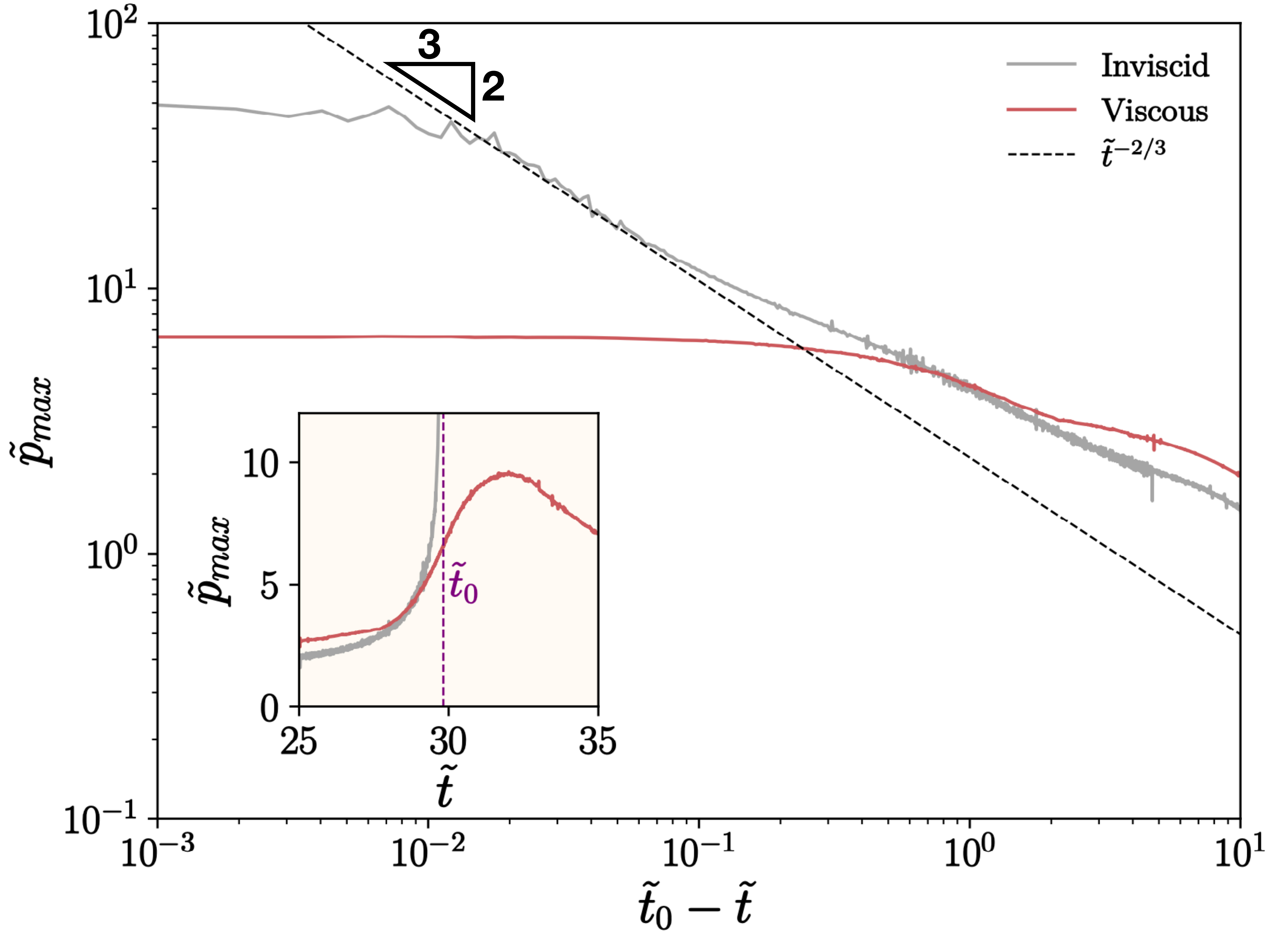


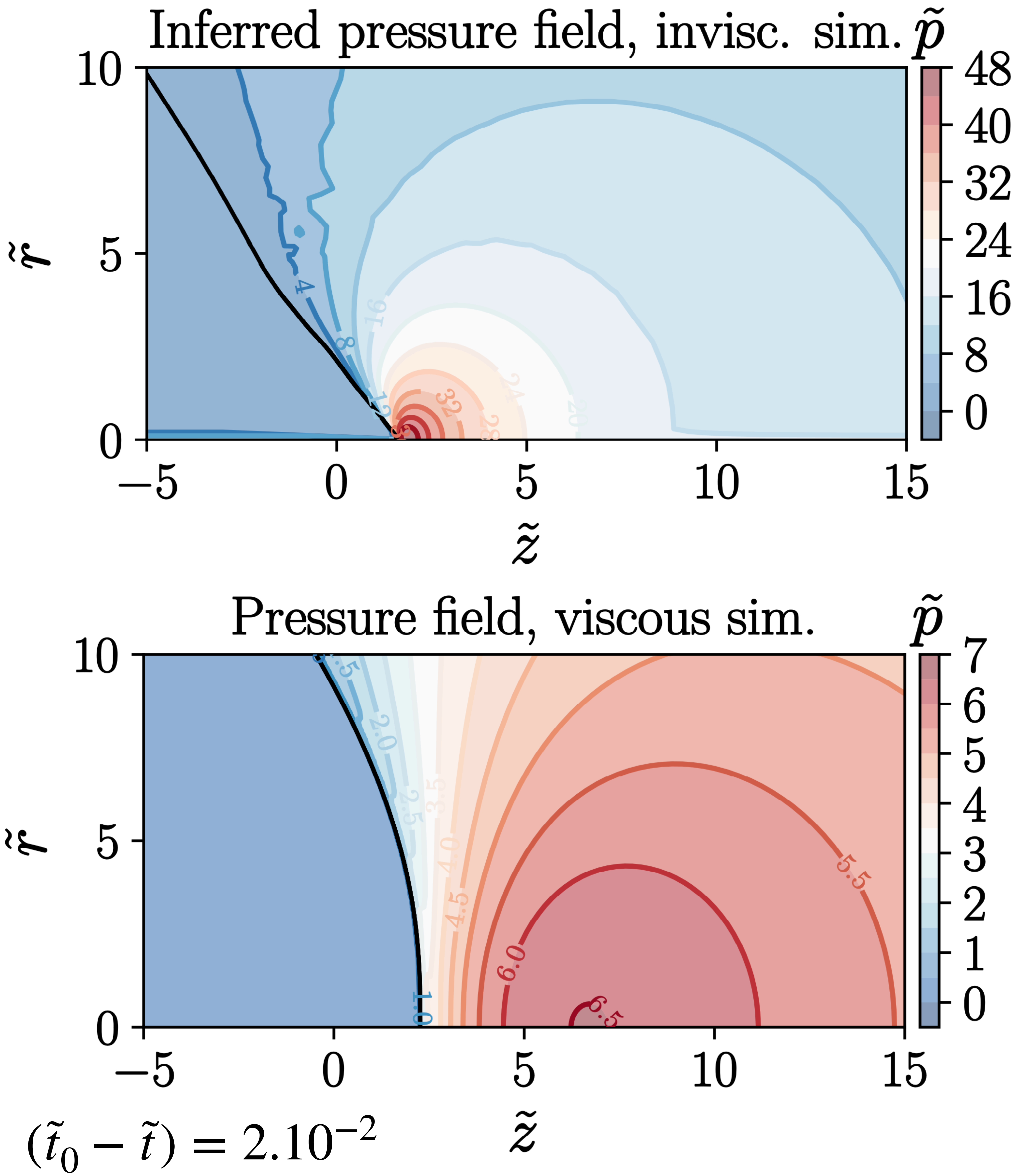
$$\frac{\rho_g}{\rho_l} = 10^{-3} \quad ; \quad \frac{\mu_g}{\mu_l} = 10^{-2} \quad ; \quad \ell_\mu = 1 \quad ; \quad t_\mu = 1$$



*Singularity horizon passed through **physically** with viscosity*

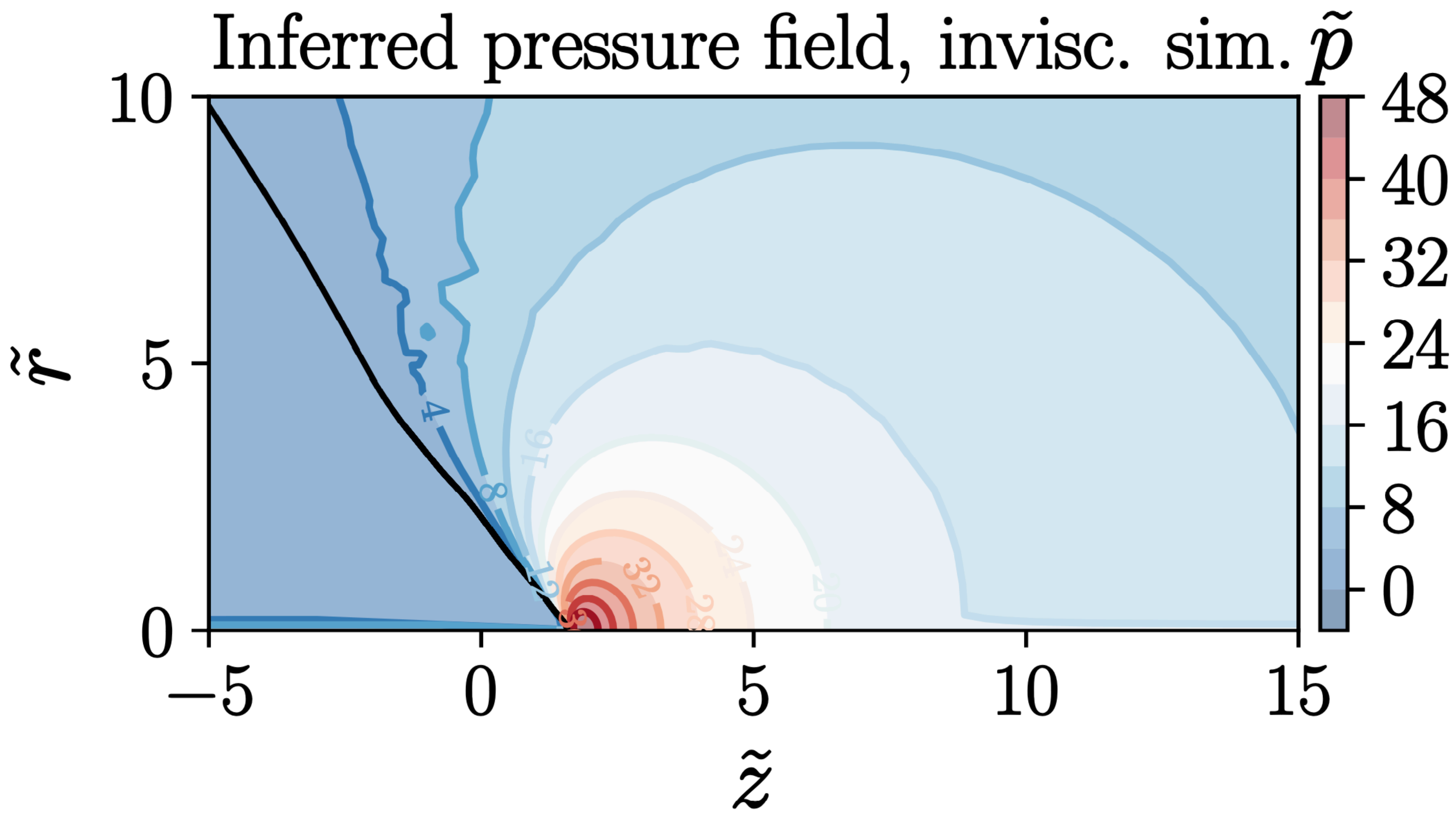

Inviscid
Viscous
$\tilde{t}^{-1/3}$
3
1
$|\tilde{u}_{axis}|$
$\tilde{t}_0 - \tilde{t}$
$\tilde{t}_0$
$\tilde{t}$

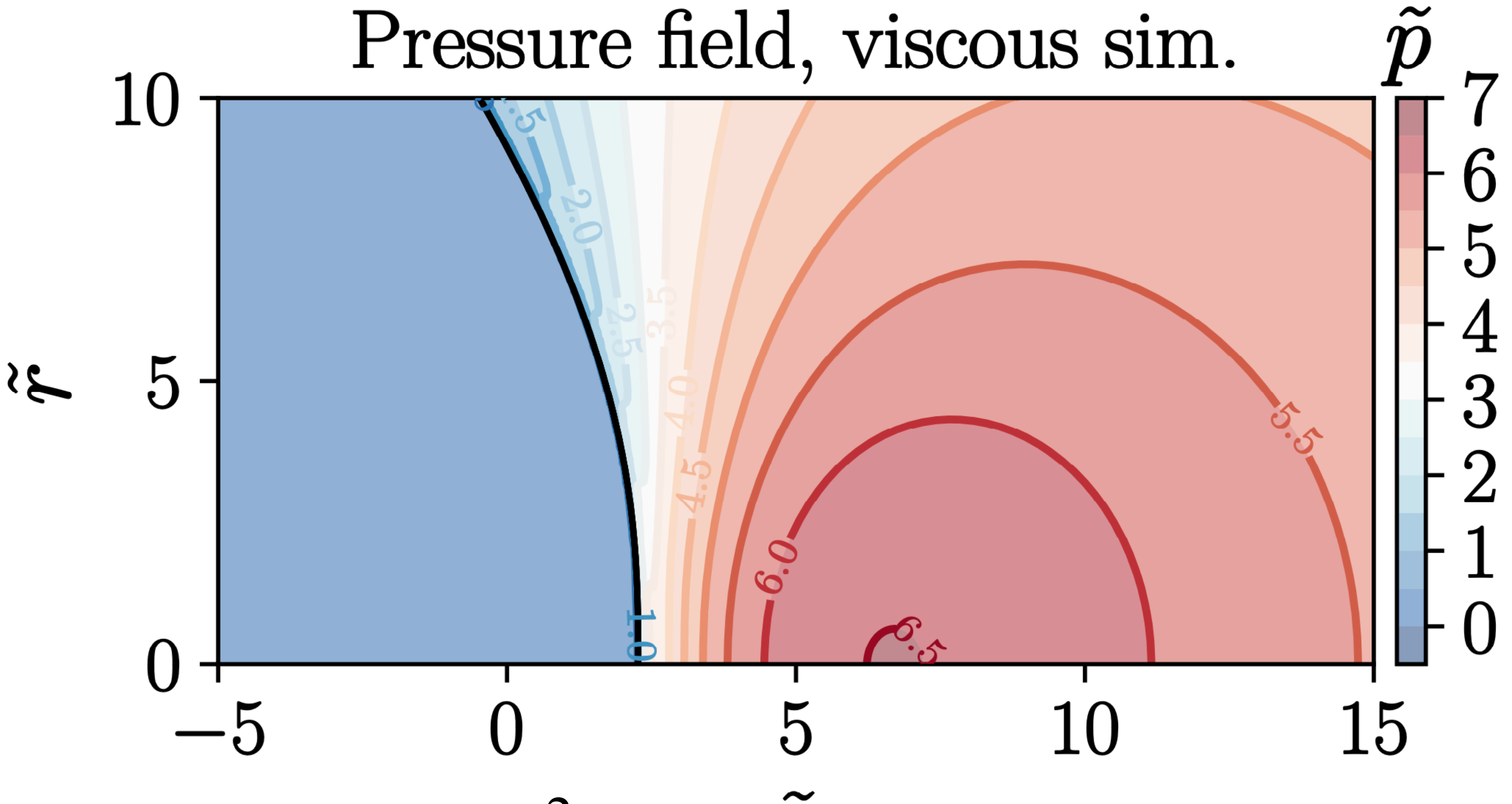

Inviscid
Viscous
$\tilde{t}^{-2/3}$
3
2
$\tilde{p}_{max}$
$\tilde{t}_0 - \tilde{t}$
$\tilde{t}_0$
$\tilde{t}$



$(\tilde{t}_0 - \tilde{t}) = 2.10^{-2}$

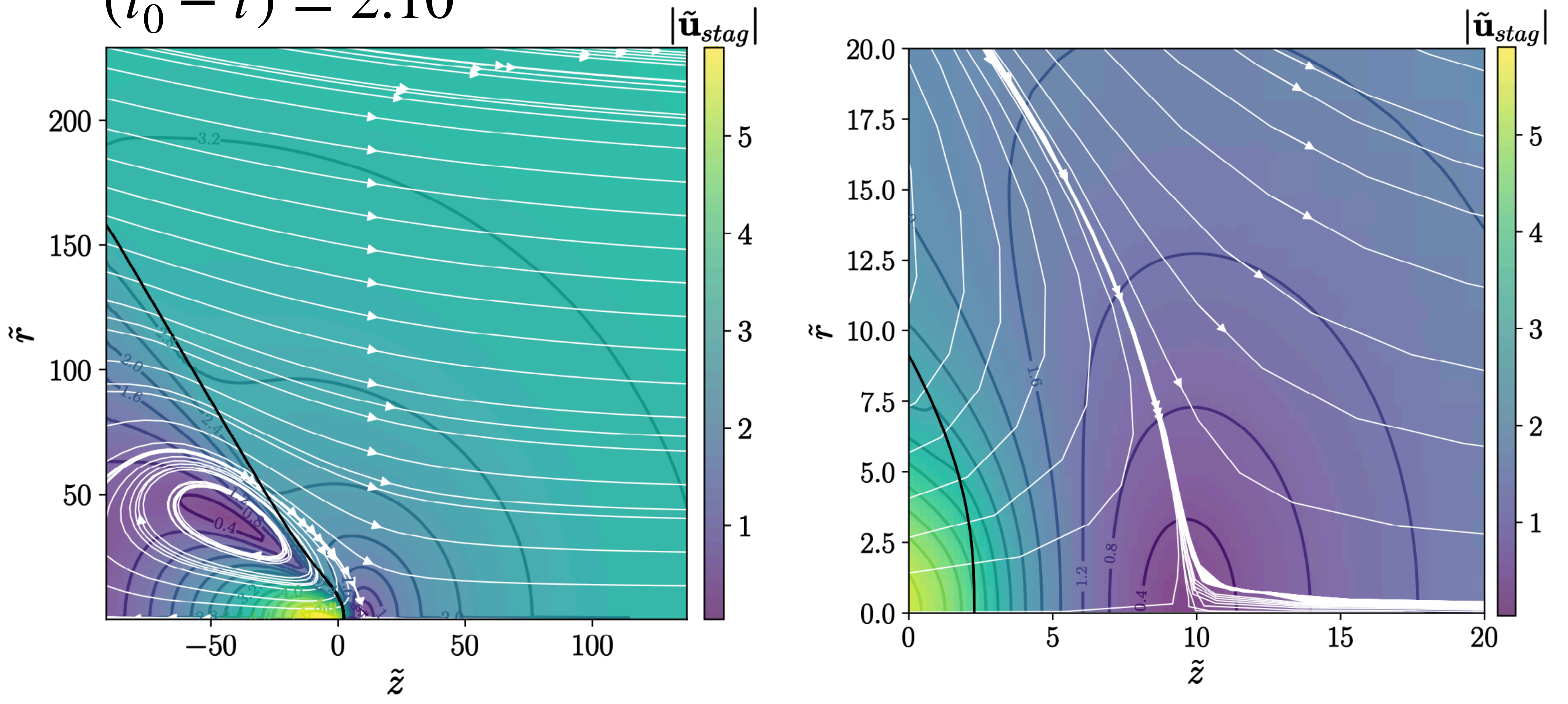
Pressure max away from the apex in the viscous sim.





$(\tilde{t}_0 - \tilde{t}) = 2.10^{-2}$

$(\tilde{t}_0 - \tilde{t}) = 2.10^{-2}$

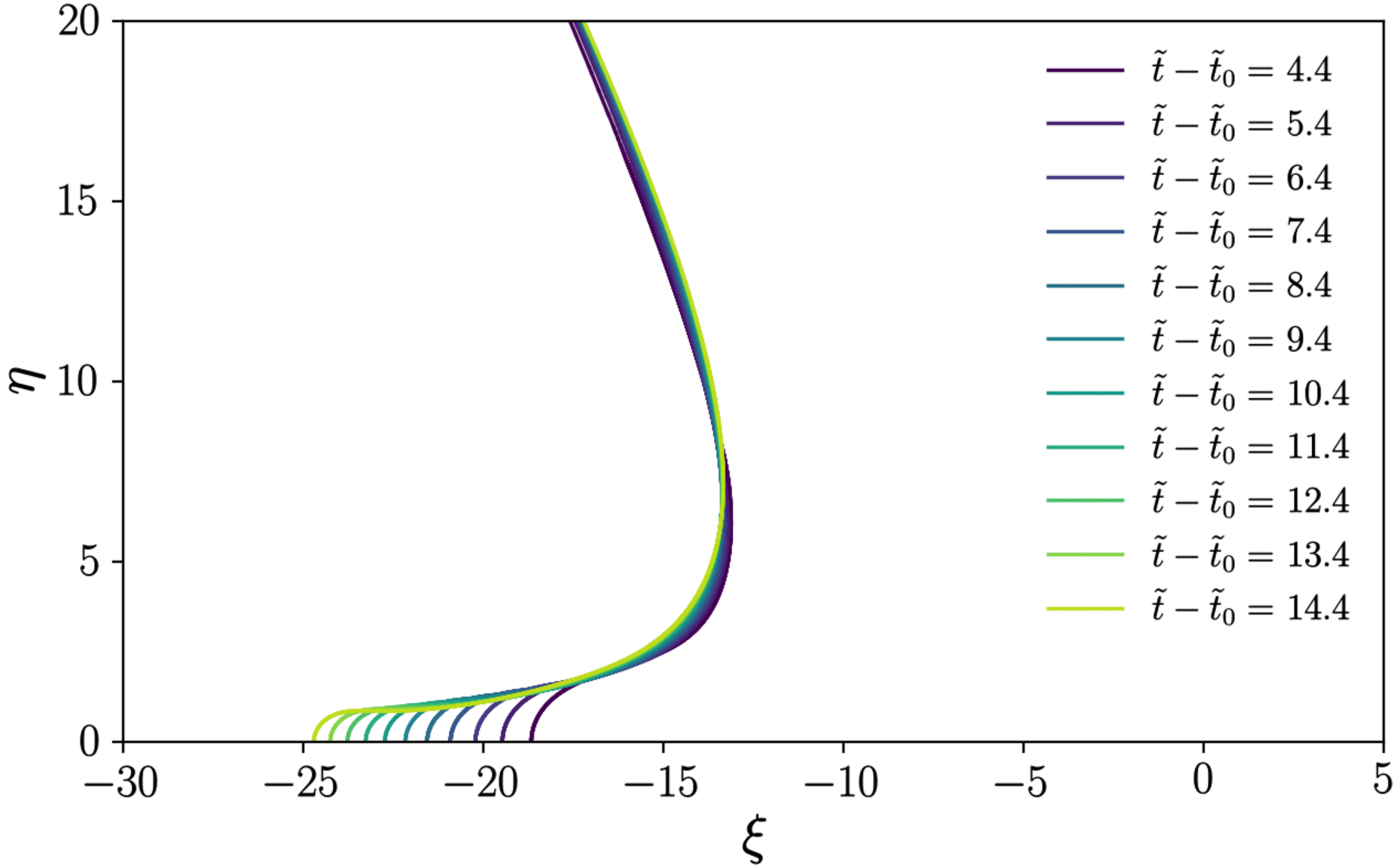


Pressure max away from the apex in the viscous sim.

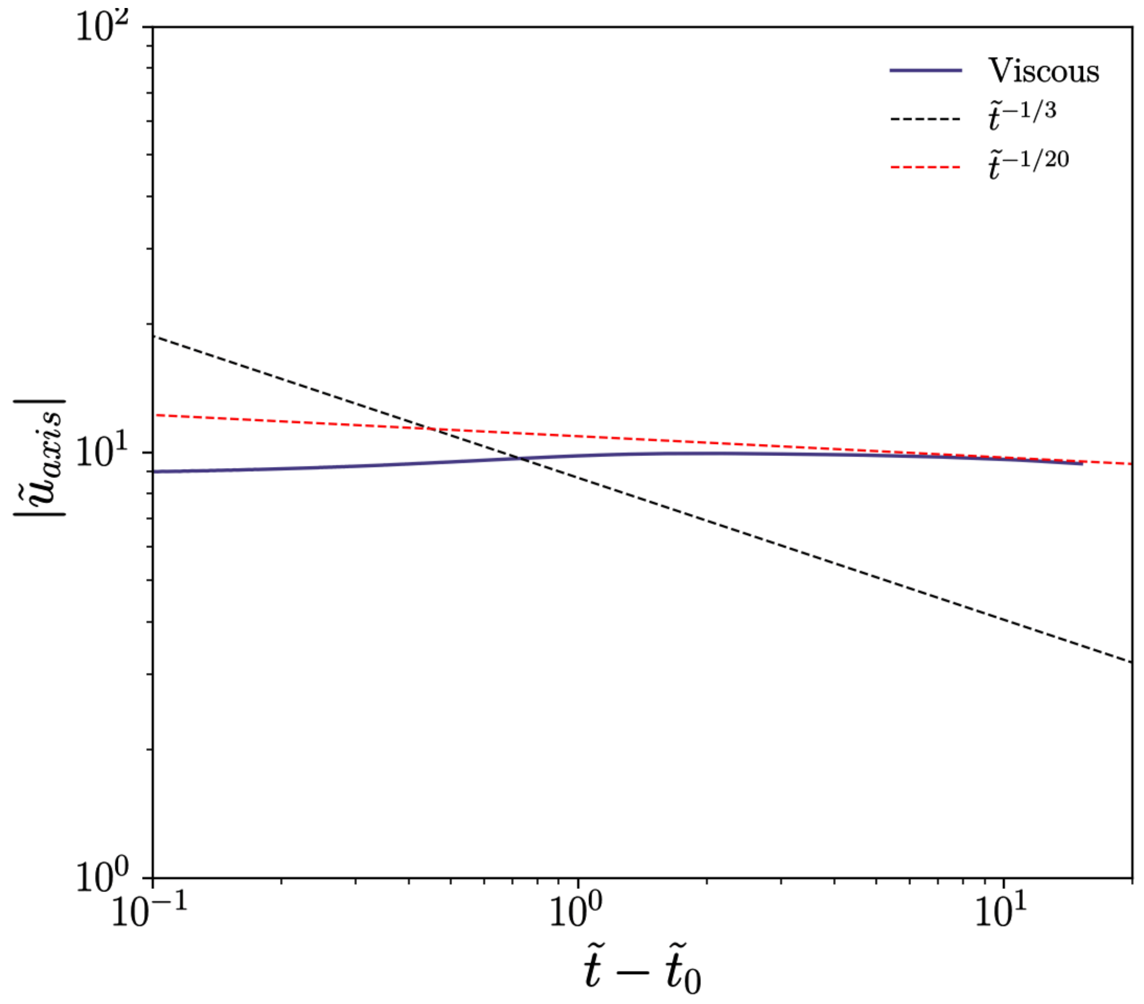
Stagnation flow $\widetilde{\mathbf{u}}_{stagn} = \widetilde{\mathbf{u}} - \widetilde{\mathbf{u}}_0$**:**

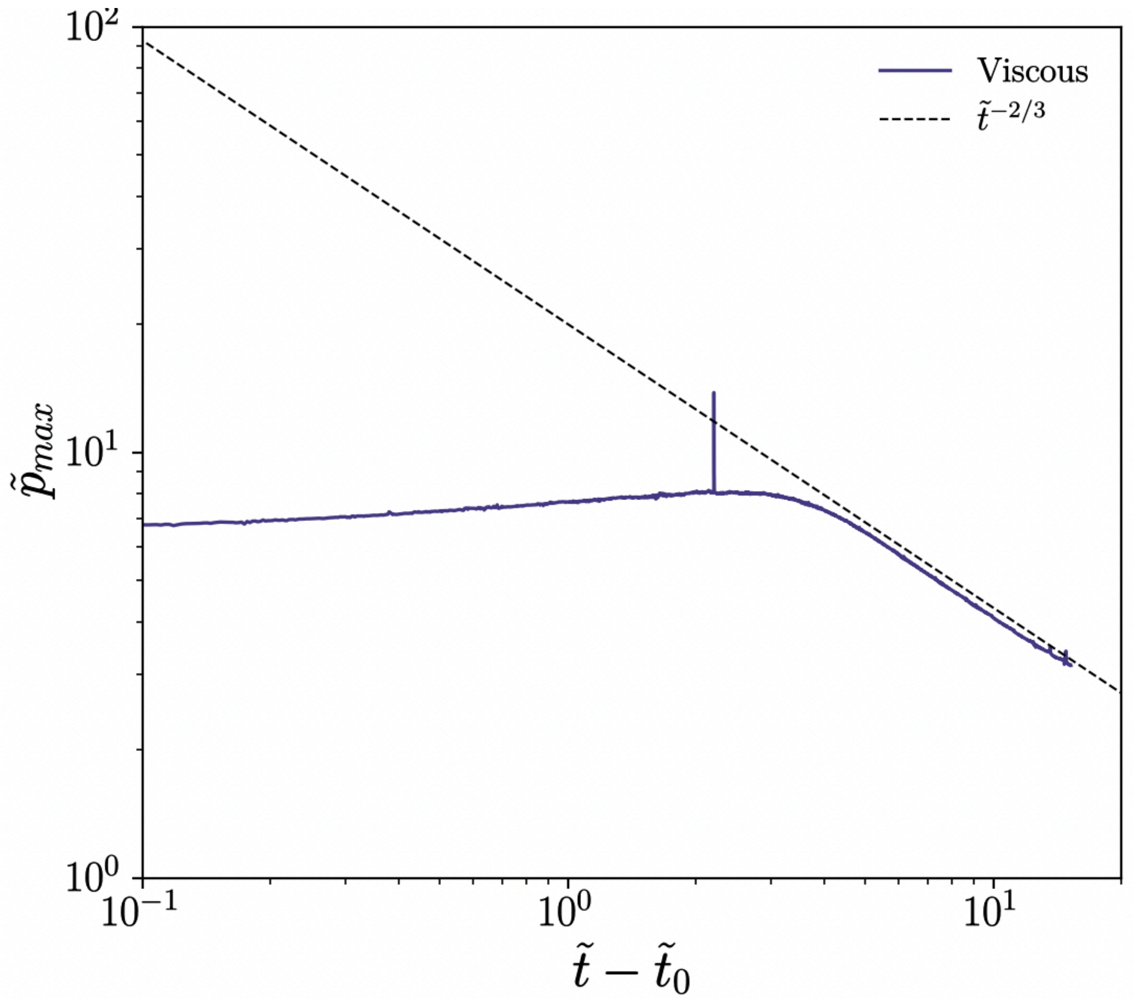
→ lateral convergence ⇒ **jet**

IV - Conical Collapsing Cavities



Post-singular jets of non-perturbed collapsed cavities are CAPILLARO-INERTIAL!





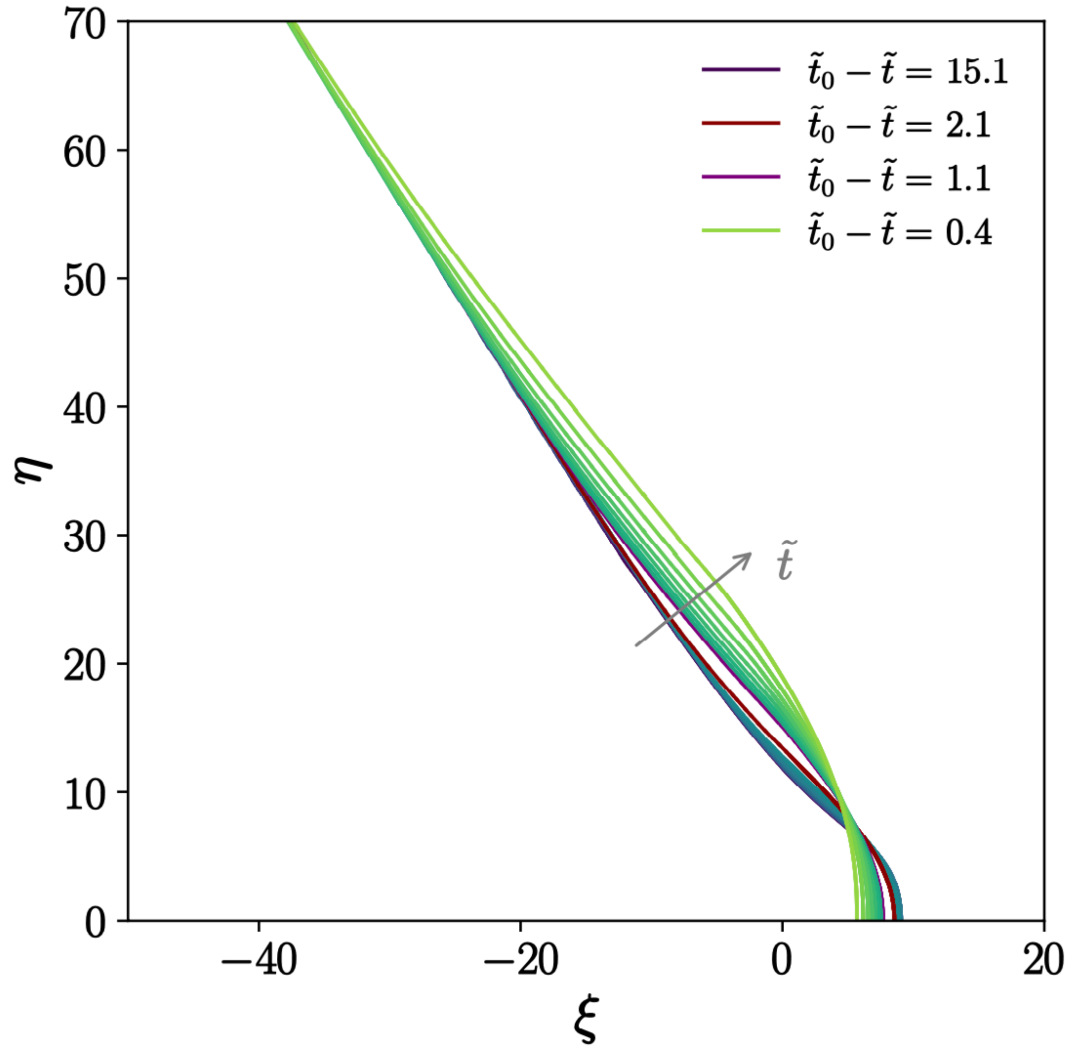
Velocity is more or less constant, as observed experimentally

Pressure field follows again a cap.-inert. Reg.

CONCLUSIONS

CONCLUSIONS

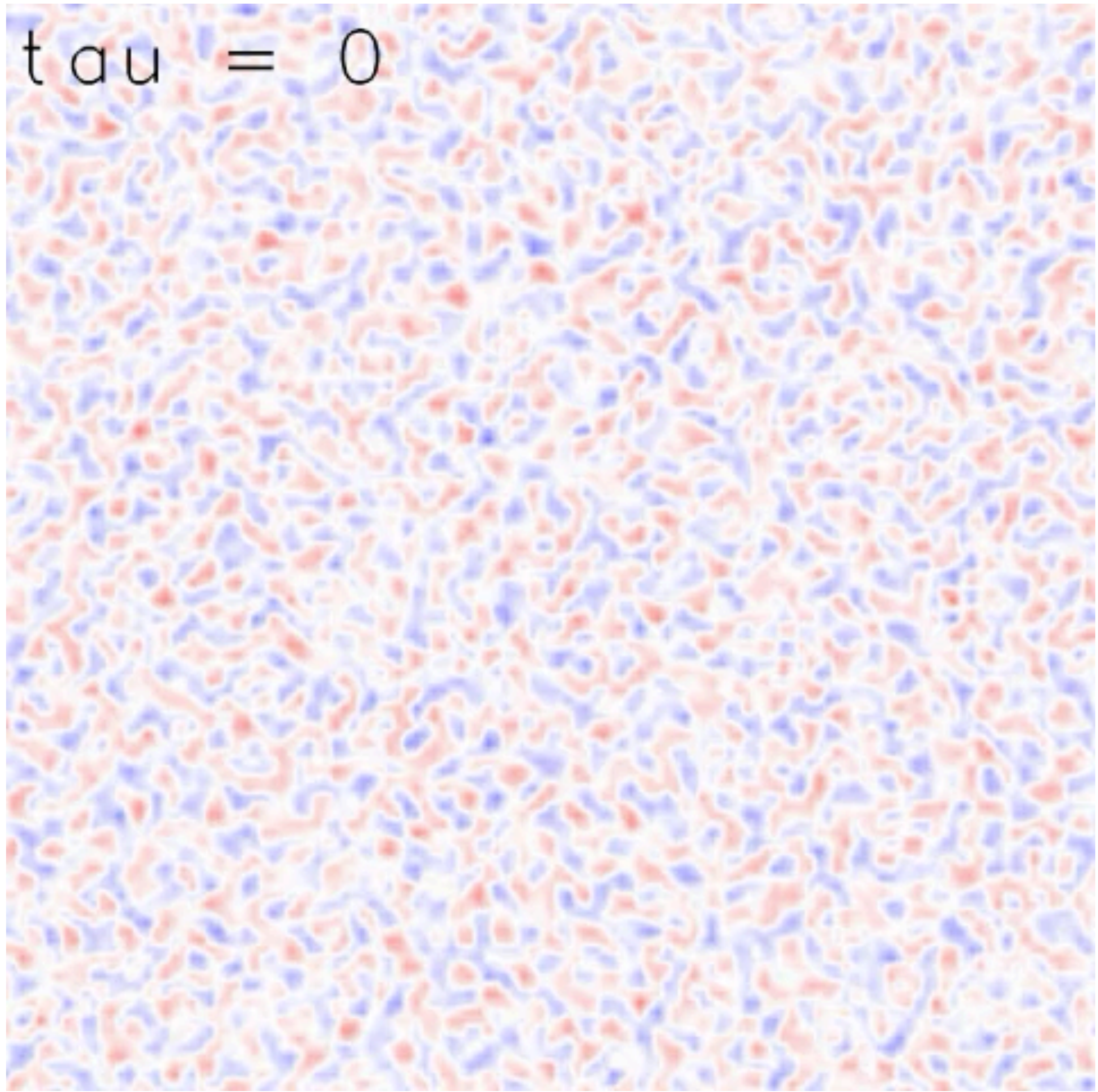
- **Collapse of a conical cavity:**
  - time reversal of a recoiling cone
  - self-similar in $t'^{2/3}$ (capillary-inertial)
  - dipolar flow → complex far-field tangential velocity discontinuities
  - family of self-similar profiles indexed by $(\theta_0, \widetilde{\mu}_0)$
  - self-similar jet profiles at high $|\widetilde{\mu}_0|$
  - singularity crossed by viscosity effects
  - stagnation point as a kinematic process for jet emission
  - variation of BCs → *inertial pinching* → *non-universality* of self-sim. sol.



Passing through the singularity with viscosity

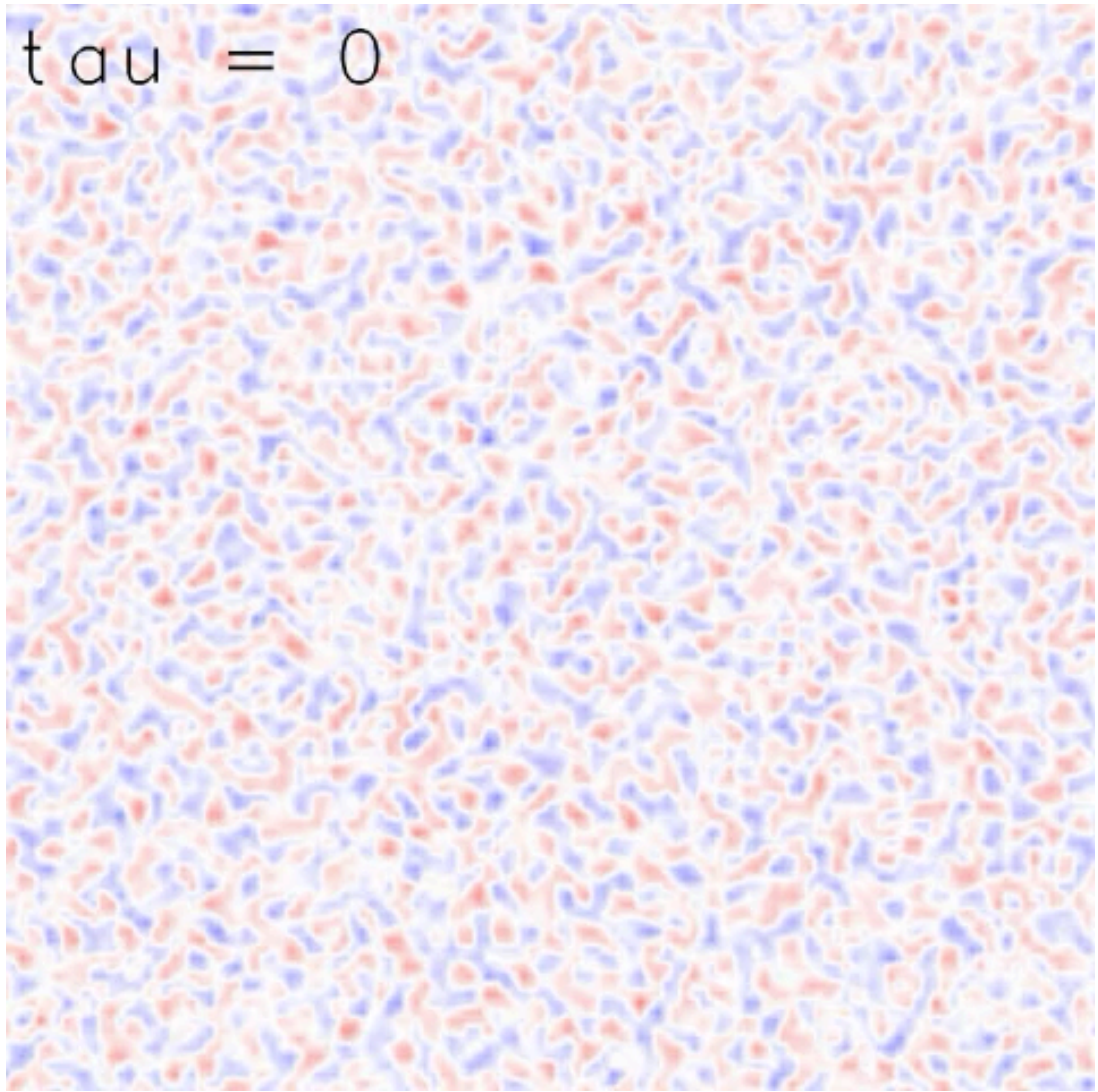
CONCLUSIONS

- Dev. of a Self-Similar Solver working *on other scale-invariant problems* (ONGOING "Huppert's heap")



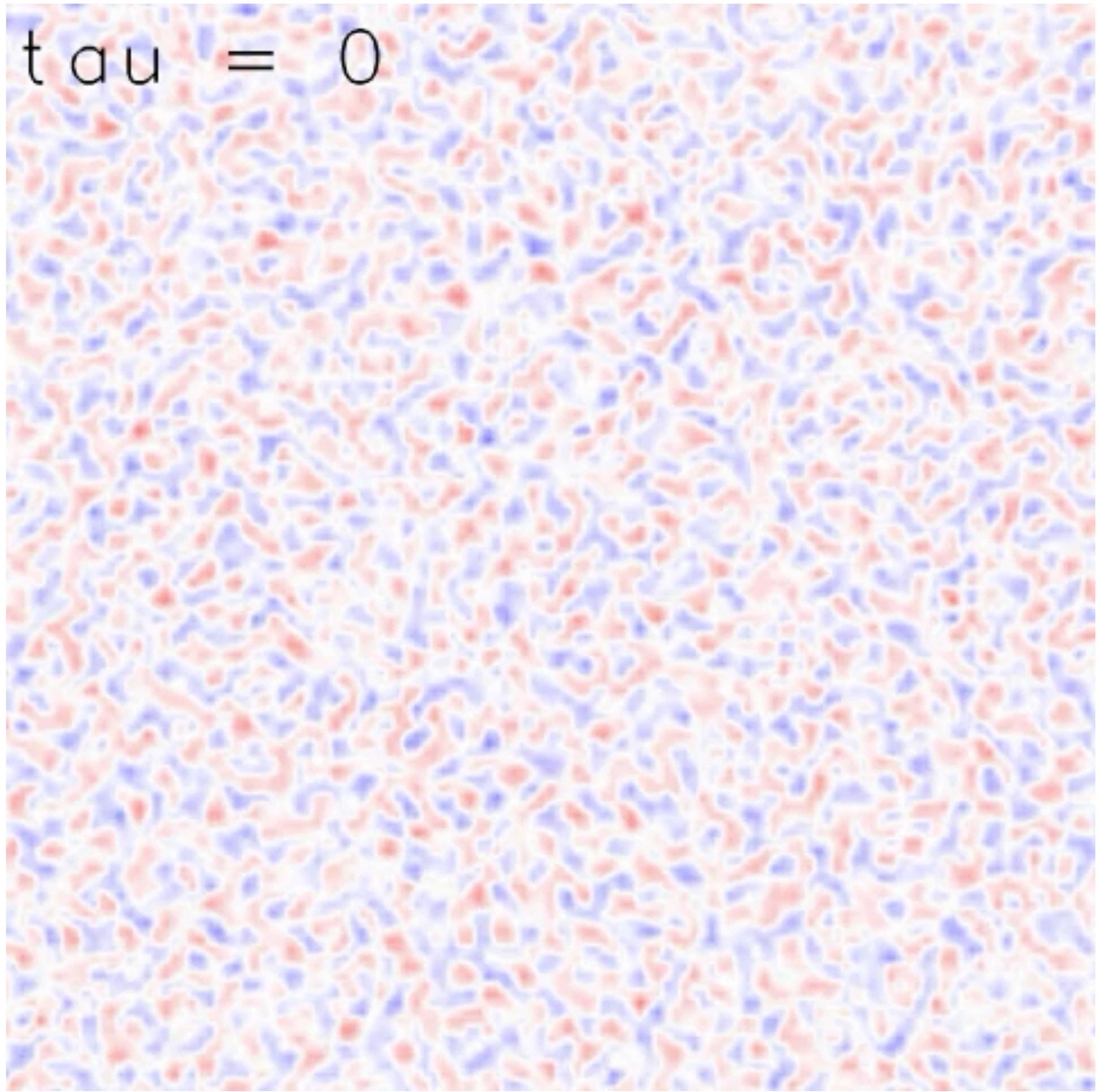
CONCLUSIONS

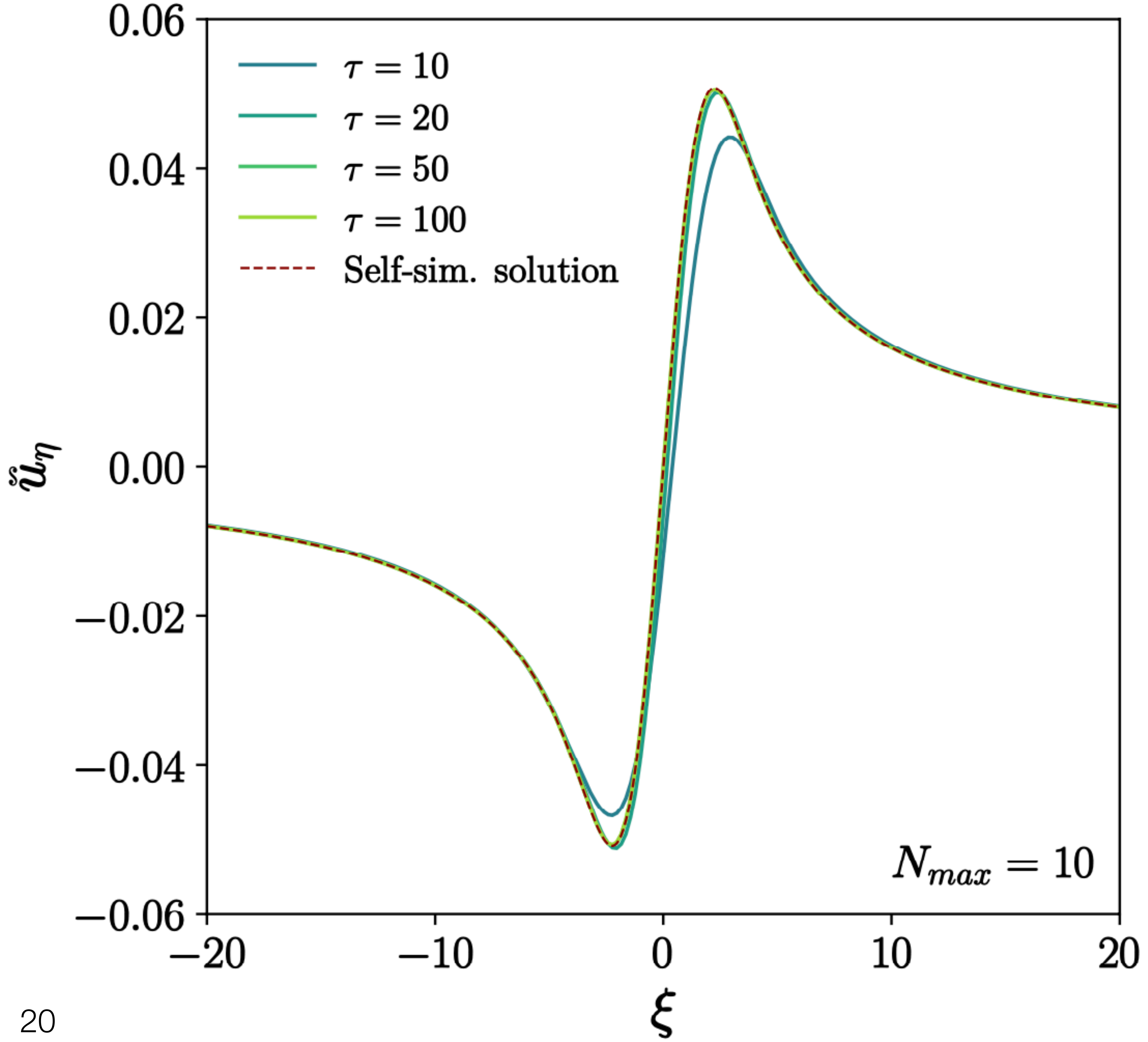
- **Dev. of a Self-Similar Solver working *on other scale-invariant problems* (ONGOING "Huppert's heap")**



CONCLUSIONS

- Dev. of a Self-Similar Solver working *on other scale-invariant problems* (ONGOING "Huppert's heap")



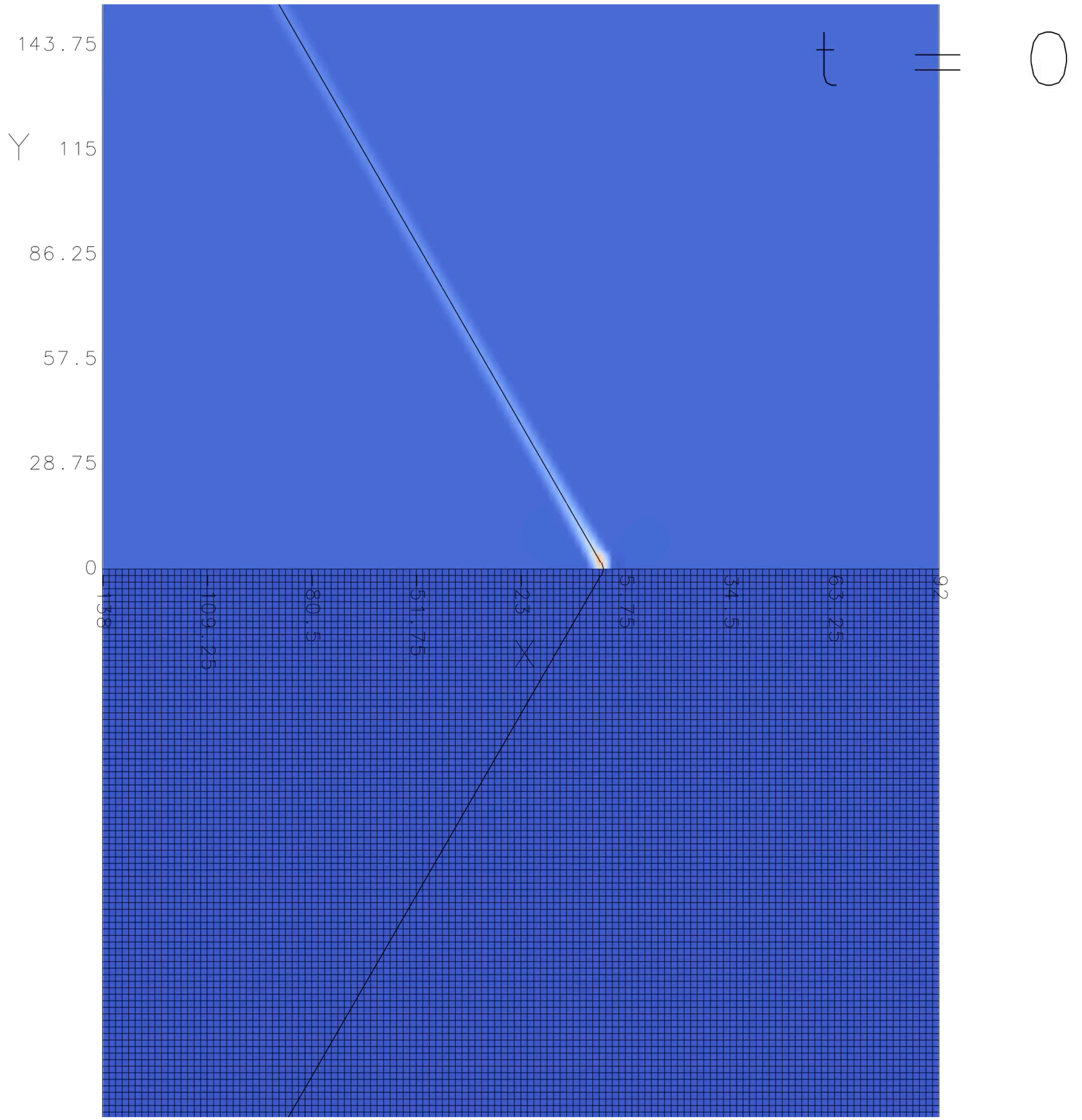


IV.8 - Perturbation of Boundary Conditions

Before: $|\widetilde{\mu}_0| = \mathrm{C}^{\mathrm{st}}$

Now: *unsteady* dipolar flow

$$|\widetilde{\mu}_0| = \begin{cases} 50, \textbf{ for } \tilde{t} < \tilde{t}_{inv} \\ 25, \textbf{ for } \tilde{t} \geqslant \tilde{t}_{inv} \end{cases}$$



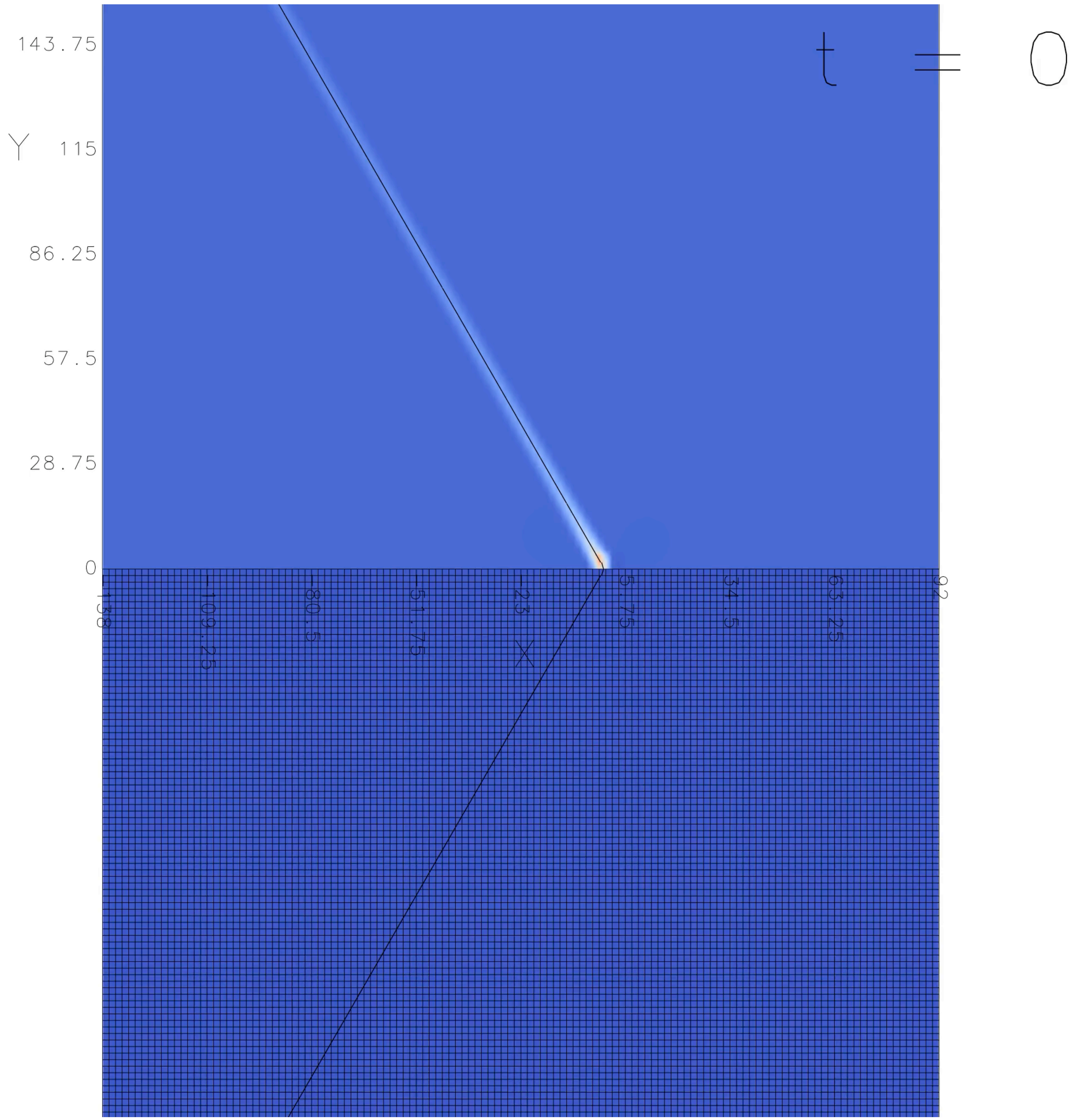
(Demo version: under resolved here)

IV.8 - Perturbation of Boundary Conditions

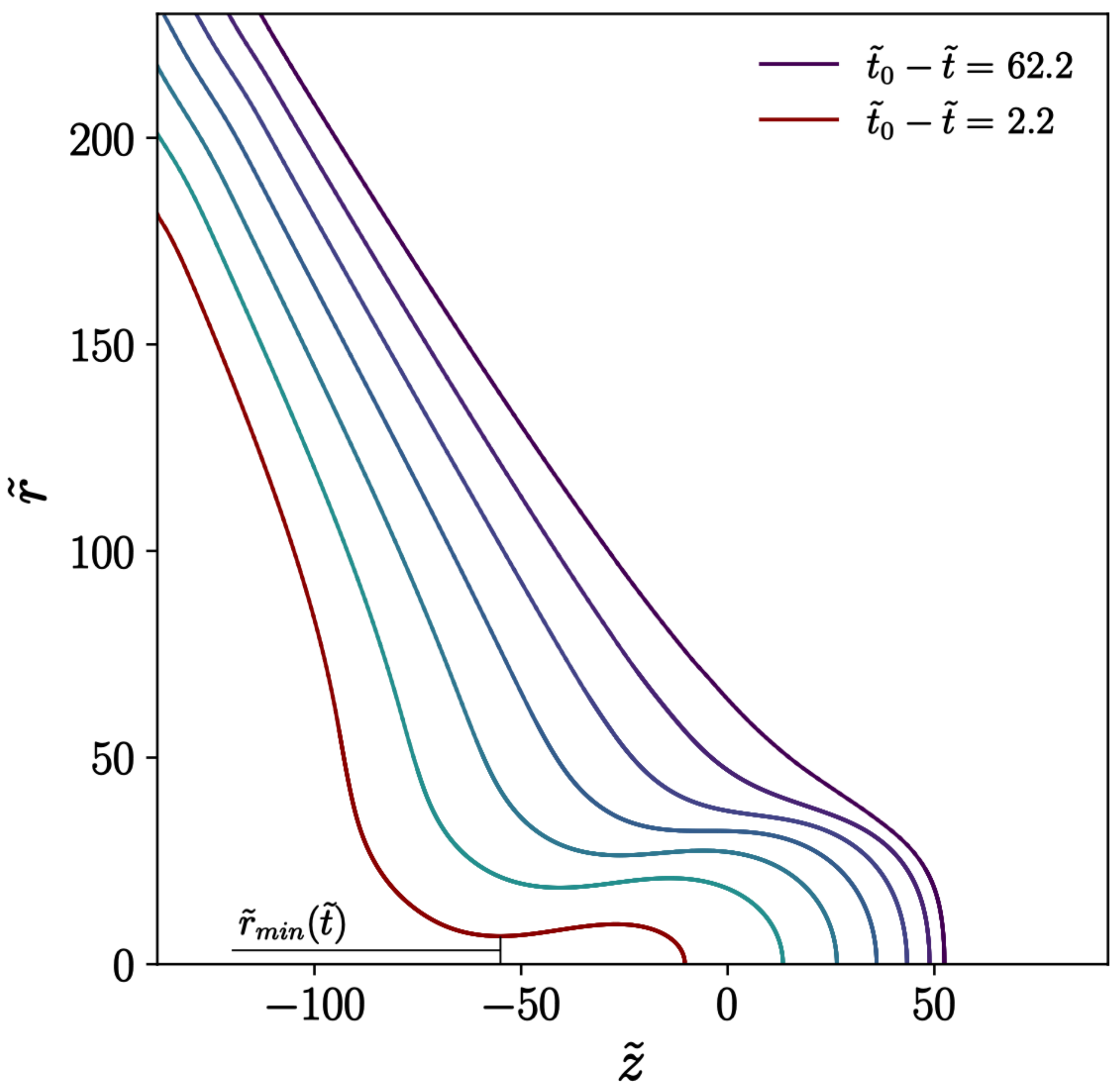
Before: $|\widetilde{\mu}_0| = \mathrm{C}^{\mathrm{st}}$

Now: *unsteady* dipolar flow

$$|\widetilde{\mu}_0| = \begin{cases} 50, \textbf{ for } \tilde{t} < \tilde{t}_{inv} \\ 25, \textbf{ for } \tilde{t} \geqslant \tilde{t}_{inv} \end{cases}$$

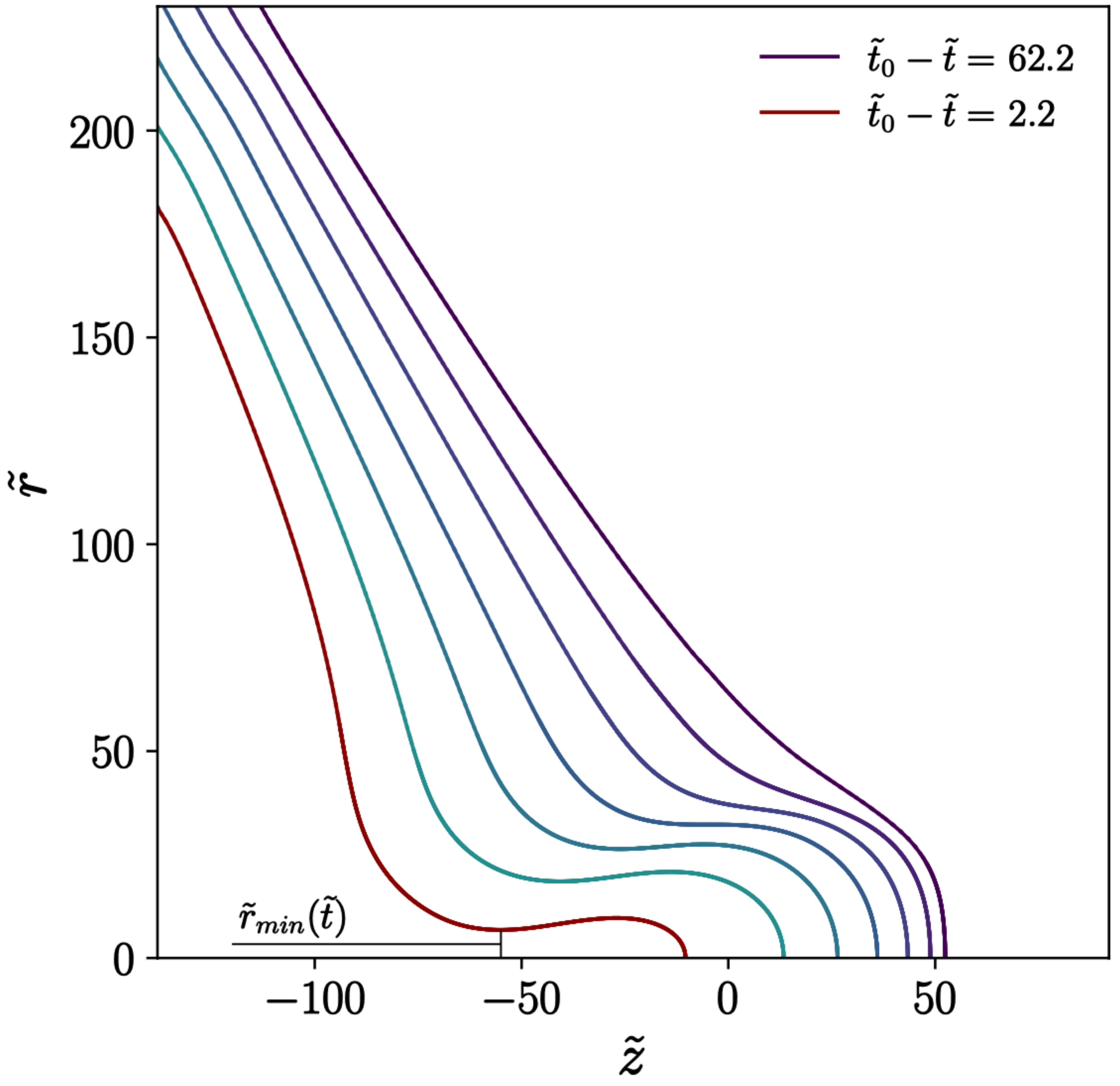


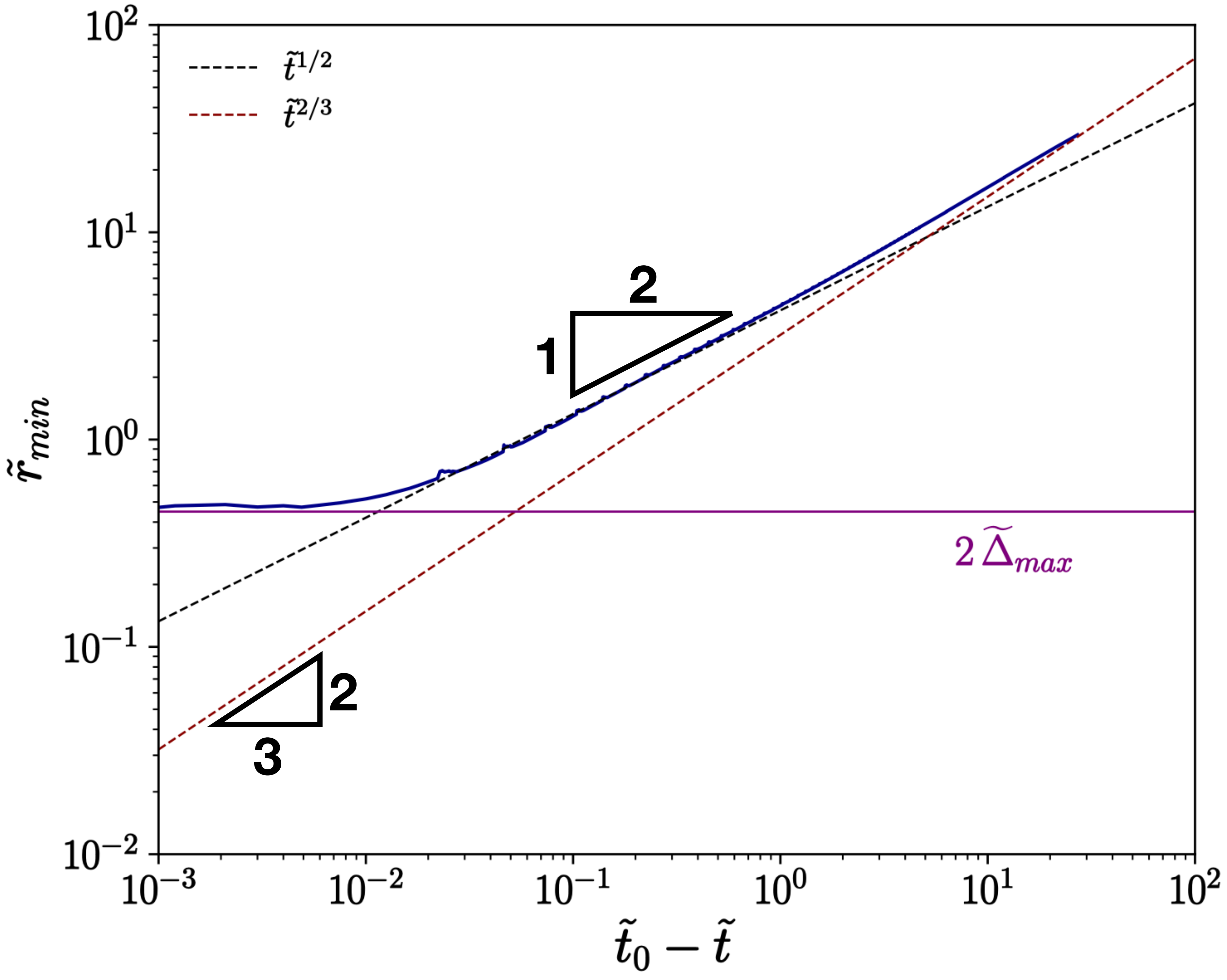
(Demo version: under resolved here)

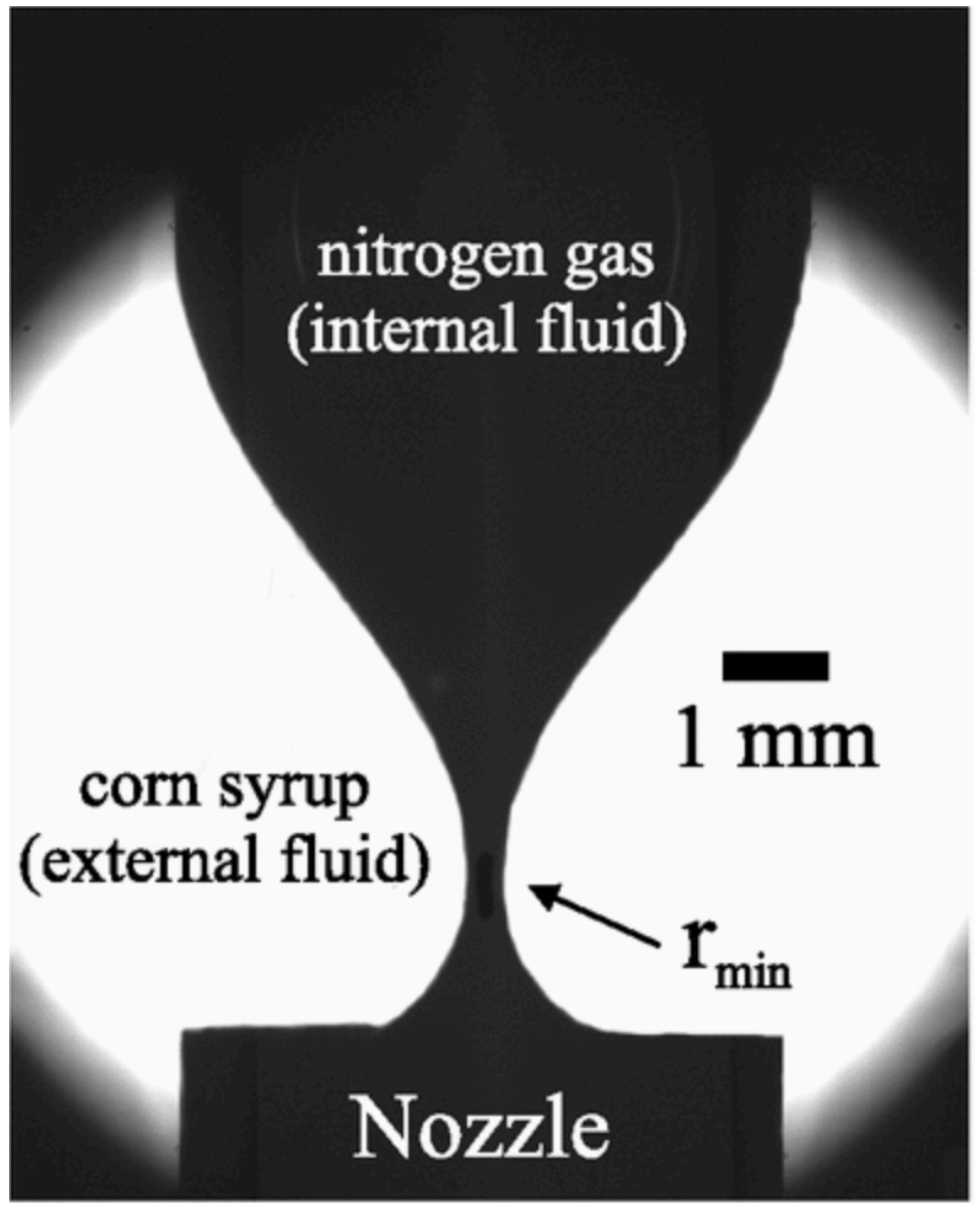


PINCHING!!

New evolution due to different self-similar flows depending on $|\widetilde{\mu}_0|$







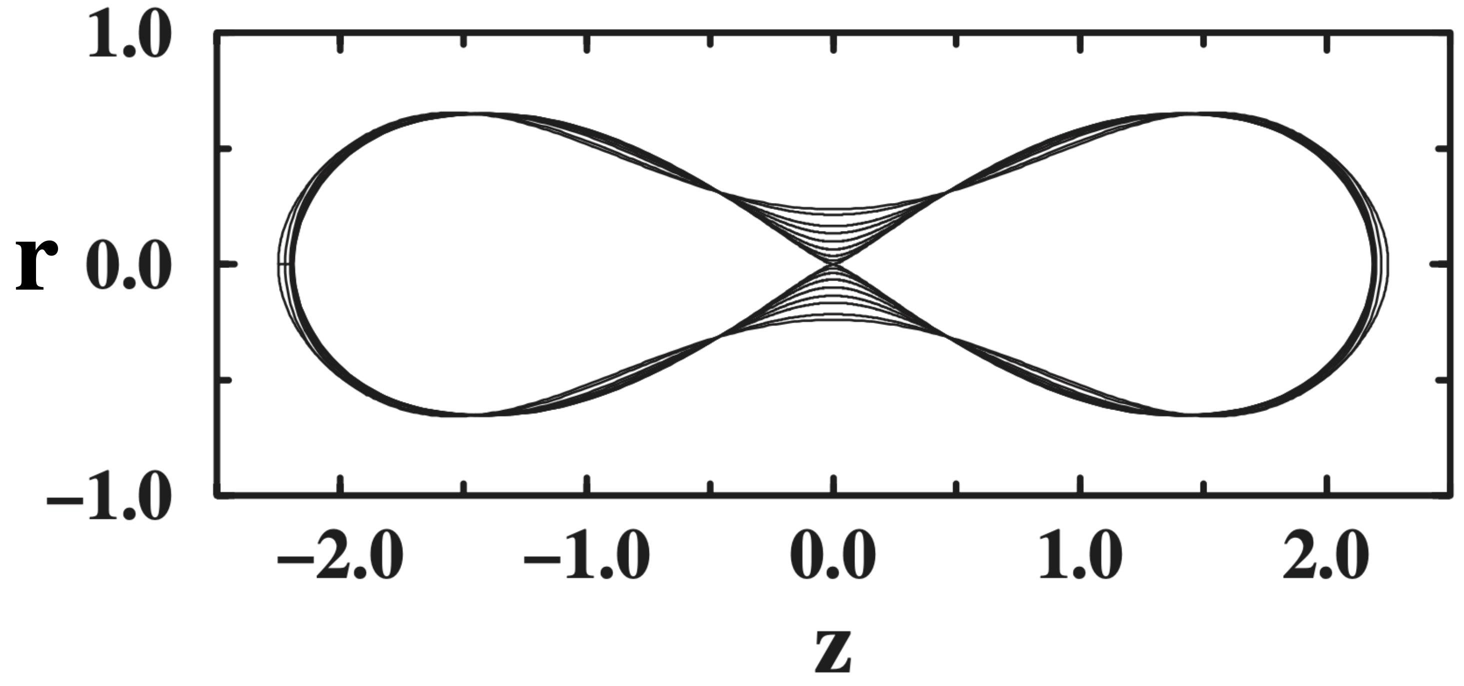
Burton *et al.* (2005)

PINCHING!!

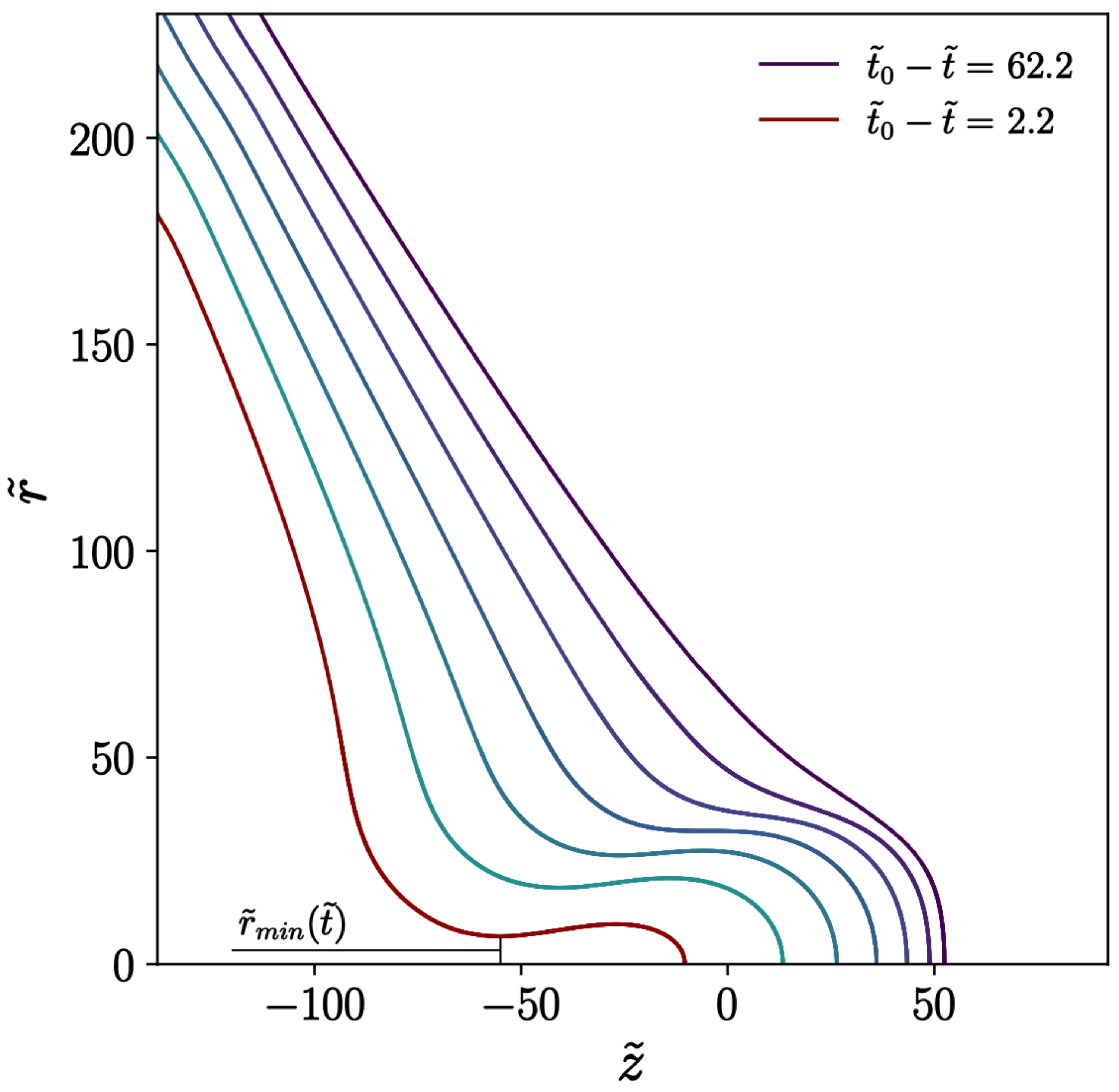
New evolution due to different self-similar flows depending on $|\widetilde{\mu}_0|$

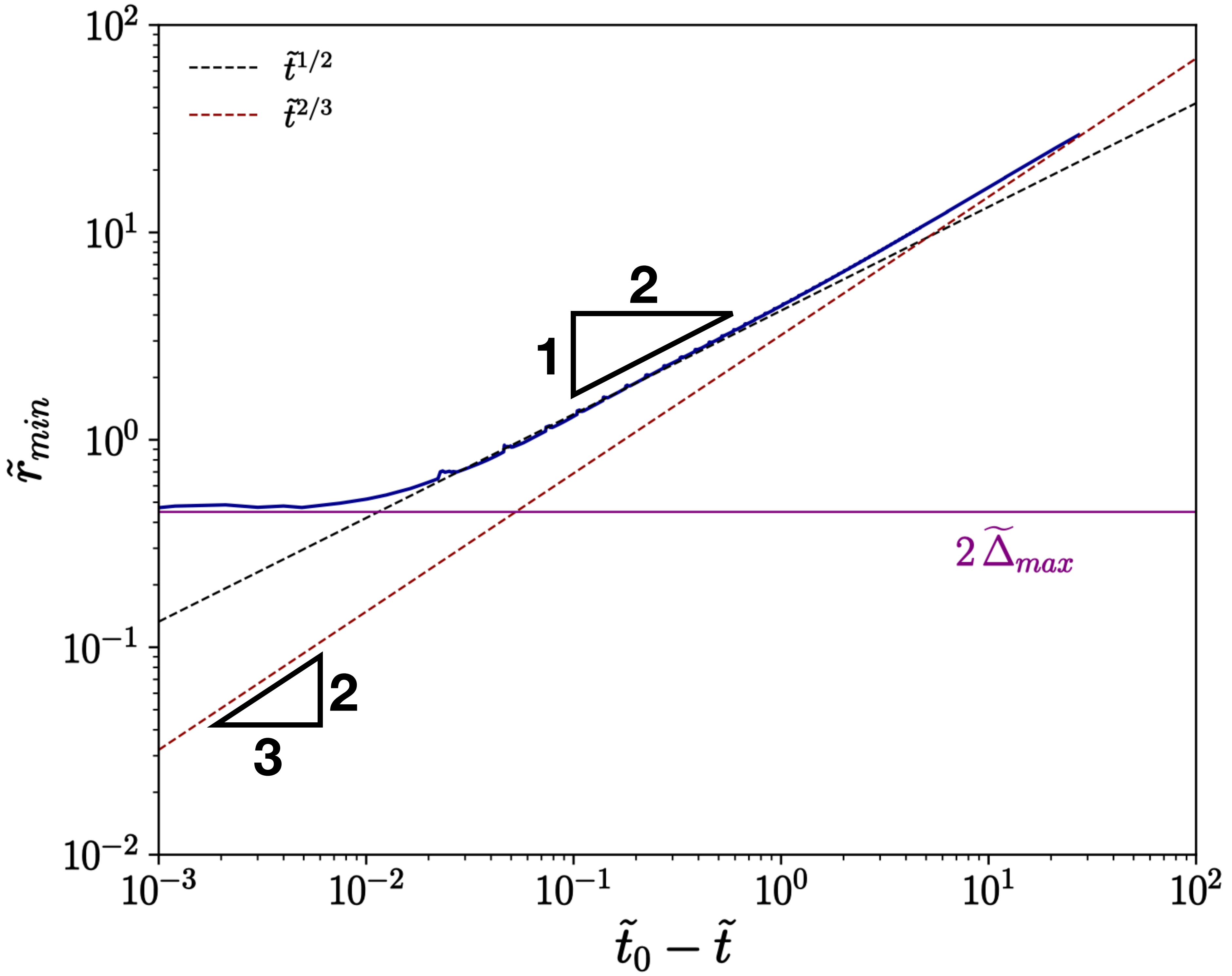
***INERTIAL* scaling!**

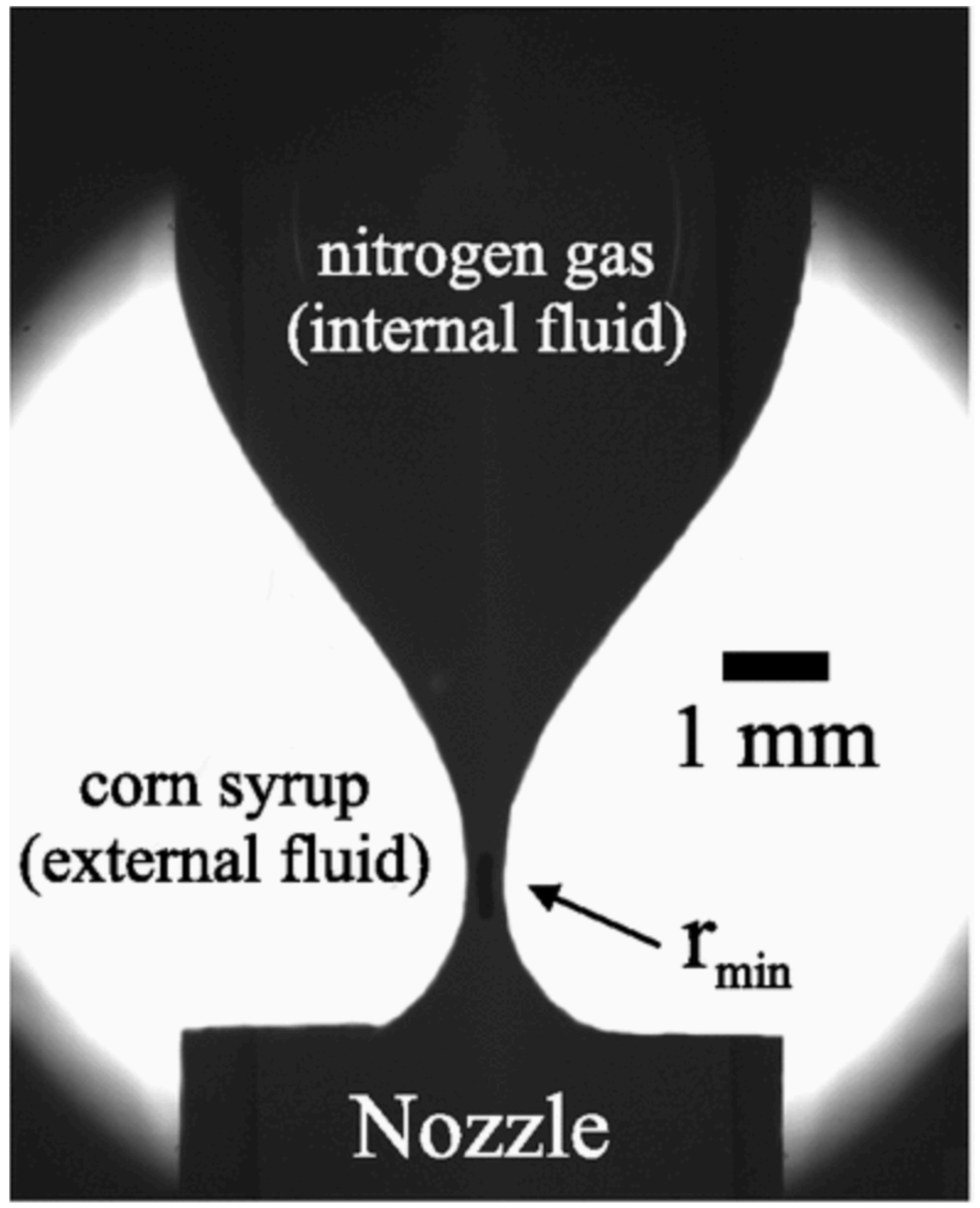
Eggers *et al.* (2007)



IV.8 - Perturbation of Boundary Conditions



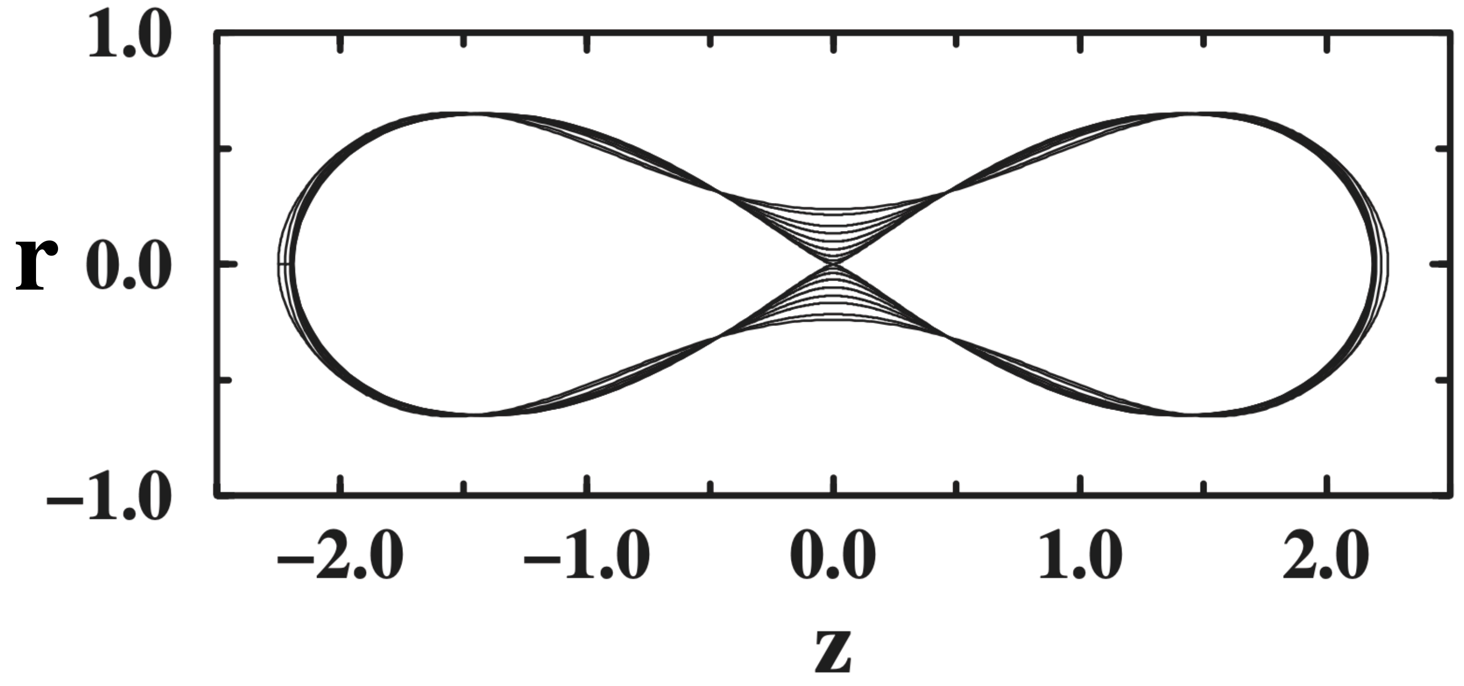




Burton *et al.* (2005)

PINCHING!!

New evolution due to different self-similar flows depending on $|\widetilde{\mu}_0|$

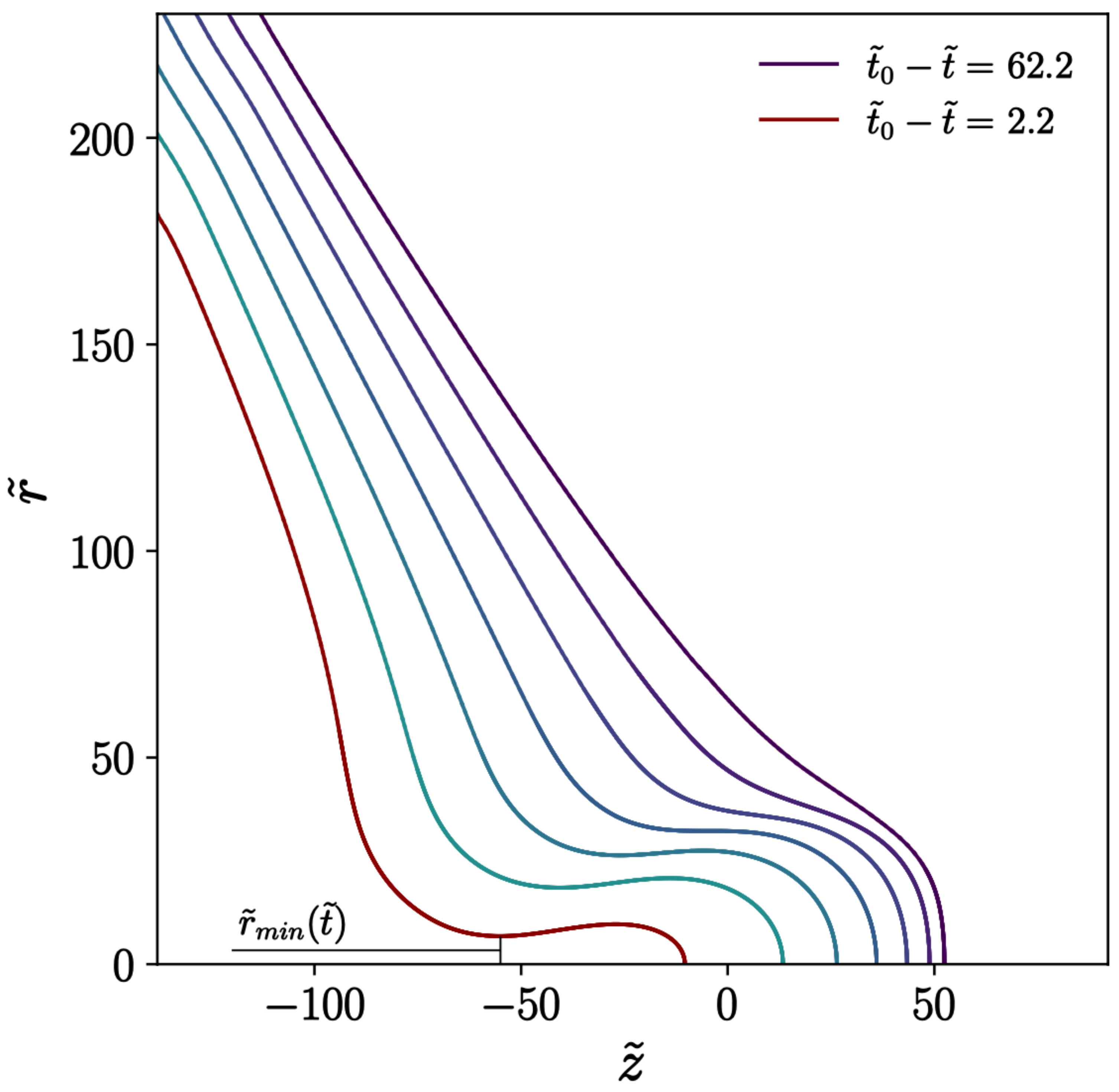


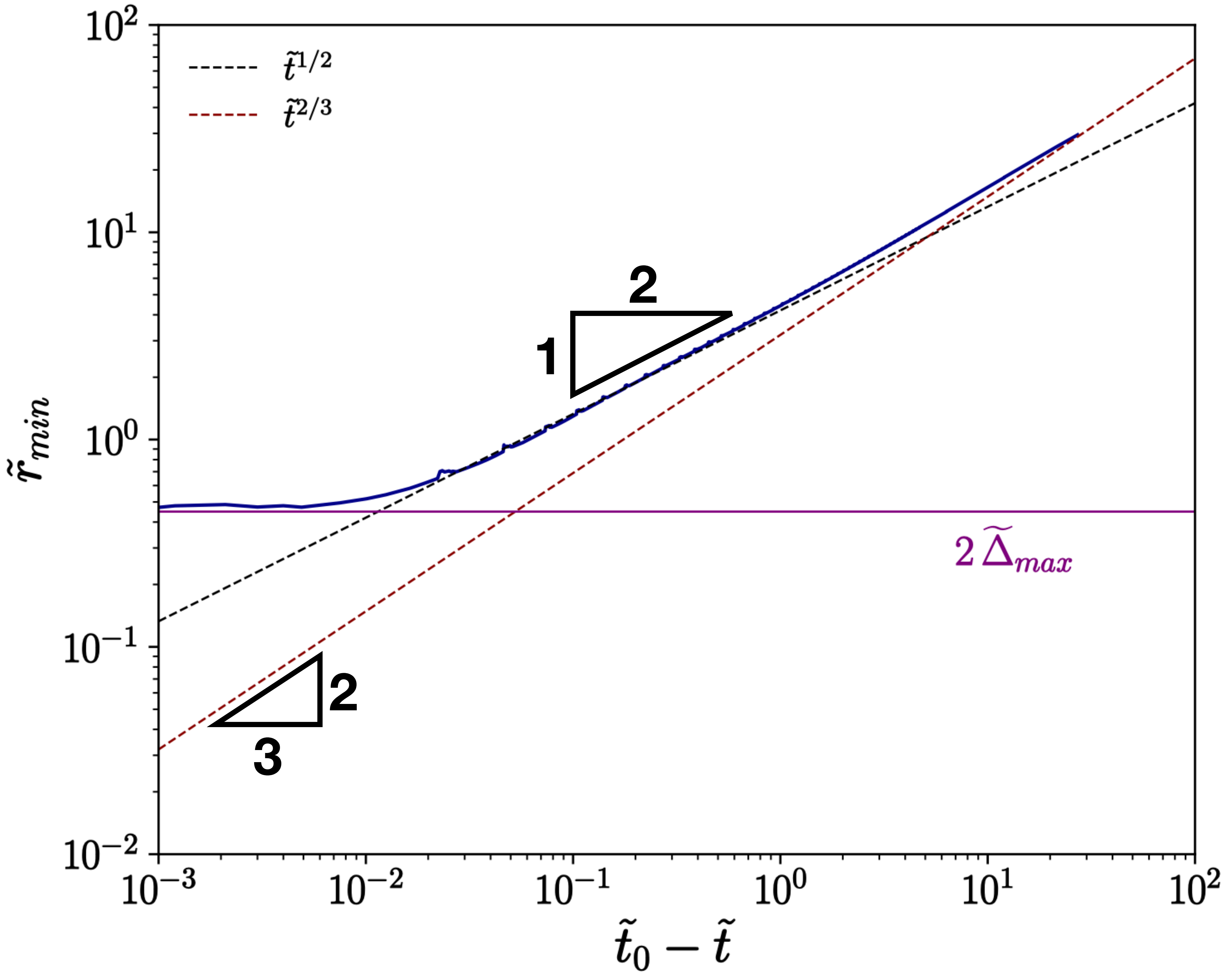
***INERTIAL* scaling!**

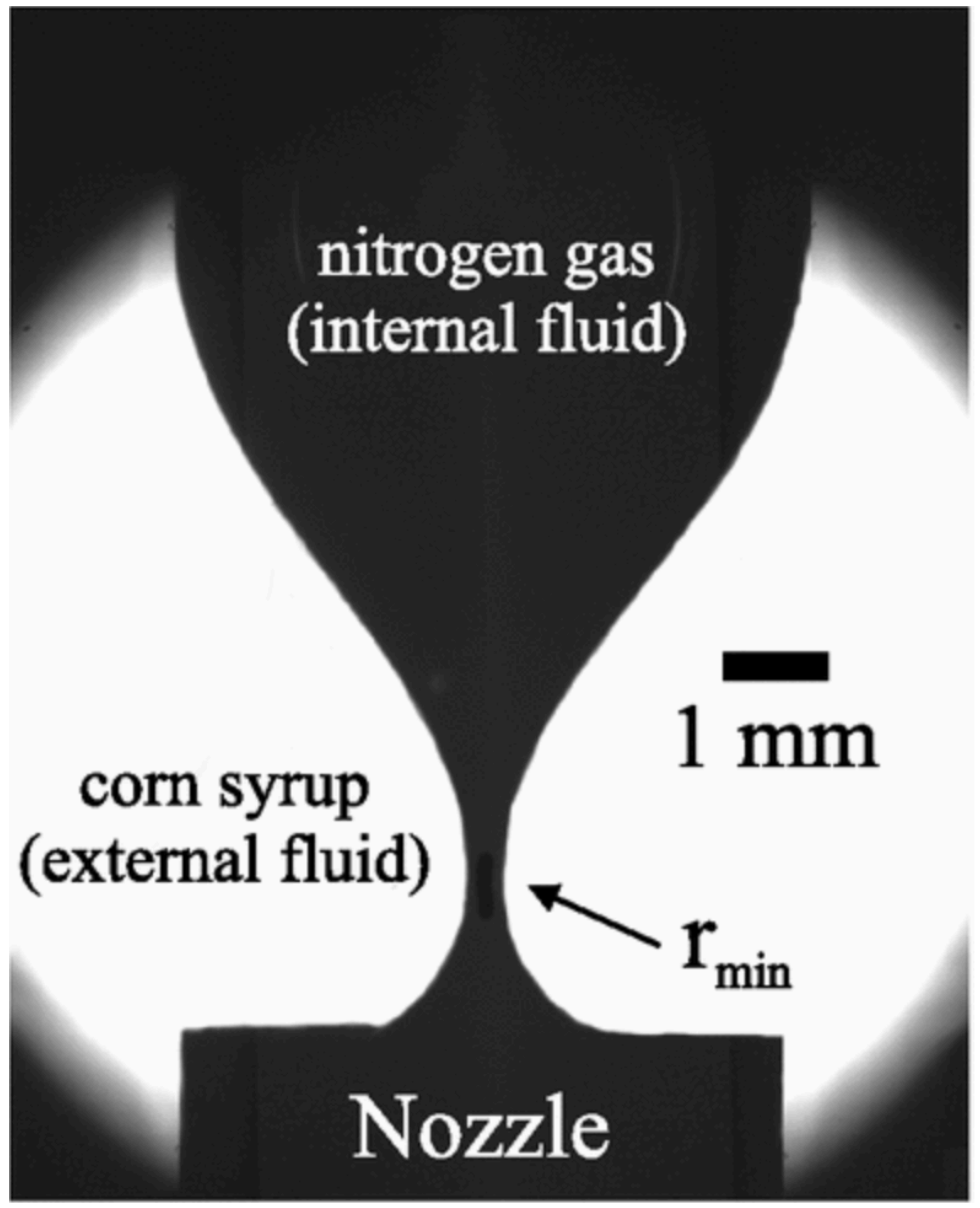
Eggers *et al.* (2007)

BCs perturbation

$\Rightarrow$ *switch in finite-time singularities*







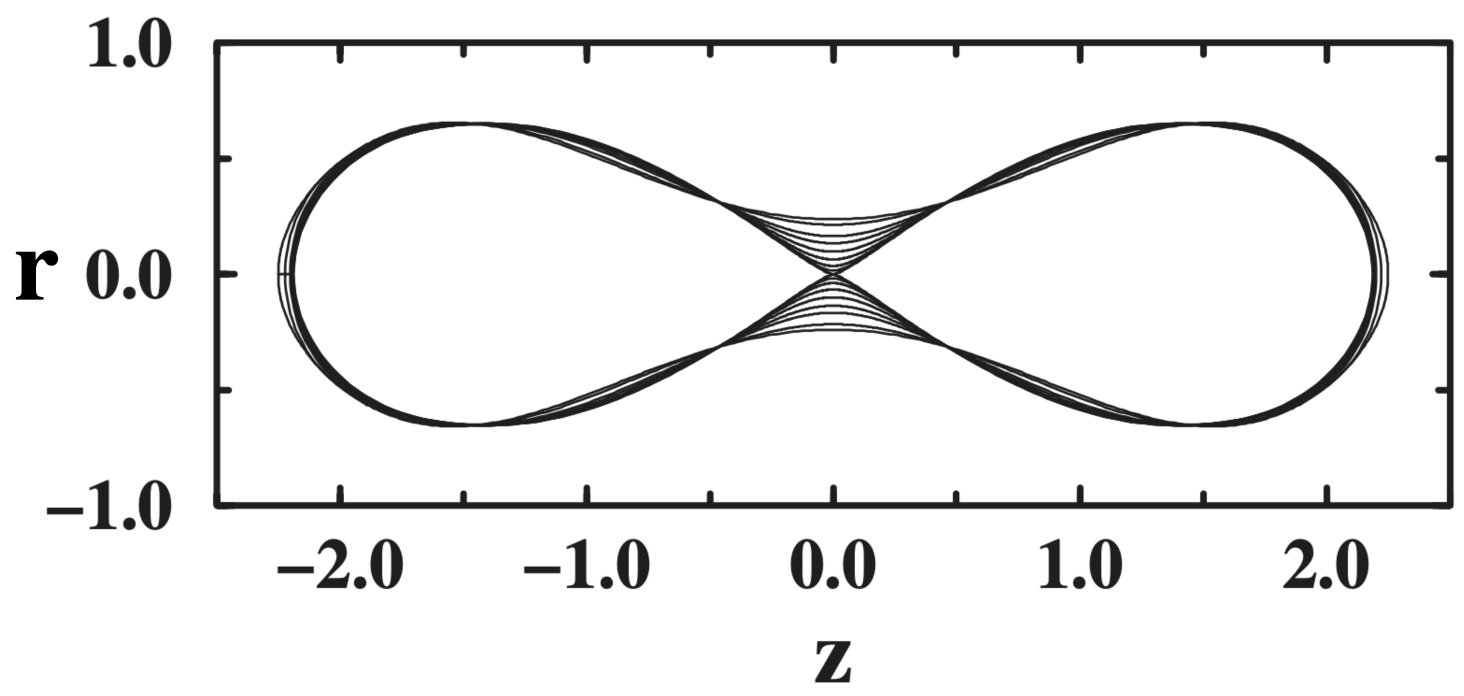
Burton *et al.* (2005)

PINCHING!!

New evolution due to different self-similar flows depending on $|\widetilde{\mu}_0|$

INERTIAL **scaling!**

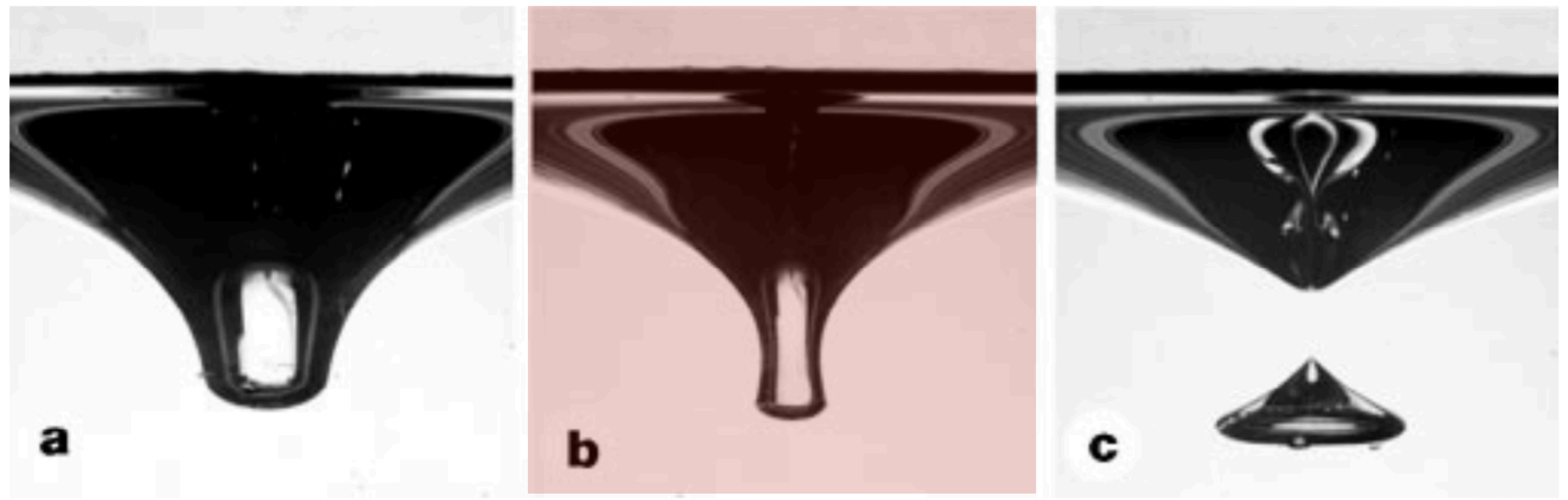
Eggers *et al.* (2007)



BCs perturbation

⇒ *switch in finite-time singularities*

More violent singularity: *velocity* $\propto \tilde{t}'^{-1/2}$



Zeff *et al.* (2000)