Implementation of a $k - \epsilon$ RANS turbulence model in Basilisk

Basilisk Gerris User Meeting 2025

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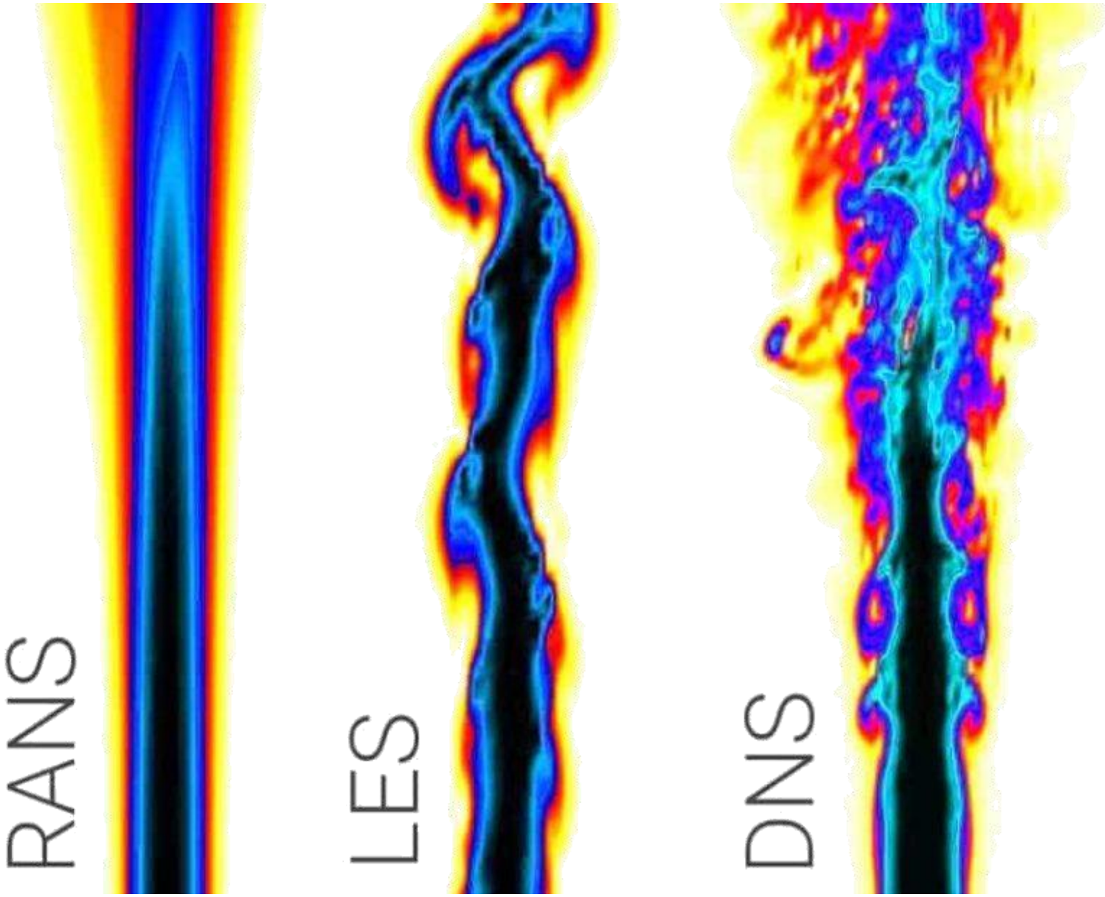
Turbulence

https://gauss.math.yale.edu

Computation of Turbulence

Modeling

Computational cost

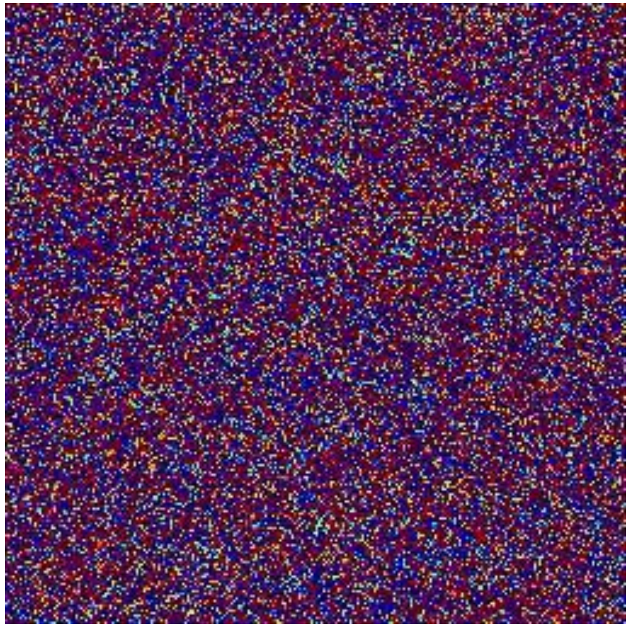


SimWorks User guide

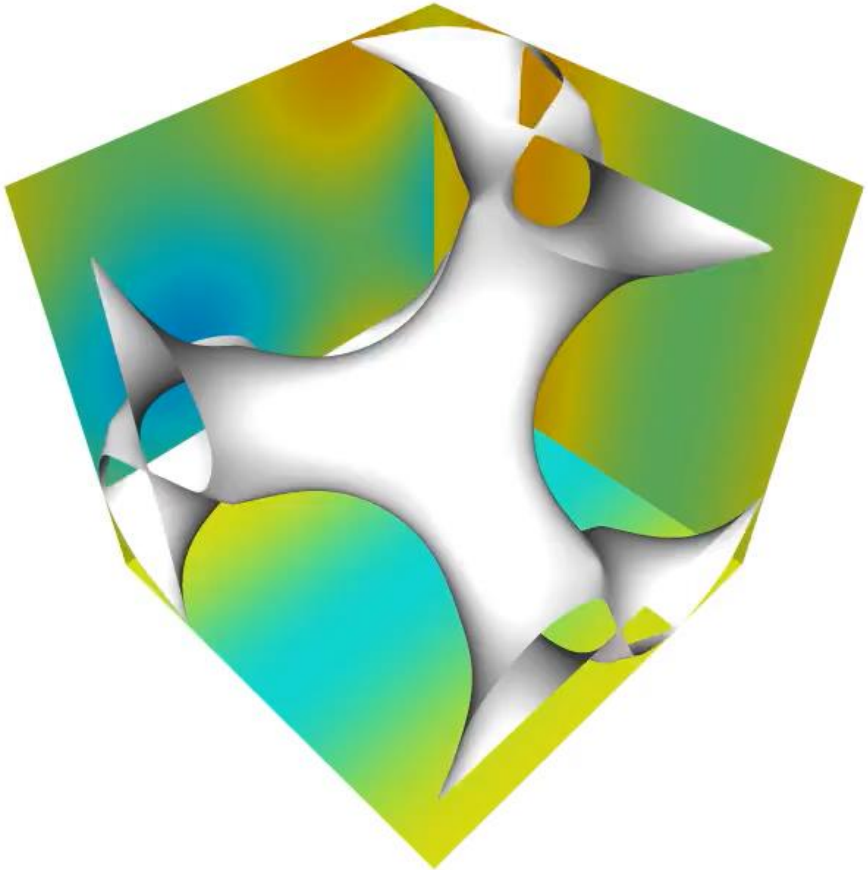
- DNS (Direct Numerical Simulation) : no model
- LES (Large Eddy Simulation) : small scales modeled
- RANS (Reynolds Average Navier-Stokes) : Average of the flow
  - Mostly used by industry

Turbulence in Basilisk

- Mostly DNS :
  - Isotropic turbulence



Examples/turbulence.c in 2D



Examples/isotropic.c in 3D

- Some case of modeling in sandboxes : Antoon van Hooft (LES)

RANS model

- Average of the Navier Stokes equation

$$\phi = \bar{\phi} + \phi' : \bar{\phi} \text{ mean part, } \phi' \text{ fluctuation}$$

$$\rho\left(\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j}\right) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \bar{u}_i}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\underbrace{\left(\rho \overline{u_j' u_i'}\right)}_{\text{Reynolds tensor}}$$

$$\text{Boussinesq hypothesis : } \rho \overline{u_j' u_i'} = \mu_t S_{ij} \quad S_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

- $k - \epsilon$ model

$$\text{Dimensional analysis : } \mu_t \propto \rho \frac{k^2}{\epsilon}$$

$$k = \frac{1}{2}\overline{u_i' u_i'} \text{ turbulent kinetic energy}$$

$$\epsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}} \text{ rate of dissipation}$$

Turbulence modeling

- Equation for the $k - \epsilon$ [1]

advection
diffusion
production
dissipation

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$$

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho k u_i}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \rho\epsilon$$

$$\frac{\partial \rho \epsilon}{\partial t} + \frac{\partial \rho \epsilon u_i}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] + C_{1\epsilon}\frac{\epsilon}{k}P_k - C_{2\epsilon}\rho\frac{\epsilon^2}{k}$$

$\mathrm{P_k} = \mu_t S^2$ with S the norm of the deformation tensor

$C_{1\epsilon} = 1.44,\ C_{2\epsilon} = 1.92,\ C_\mu = 0.09,\ \sigma_k = 1,\ \sigma_\epsilon = 1.3$

[1] The Numerical Computation of Turbulent Flows, B. E. Launder, D. B. Spalding, 1973

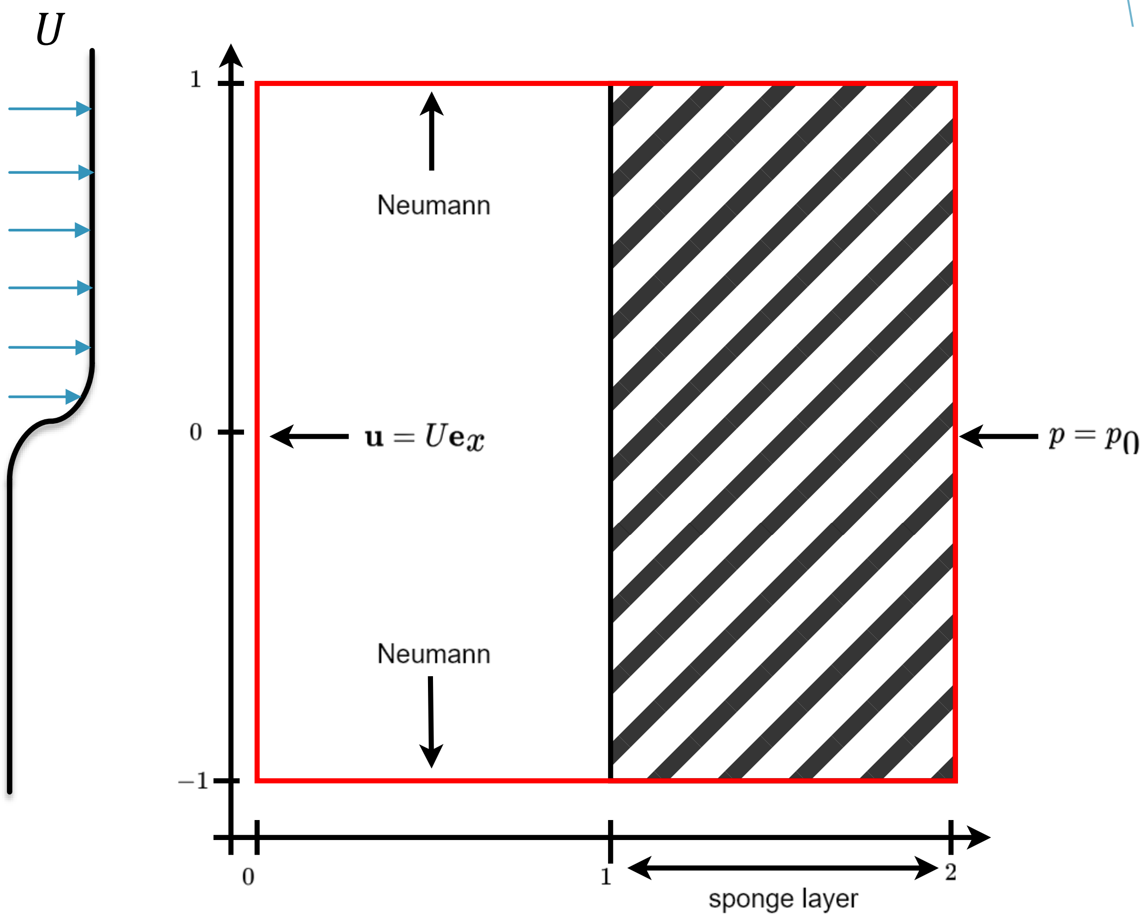
Test on a 2D Mixing Layer

Upstream velocity:

$$U = U_1\left(\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{y}{\delta_i}\right)\right)$$

$U_1 = 18 m.s^{-1}$ [1]

$\delta_i = 0.1m$

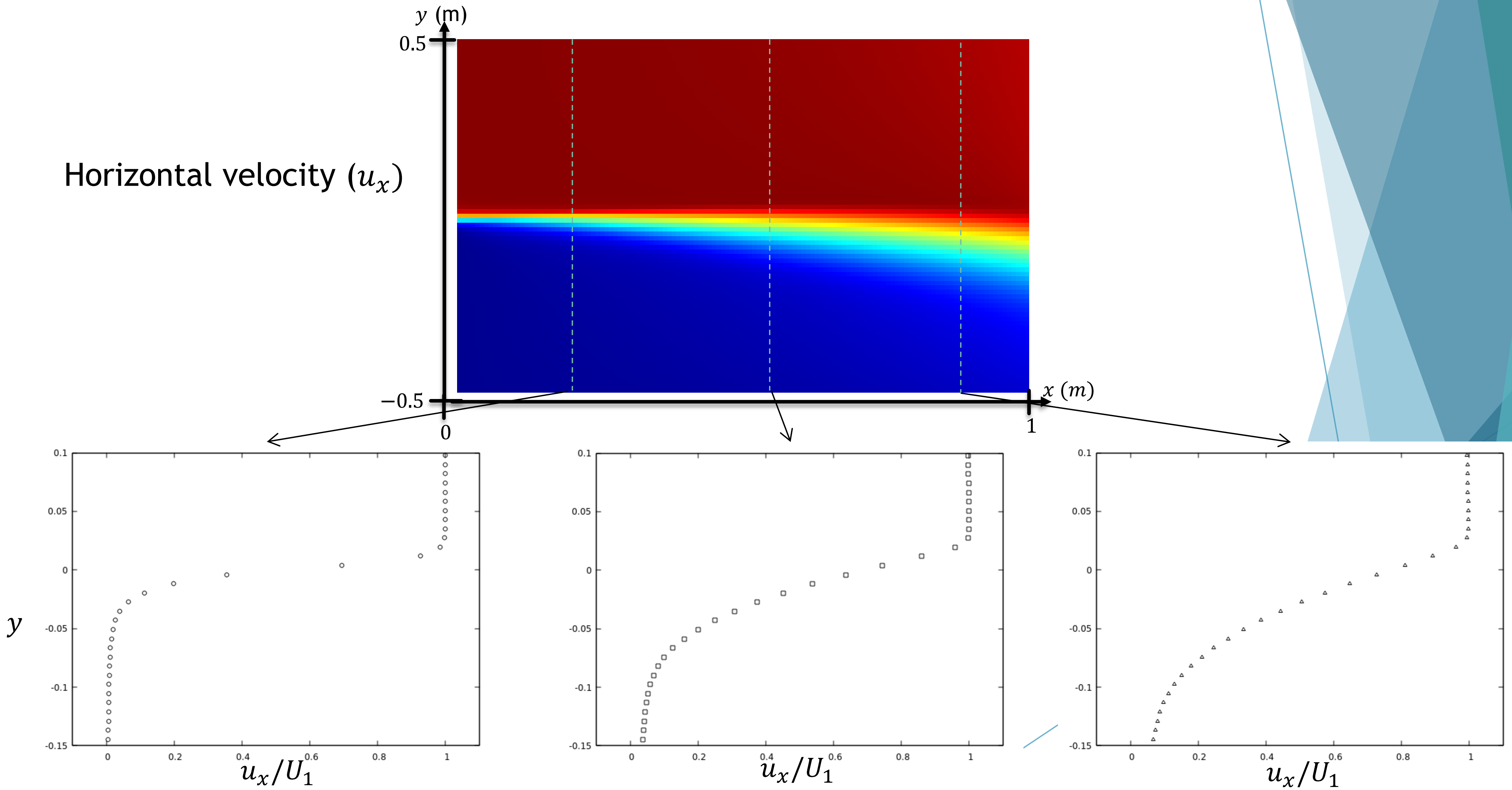


$$Re = \frac{\rho U_1 1}{\mu} = 2.34 \times 10^6$$

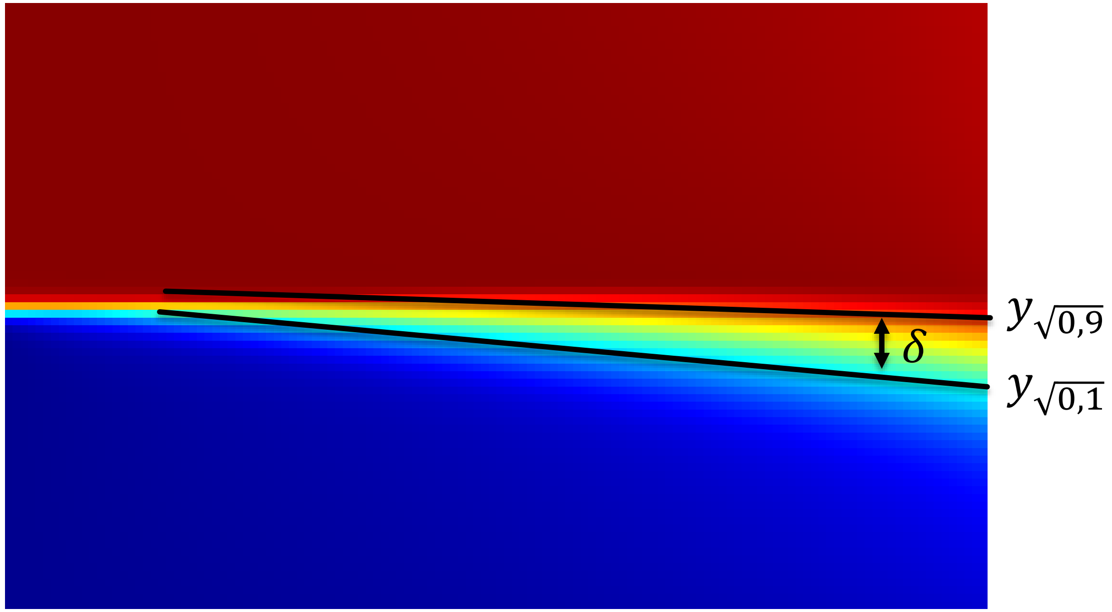
[1] Investigations of free turbulent mixing, H. W. Liepmann and J. Laufer, 1947
[2] Turbulence Modeling Validation, Testing, and Development, J. E. Bardina, P. G. Huang, T. J. Coakley, 1997

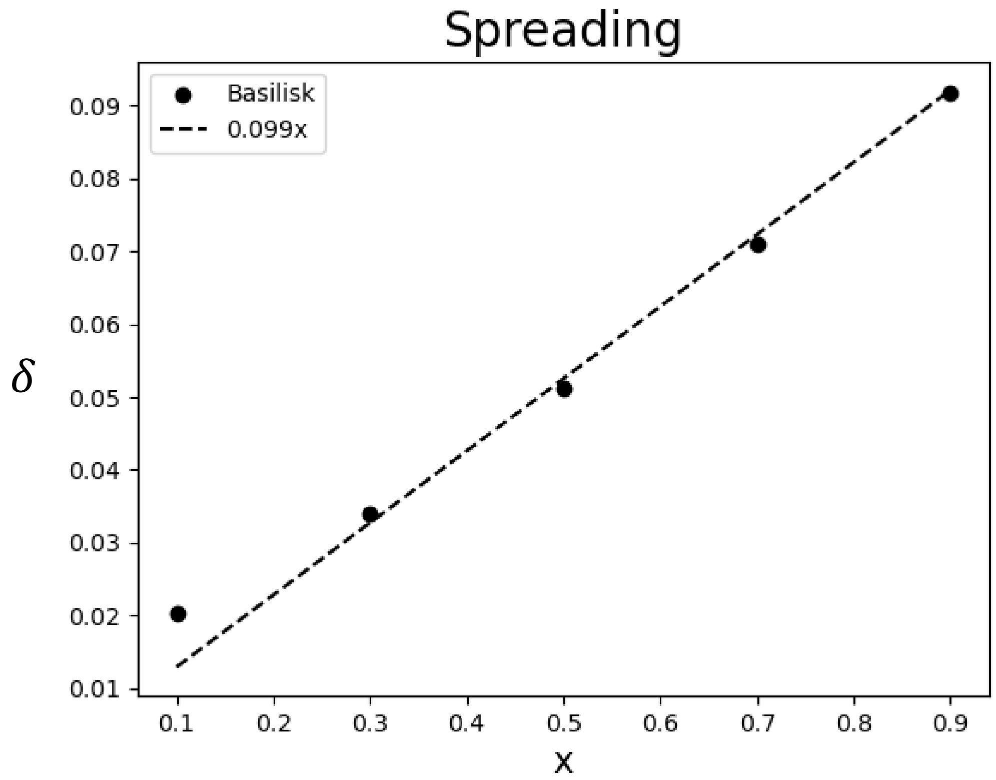
2D Mixing Layer : Results

Horizontal velocity (u_x)



2D Mixing layer : Spreading





$$u_x(y_\alpha) = \alpha U_1$$

Spreading : $\delta = y_{\sqrt{0,9}} - y_{\sqrt{0,1}}$

Model	Spreading Rate
Experiment	0.115 ± 0.015
Launder-Sharma $k - \epsilon$ [1]	0.099
Basilisk	0.099

[1] Turbulence Modeling Validation, Testing, and Development, J. E. Bardina, P. G. Huang, T. J. Coakley, 1997

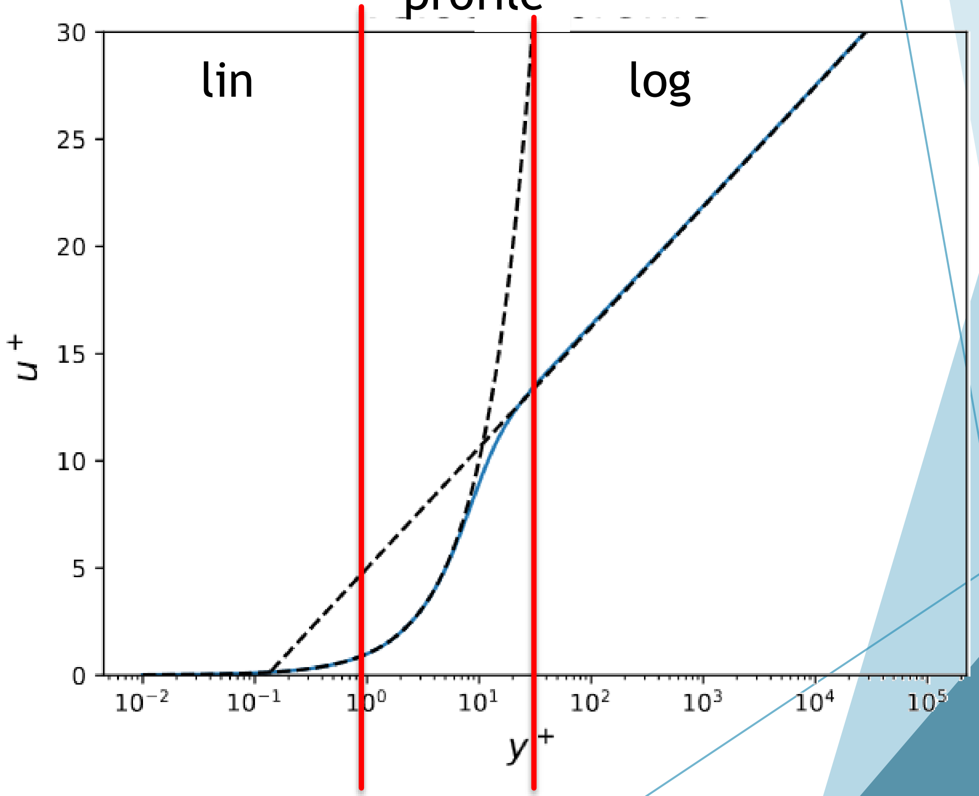
Boundary Layer close to a Solid

y^+, u^+ inner non-dimensional variable

y^+ : vertical position

u^+ : tangential velocity

Theoretical boundary layer profile



2 kinds of model depending on where is the first point above the wall

$y^+ > 30$: zone log →High Reynolds model

$y^+ < 1$: zone lin →Low Reynolds model

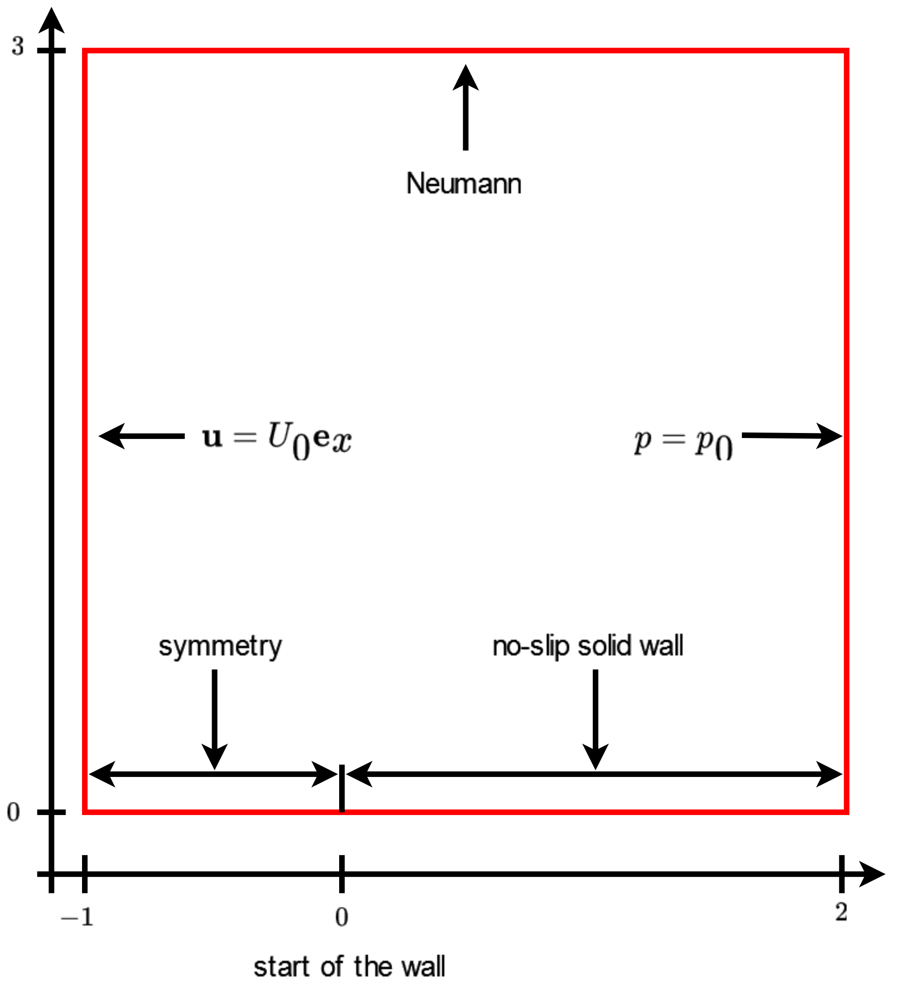
Basilisk classical solver uses isotropic mesh : use of high Reynolds model because $y^+ > 30$

Test on a 2D Flat Plate

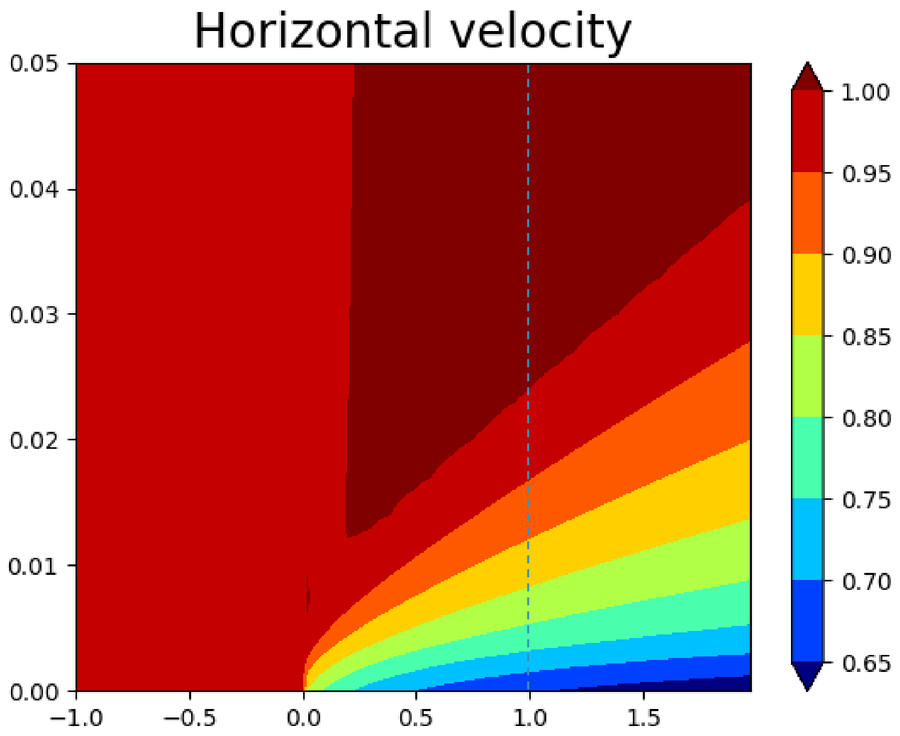
Examples from the Turbulence Modeling Resource of the NASA

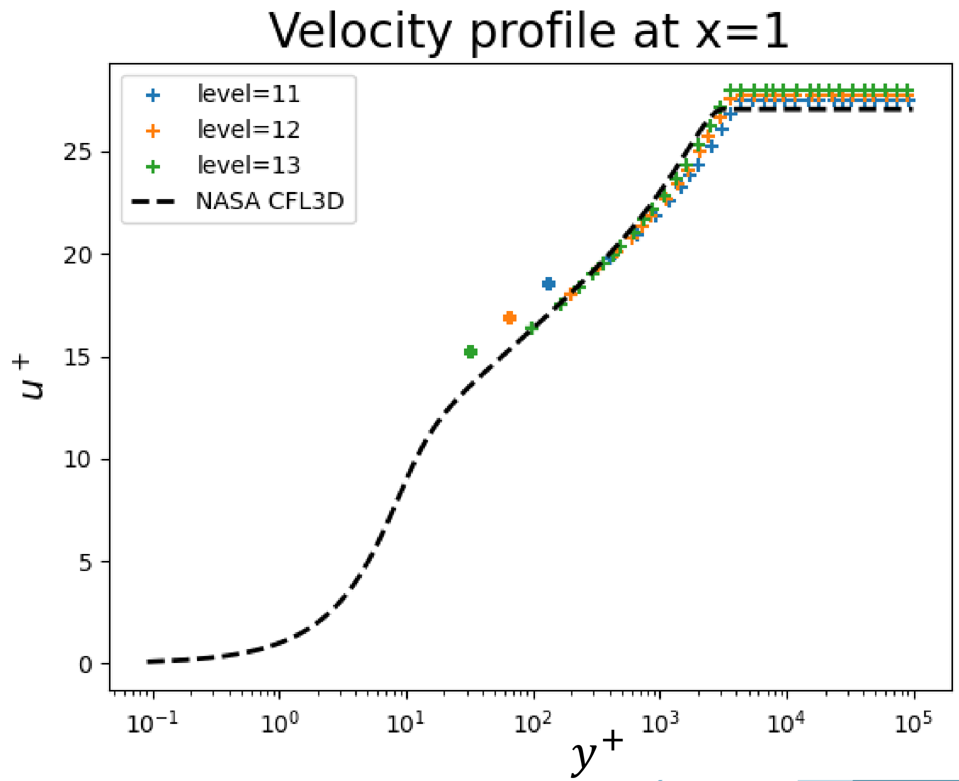
Maxlevel	Minlevel	y^+ at $x = 1$	Number of cells
11	5	271	~16000
12	5	135	~60000
13	5	**68**	~132000

$$Re = \frac{\rho U_0 1}{\mu} = 5 \times 10^6$$

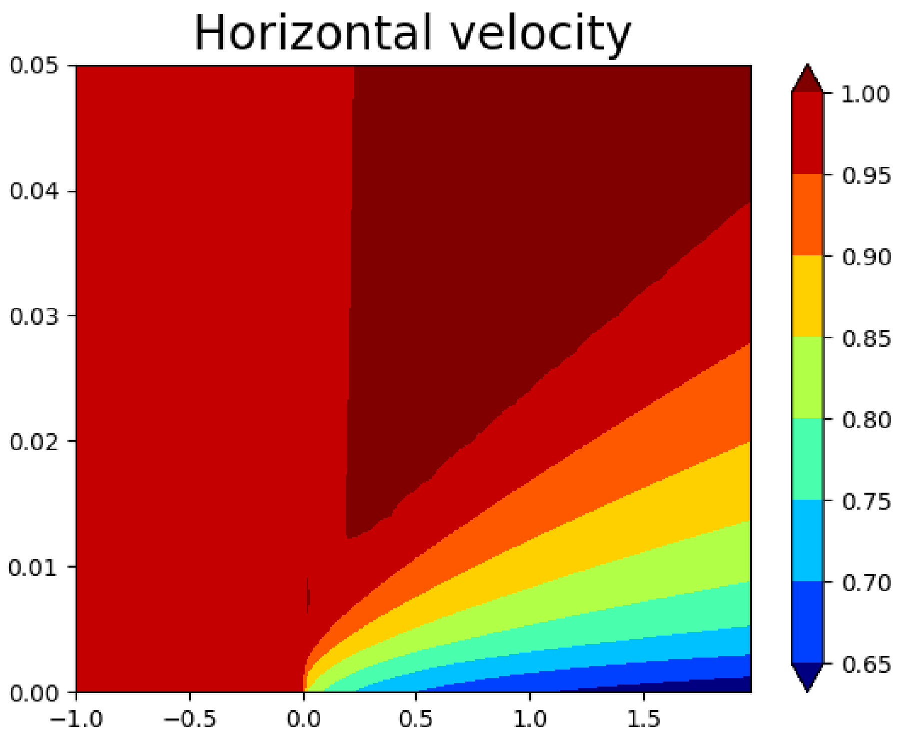


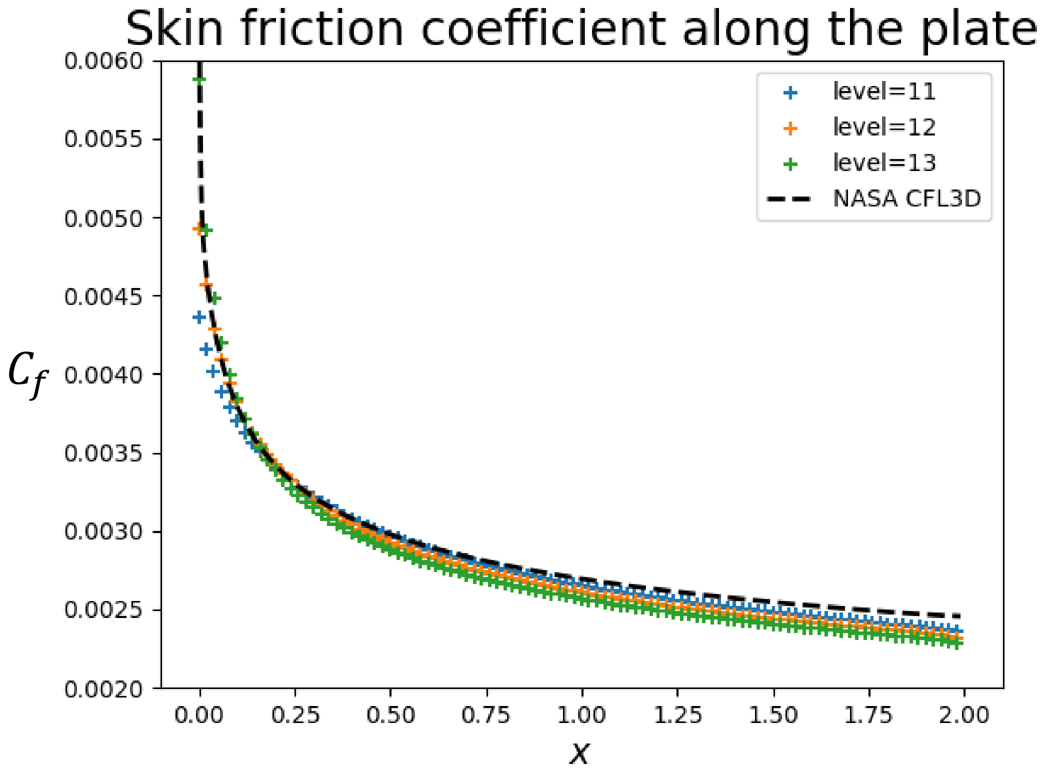
2D Flat Plate with High Reynolds Model





2D Flat Plate with High Reynolds model

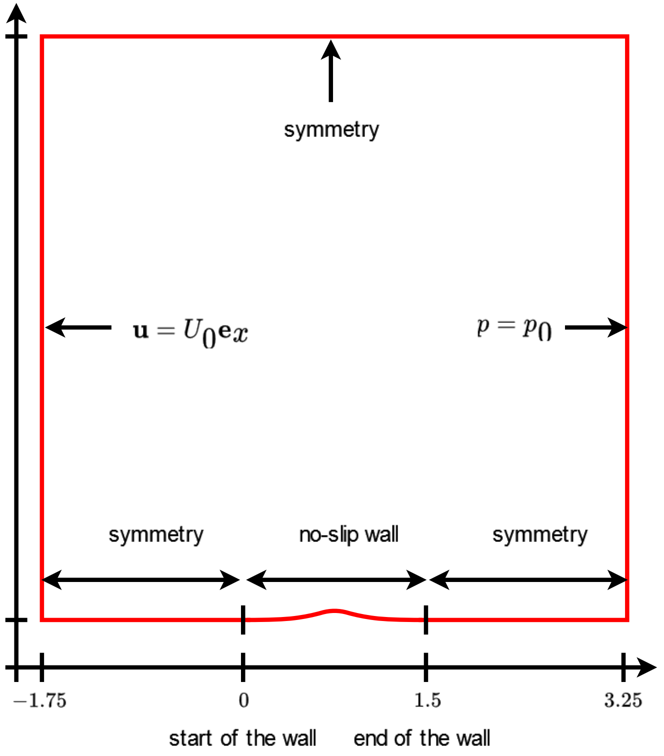


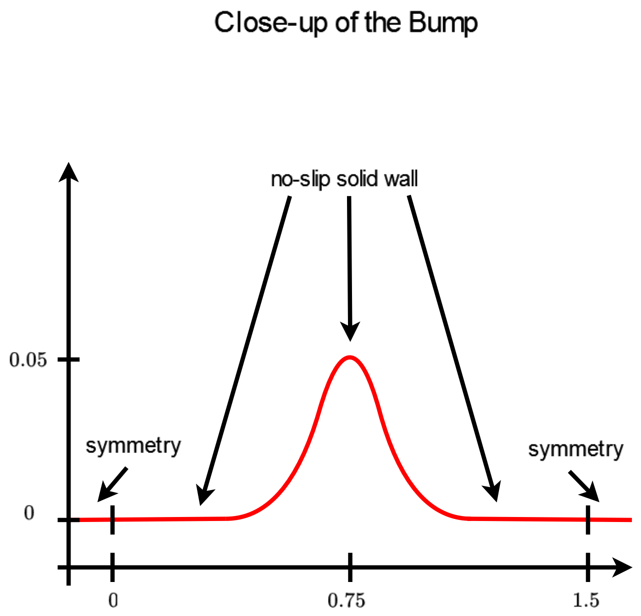


$$C_f = \frac{\mu \frac{\partial u_t}{\partial y}}{\frac{1}{2}\rho U_0^2}$$

2D Bump in a channel

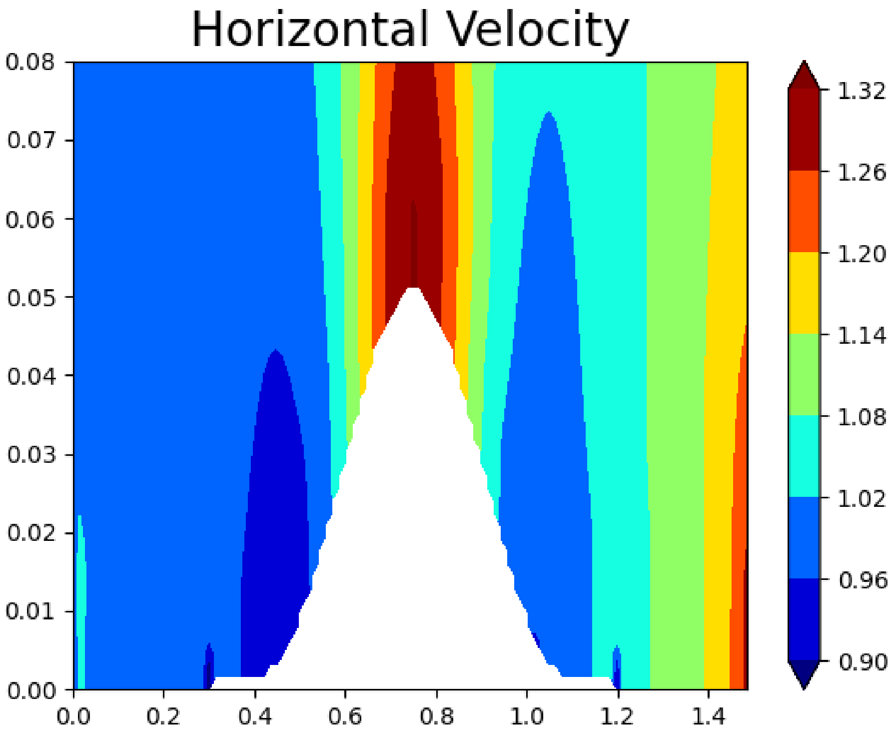
$$Re = \frac{\rho U_0 1}{\mu} = 3 \times 10^6$$

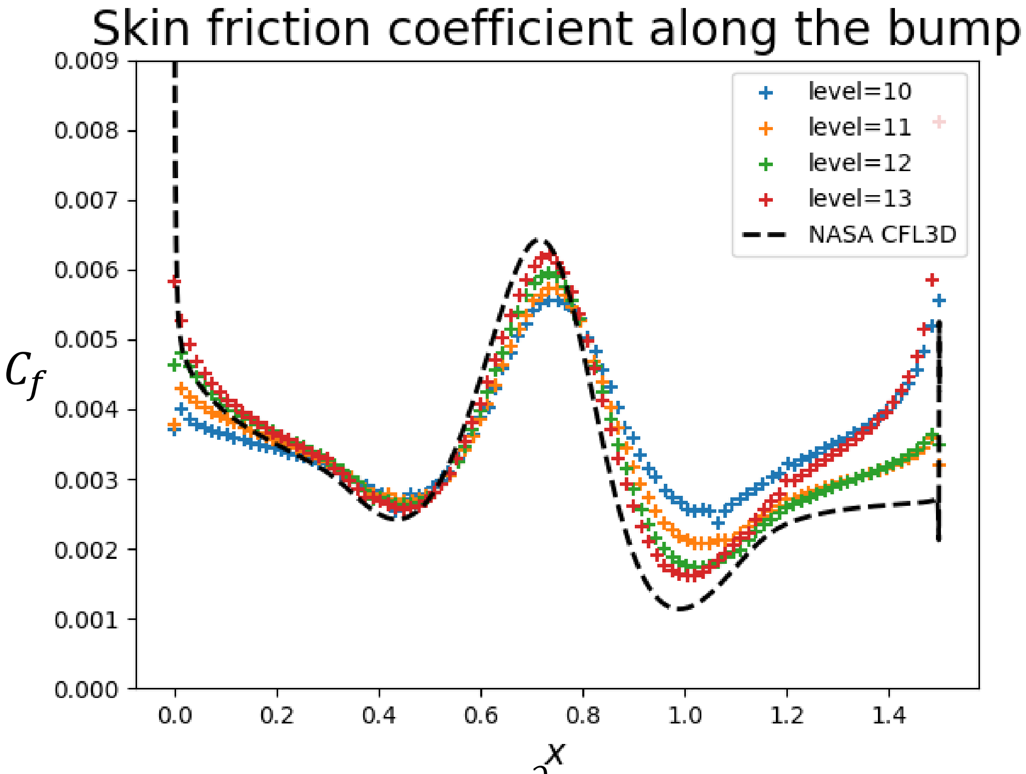




Maxlevel	Minlevel	y^+ at $x = 0{,}75$	Number of cells
10	5	1611	~15000
11	5	806	~31000
12	5	403	~52000
13	5	**201**	~92000

2D Bump in a channel

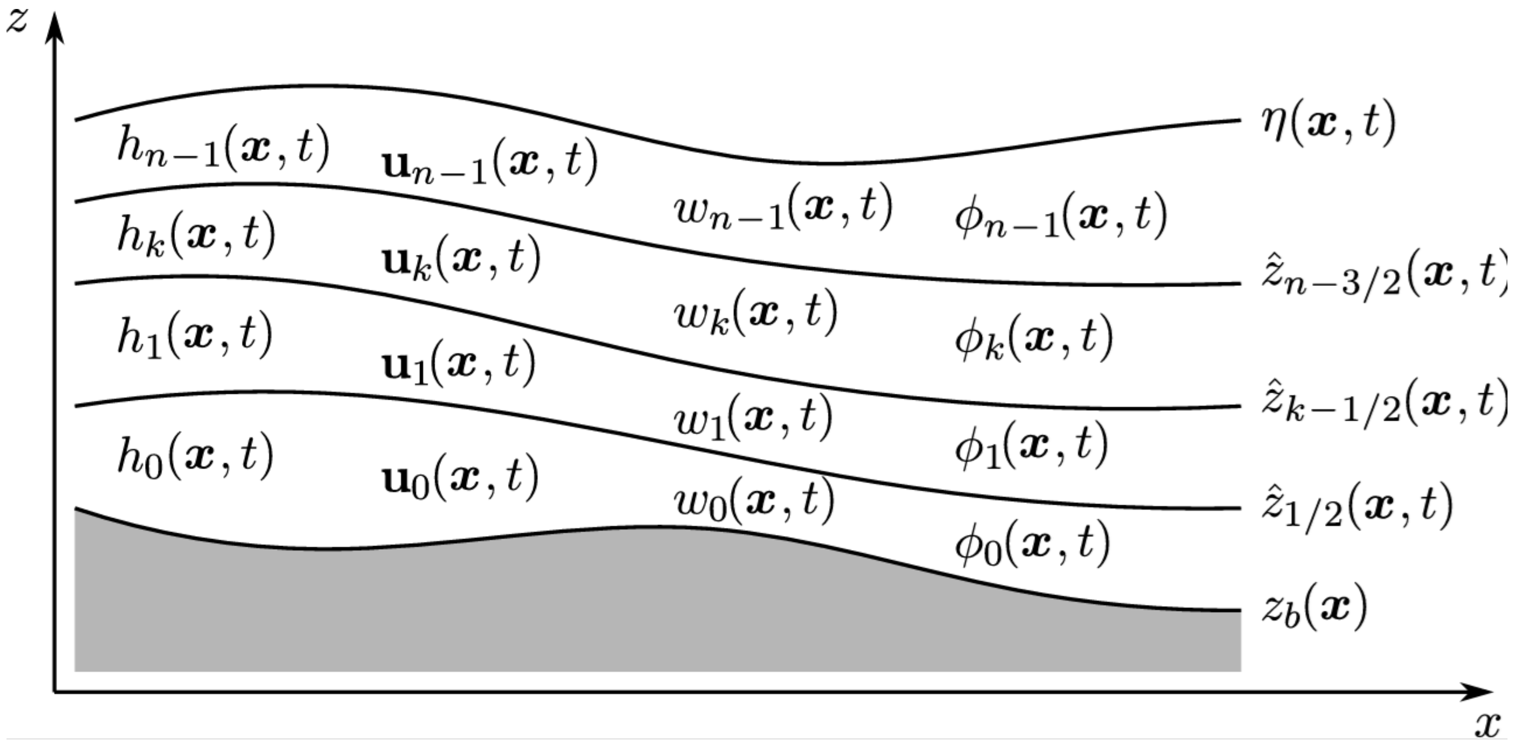




$$C_f = \frac{\mu \frac{\partial u_t}{\partial y}}{\frac{1}{2}\rho U_0^2}$$

Low Reynolds model

- Use of the mutlilayer solver : allows elongated mesh close to the solid → lower y^+



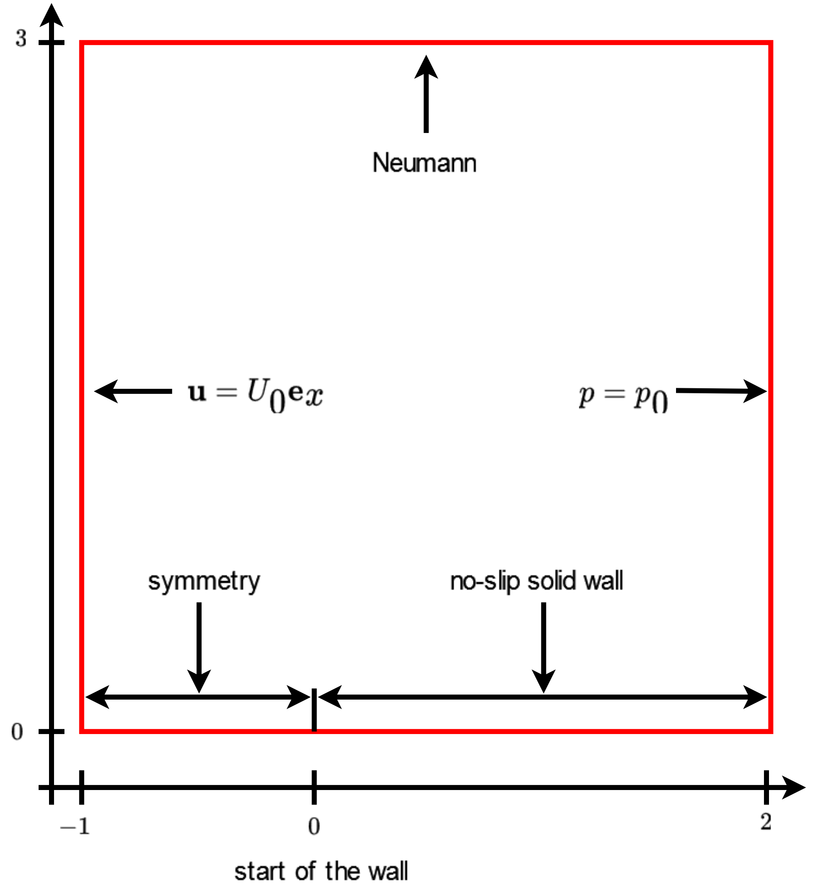
- Low Reynolds model of Chien [1]

[1] Predictions of Channel and Boundary-Layers Flows with a Low-Reynolds-Number Turbulence Model, K.-Y. Chien, 1982

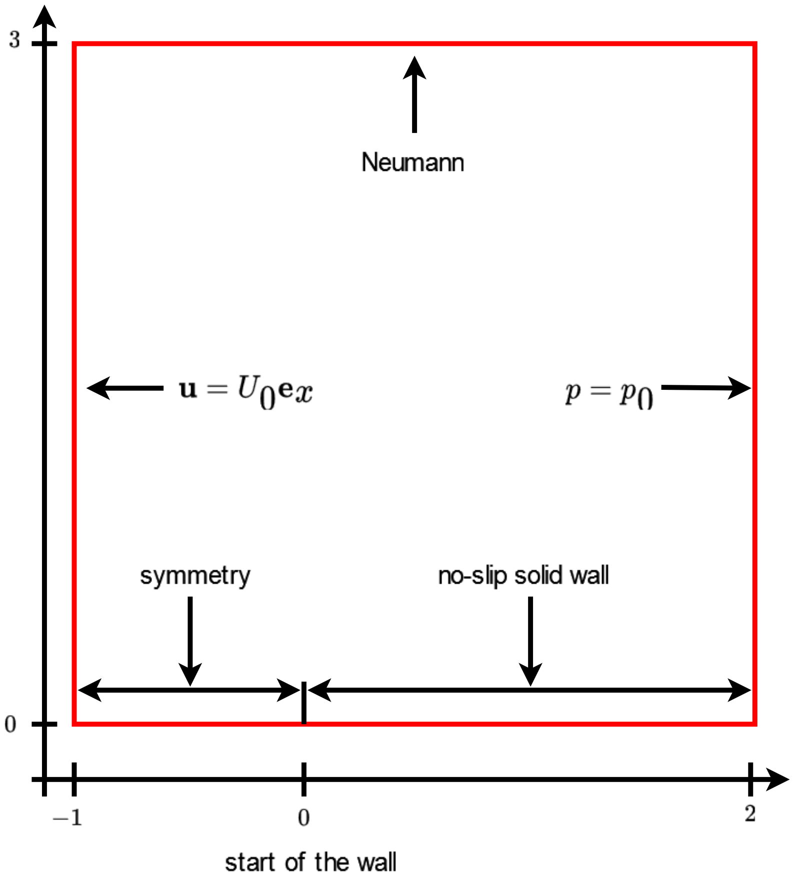
2D Flat Plate with Low Reynolds Model

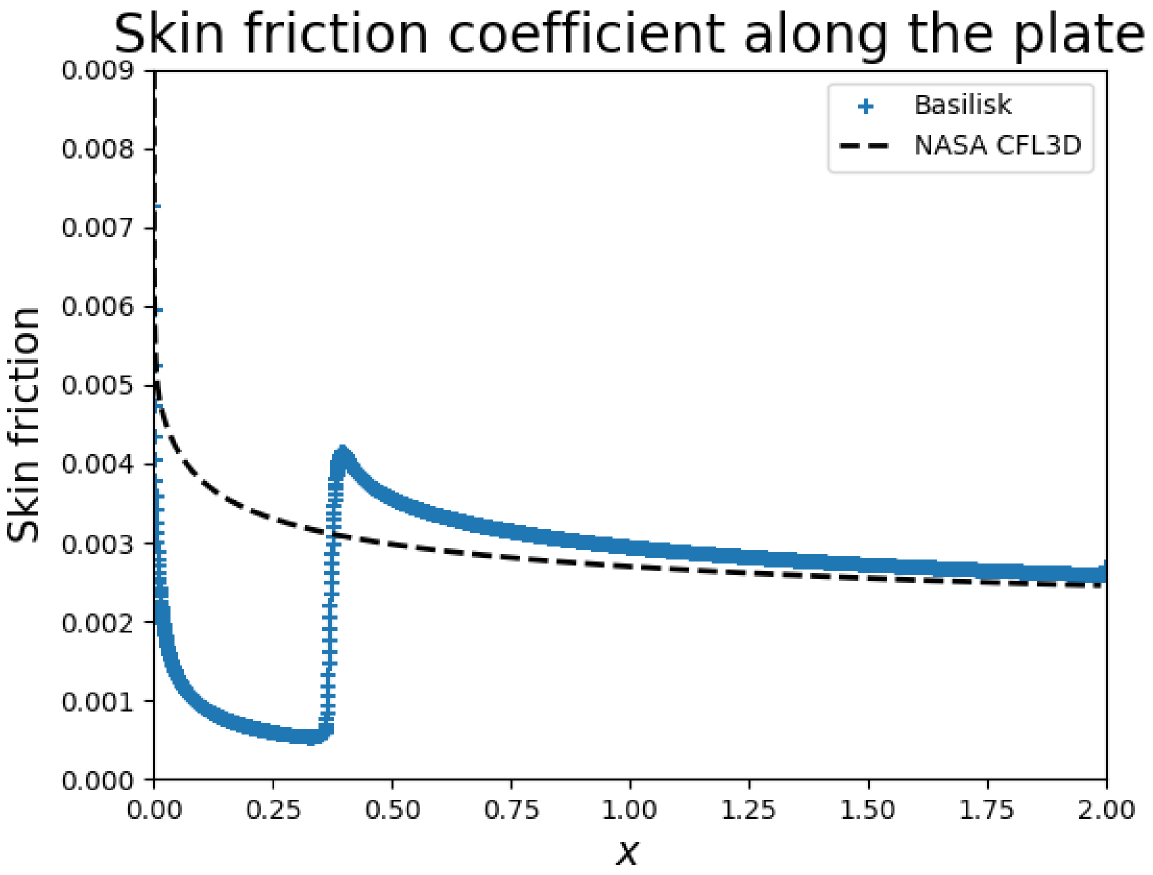
Same case as for high Reynolds model

Maxlevel	Minlevel	y^+ at $x = 1$	Number of cells
13	5	68	132028
Multilayer	$nl = 50$ layers	**1.2**	204800

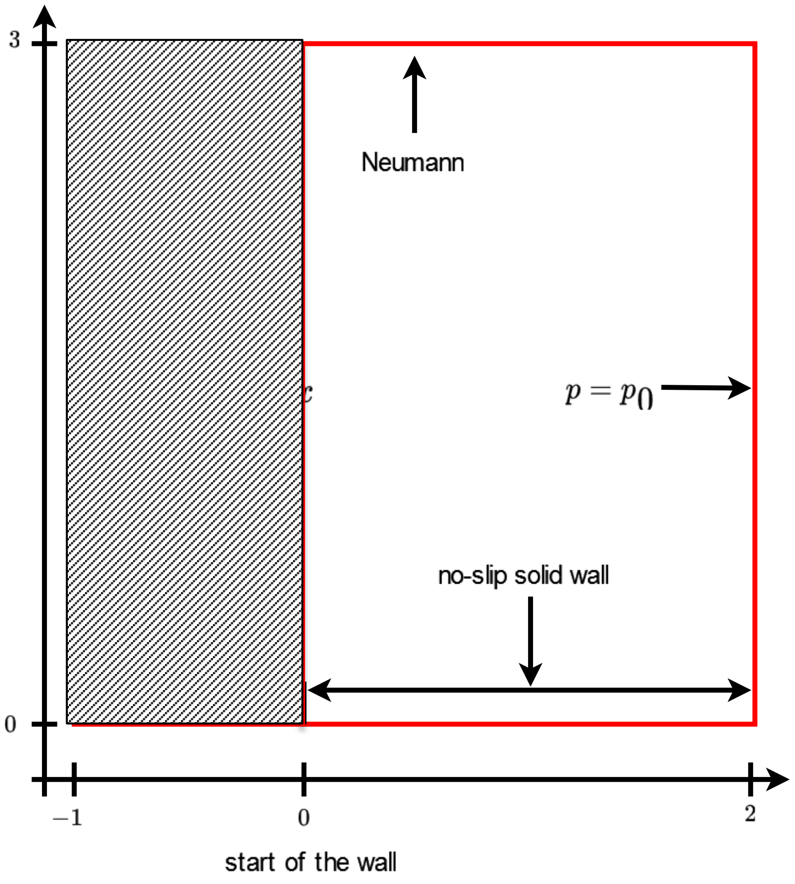


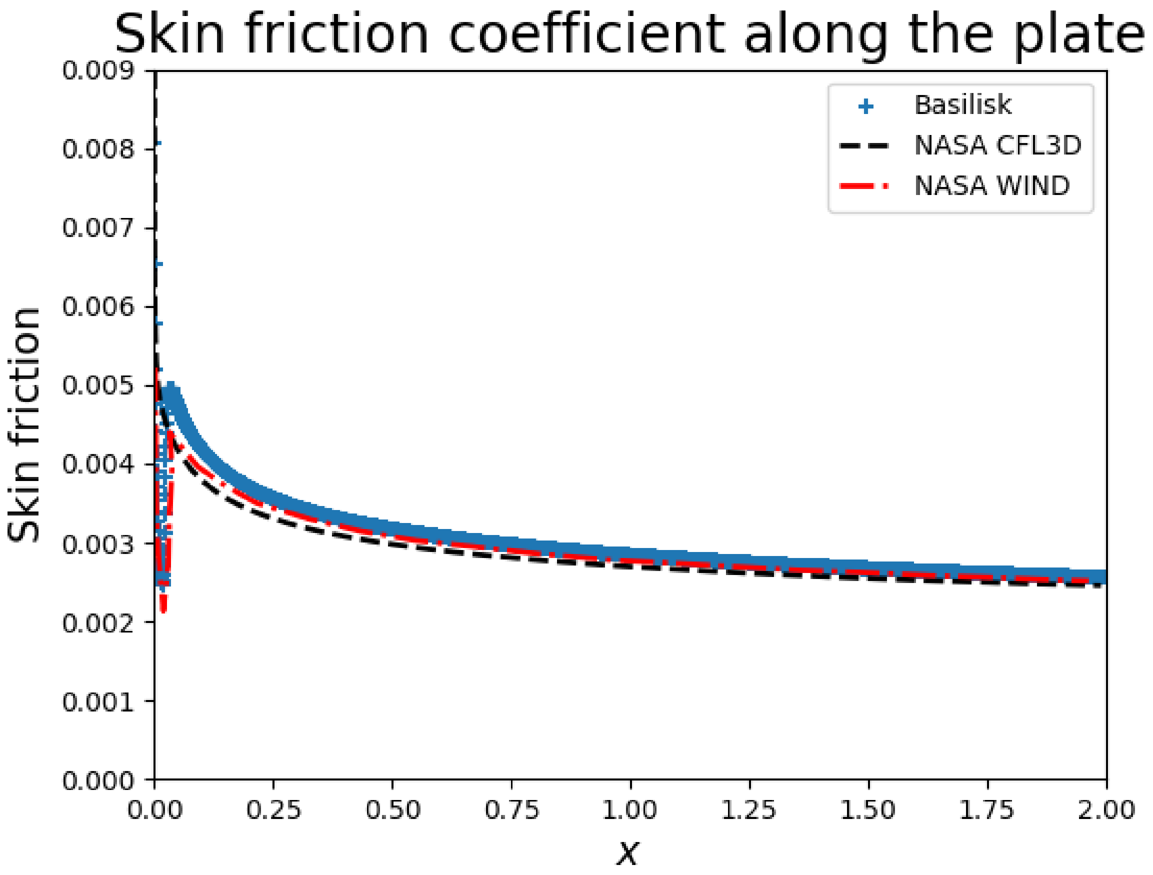
2D Flat Plate with Low Reynolds Model





2D Flat Plate with Low Reynolds Model

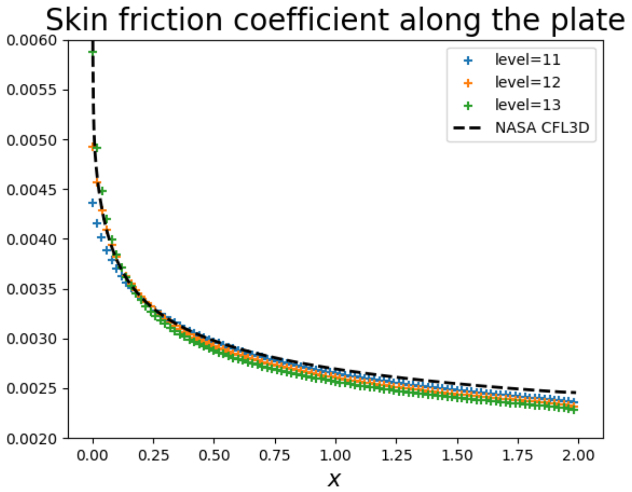


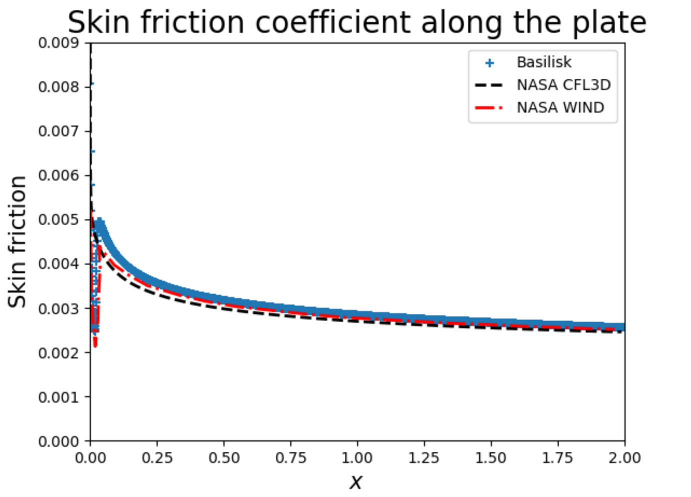


Conclusion and Perspectives

Implementation of a $k - \epsilon$ turbulence model in Basilisk :

- Good agreement with existing results for the mixing layer and the flat plate with the high-Re model
- More discrepancies for the bump and the flat plate with the low-Re model

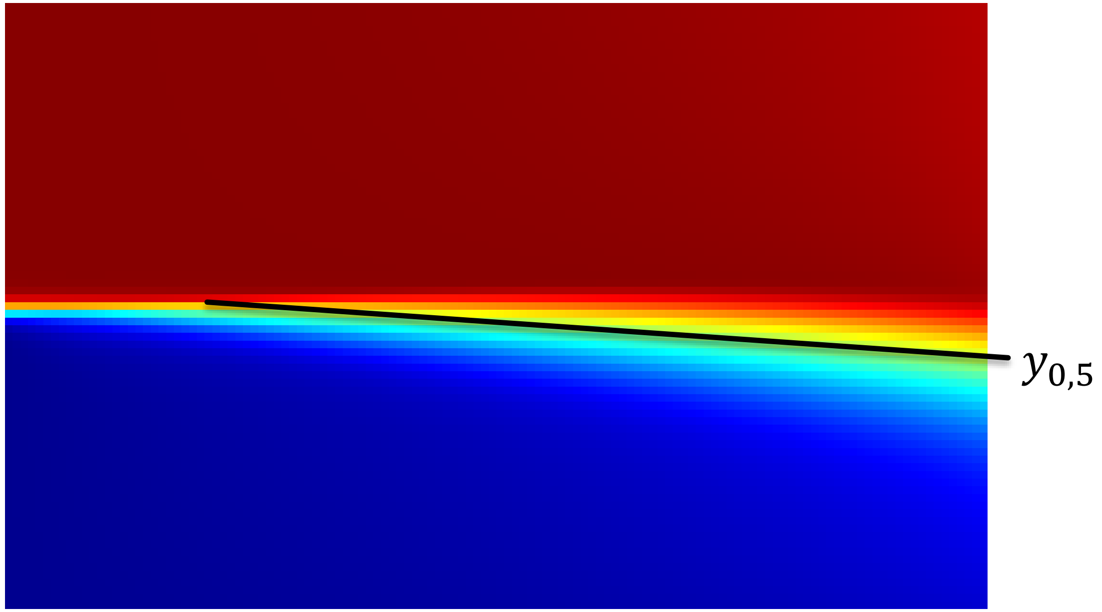




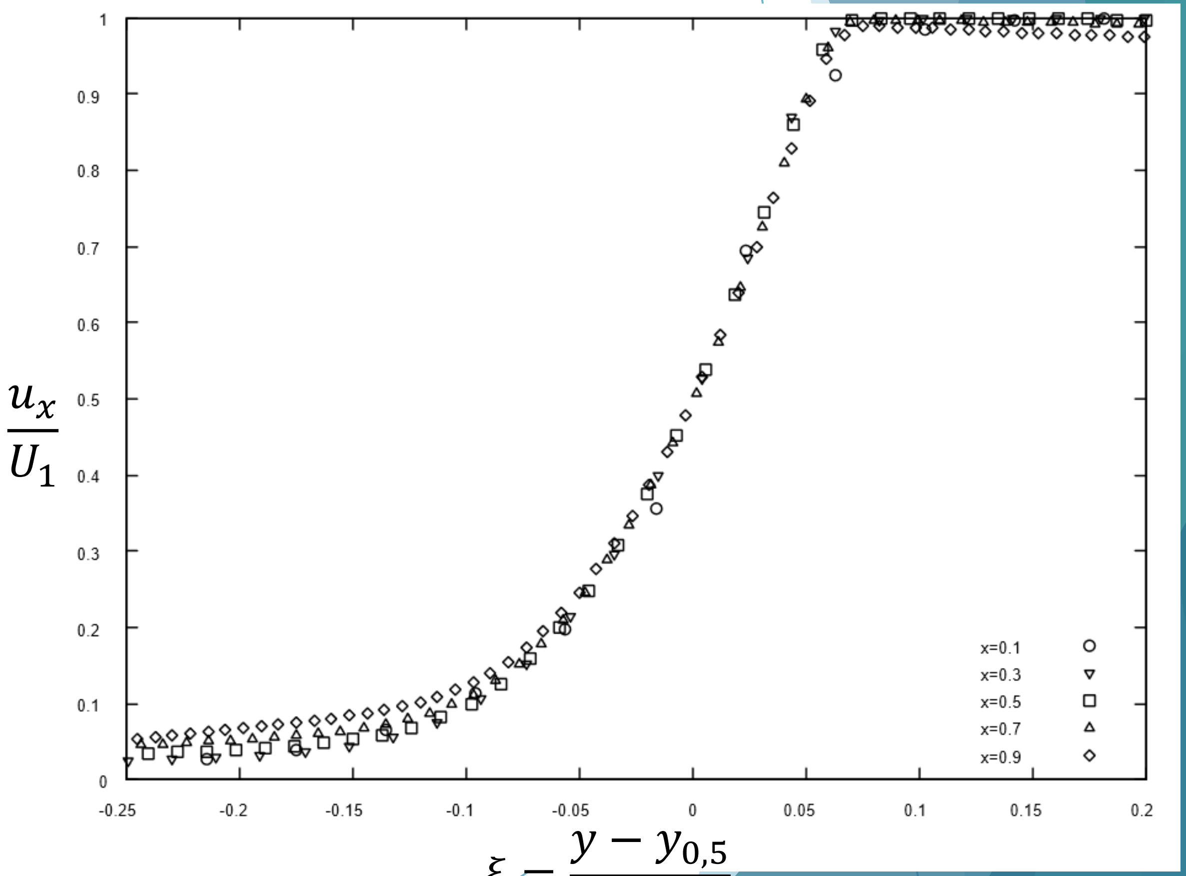
What’s next : - try different geometries with recirculation zone
- extension to 3D
- Add multiphasics model

All the codes are available here : sandbox/aaubert

2D Mixing Layer : Self Similarity



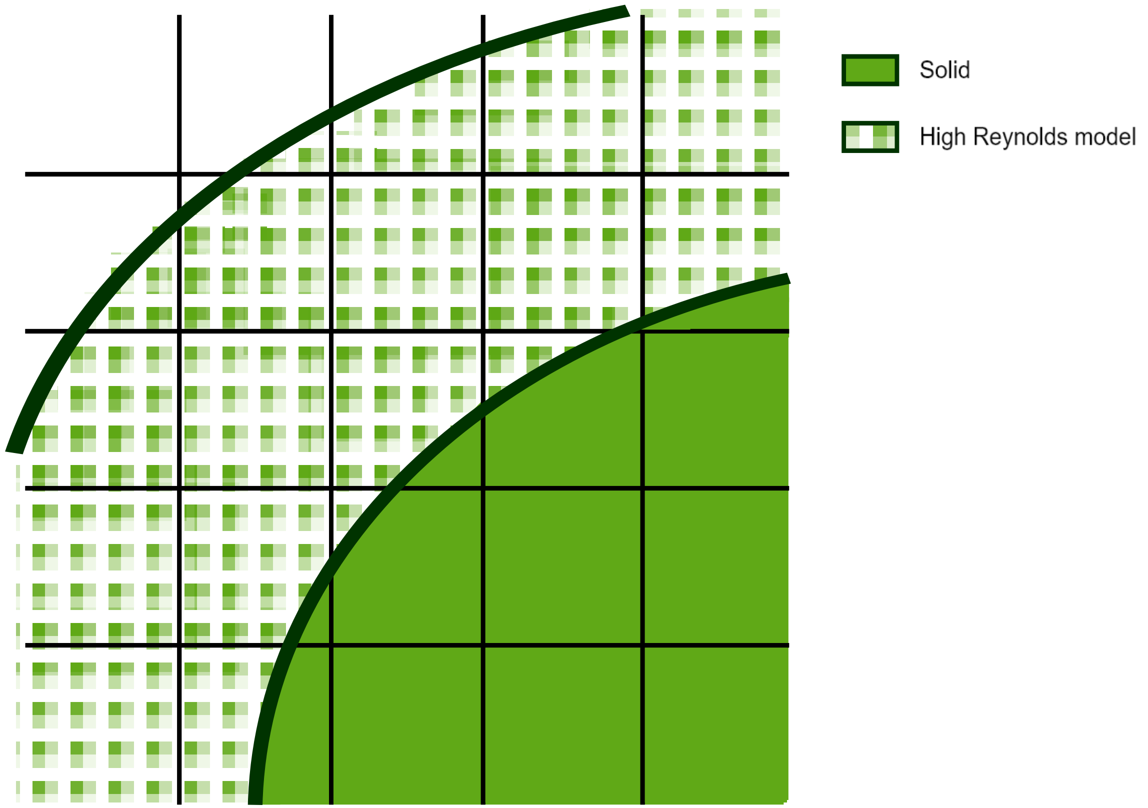
$$u_x(y_{0,5}) = \frac{U_1}{2}$$



$$\xi = \frac{y - y_{0,5}}{x}$$

High Reynolds model

- Basilisk classical solver uses isotropic mesh : use of high Reynolds model because $y^+ > 30$



Low Reynolds model

- Low Reynolds model of Chien [1]

$$\mu_t = \rho C_\mu f_\mu \frac{k^2}{\epsilon}$$

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho k u_i}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) + P_k - \rho\epsilon - 2\frac{\mu k}{d^2}$$

$$\frac{\partial \rho \epsilon}{\partial t} + \frac{\partial \rho \epsilon u_i}{\partial x_i} = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right) + C_{1\epsilon} f_1 \frac{\epsilon}{k} P_k - \rho C_{2\epsilon} f_2 \frac{\epsilon^2}{k} - 2\frac{\mu\epsilon}{d^2} e^{-\frac{d^+}{2}}$$

$k_{wall} = 0$ and $\epsilon_{wall} = 0$

$f_1 = 1$, $f_2 = 1 - \frac{0.4}{1.8} e^{-\frac{Re_t^2}{36}}$ and $f_\mu = 1 - e^{-0.0115 d^+}$ with $Re_t = \frac{\rho k^2}{\mu\epsilon}$

$C_{1\epsilon} = 1.35$, $C_{2\epsilon} = 1.80$, $C_\mu = 0.09$, $\sigma_k = 1$, $\sigma_\epsilon = 1.3$

[1] Predictions of Channel and Boundary-Layers Flows with a Low-Reynolds-Number Turbulence Model, K.-Y. Chien, 1982