Study of the effect of Regularization Errors on the developpement of Multiphase Fluid Instabilities

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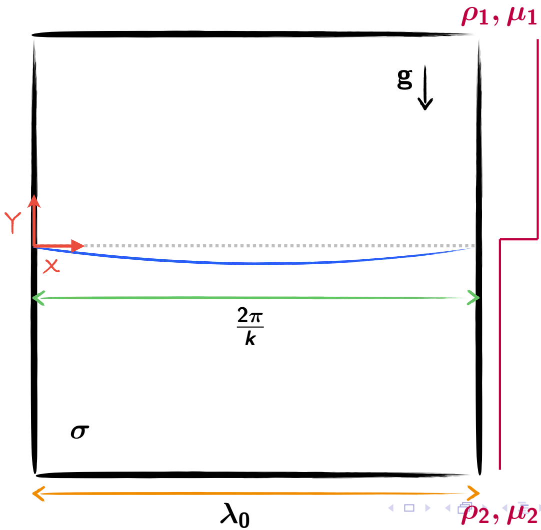
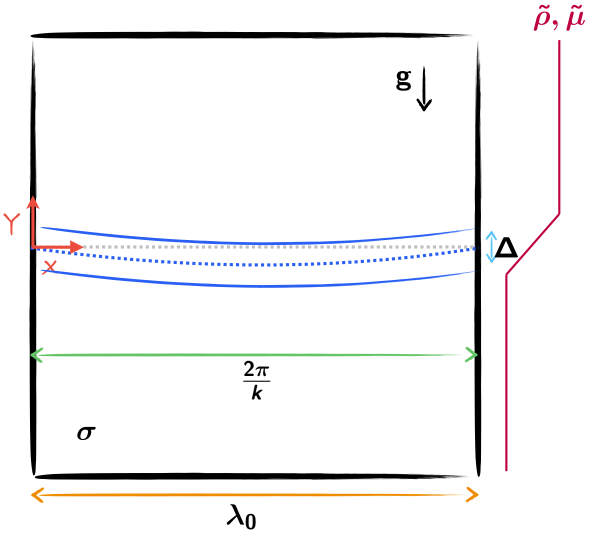
Theoretical Setup : Classical Sharp Case

- Interfacial instability
- $\rho_1 > \rho_2$
- $g = Cste$
- Incompressible flow



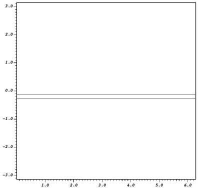
Theoretical Setup : Diffuse Case



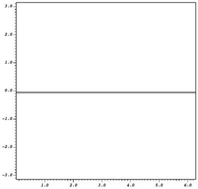
Simulation of the Classical One Fluid Model

$N = \frac{\lambda_0}{\Delta_x}$

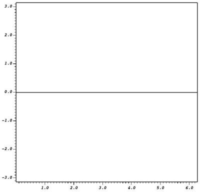
- Volume of Fluid Method
- $\Delta = \Delta_x$
- Density Ratio $R = \frac{\rho_2}{\rho_1} = 0.7$
- Sinusoidal Pertubation of the interface : $h = 10^{-3}\cos(2\pi x)$



$N = 16$



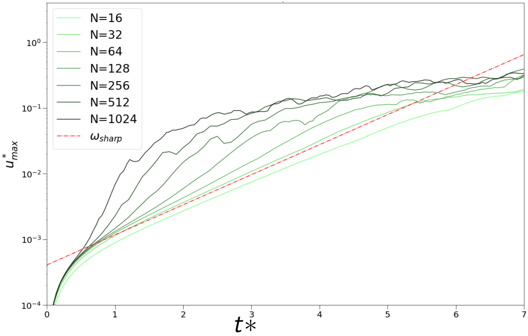
$N = 64$



$N = 512$

Simulation of the Classical One Fluid Model

- $R = 0.7$



Questions:

- Non linear regime?
- Kelvin Helmoltz instability?
- Numerical Noise?

Linear Stability Study:

Main goals:

- Analytical Analysis
- Numerical Validation

Fundamental Equations For The Problem:

$$\frac{\omega_{Visc,Diff}}{\sqrt{gk}} = f\left(R = \frac{\rho_2}{\rho_1}, Re = \frac{\rho_1 g^{\frac{1}{2}}}{\mu_1 k^{\frac{3}{2}}}, M = \frac{\mu_1}{\mu_2}, We = \frac{\rho_1 g}{\sigma k^2}, \Delta k\right)$$

Simplified problem ($\mu_1 = \mu_2 = \mu$):

$$\frac{\omega_{Visc,Diff}}{\sqrt{gk}} = f\left(R = \frac{\rho_2}{\rho_1}, Re, We, \Delta k\right)$$

$$\boxed{\nabla \cdot \overline{\mathbf{u}} = 0}$$

$$\boxed{\overline{\rho}\left(\frac{\partial \overline{\mathbf{u}}}{\partial t} + \overline{\mathbf{u}}\nabla\overline{\mathbf{u}}\right) = -\nabla\overline{p} + \overline{\rho}\mathbf{g} + \overline{\mu}\Delta\overline{u} + \sigma\kappa\nabla f}$$

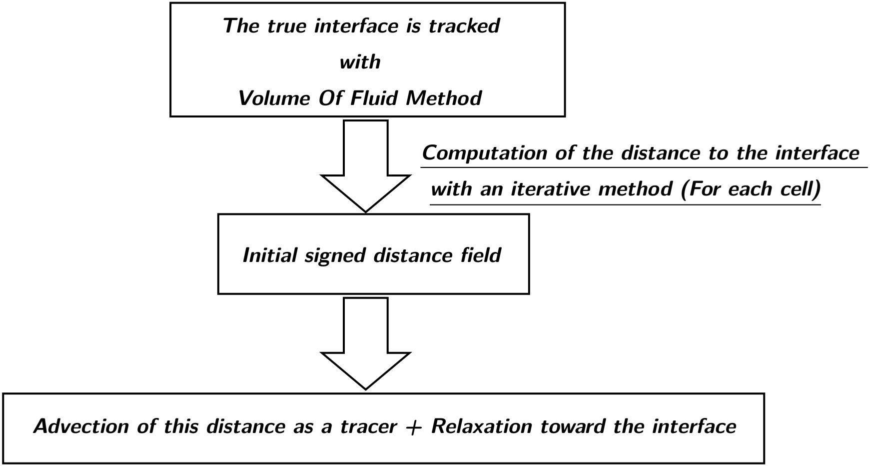
$$\boxed{\frac{\partial d}{\partial t} + \overline{\mathbf{u}} \cdot \nabla d = 0}$$

d = distance to interface:

$$f = \begin{cases} 1 & d \geq \frac{\Delta}{2} \\ \frac{d}{\Delta} + \frac{1}{2} & -\frac{\Delta}{2} \leq d \leq \frac{\Delta}{2} \\ 0 & d \leq -\frac{\Delta}{2} \end{cases}$$

And f = fraction field

Numerical Method for the Interface Diffusion

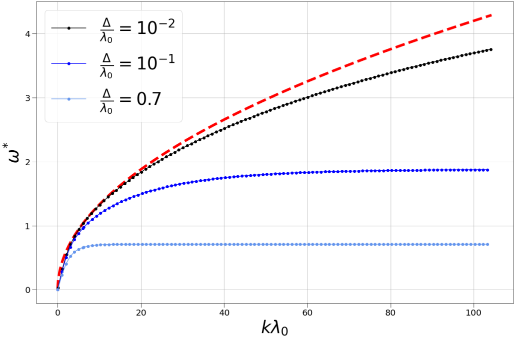


Theoritical Results for the Inviscid with no Surface Tension Problem (FILTRED Regime)

$$\lim_{k \to \infty} \omega^2_{sharp} = \infty$$

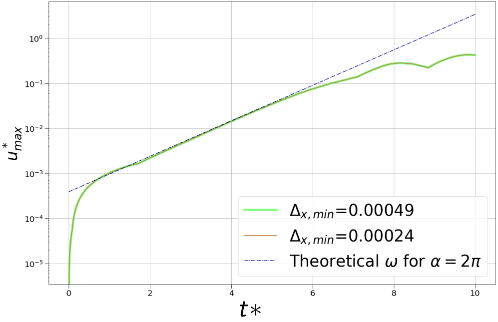
$$\lim_{\Delta k \to \infty} \omega^2_{Diff} = 2\frac{1-R}{1+R}\frac{g}{\Delta} = \frac{2\omega^2_{sharp}}{k\Delta}$$

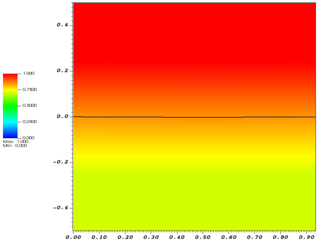
$$\lim_{k\Delta \to 0} \omega_{Diff} = \omega_{Sharp}$$



Simulation of the Inviscid with no Surface Tension Problem

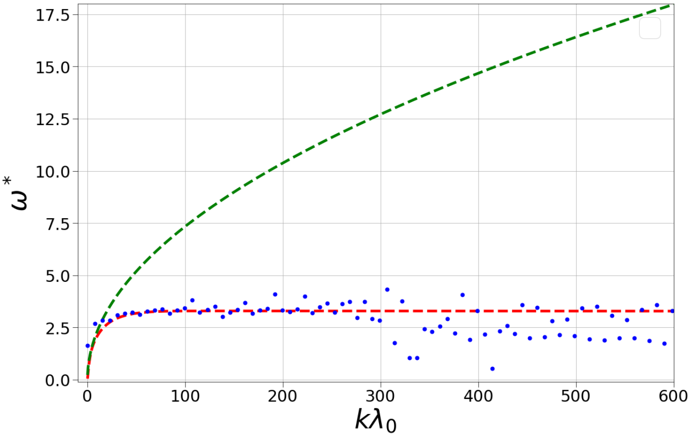
- $\frac{\Delta}{\lambda_0} = 0.2$ and $R = 0.7$





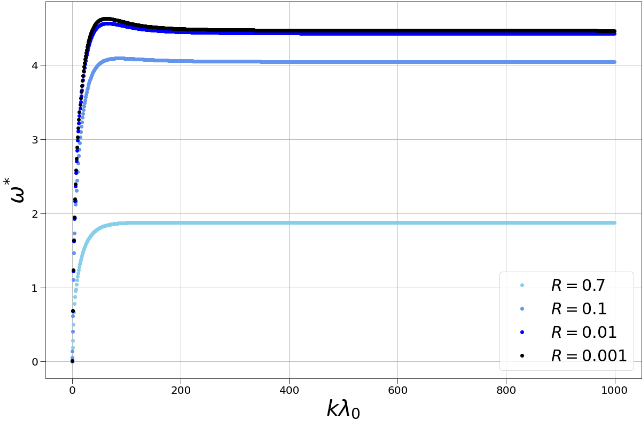
Simulation of the Inviscid with no Surface Tension Problem

- $\frac{\Delta}{\lambda_0} = 0.1$ and, $R = 0.3$



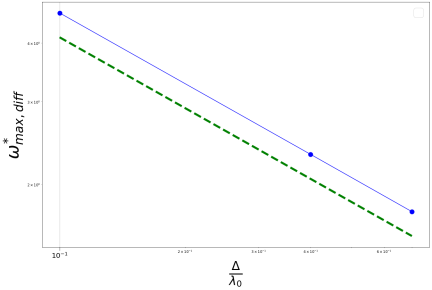
Theoritical Results for the Inviscid with no Surface Tension Problem (Monotonicity)

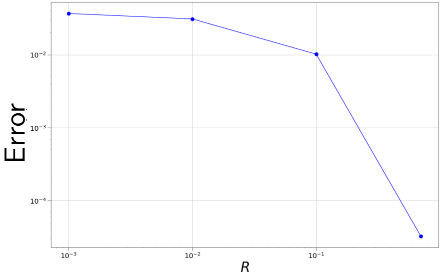
- $\omega^*(k\lambda_0)$ not monotonic



Evolution of ω^*_{max}:

- $\omega_{max,diff} \sim \mathcal{O}\left(\Delta^{-\frac{1}{2}}\right)$
- $\omega_{max,diff} \nsim \mathcal{O}\left(A_t^{\frac{1}{2}}\right)$

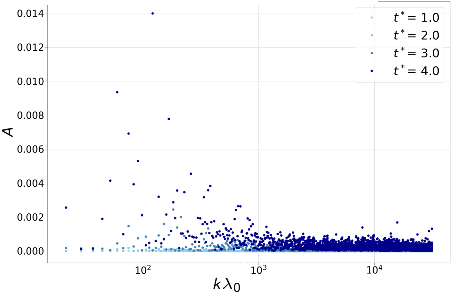


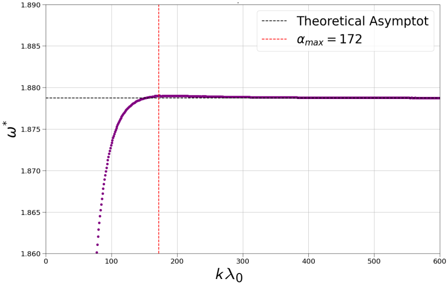


Simulation of the Inviscid with no Surface Tension Problem

- R=0.7
- $\frac{\Delta}{\lambda_0} = 0.02$

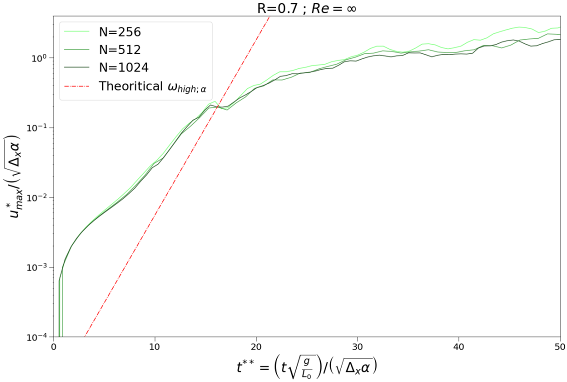
Numerical $k_{max} \approx$ Theoretical $k_{max} = 172$





Back to the Classical Volume Of Fluid Method

- $\frac{\Delta_x}{\lambda_0} = \frac{1}{N}$ and $R = 0.7$

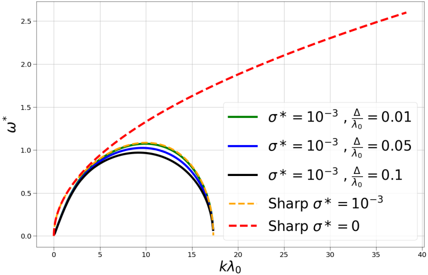


What's next?

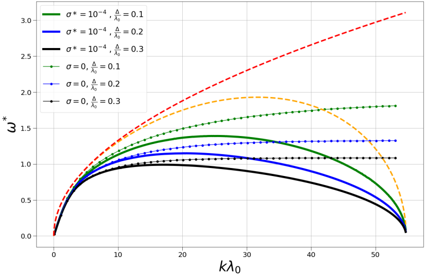
- What if we add some physical characteristic lengths to the system?

Theoretical Results With Surface Tension:

Regime 1:DNS Regime



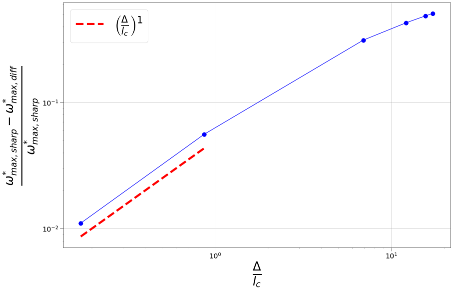
Regime 2:FILTERED Regime

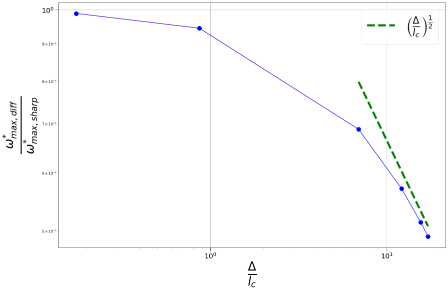


Theoretical Results With Surface Tension:

$$l_c = \sqrt{\frac{\sigma}{(\rho_1 - \rho_2)g}}$$

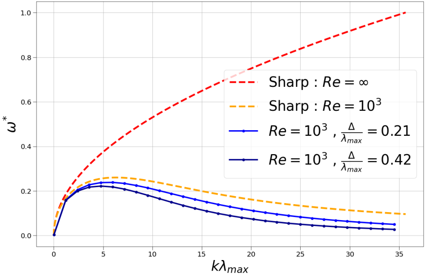
$\frac{\Delta}{l_c} \leq 1$ to achieve the DNS Regime



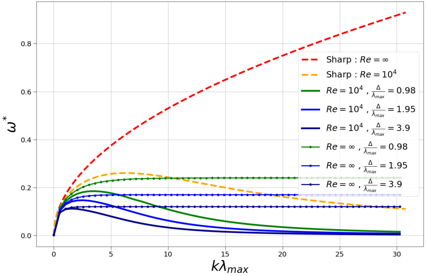


Theoretical Results With Viscosity:

Regime 1:DNS Regime



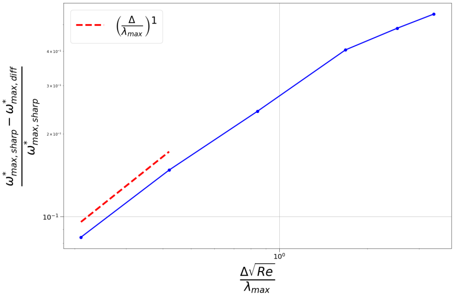
Regime 2:FILTERED Regime

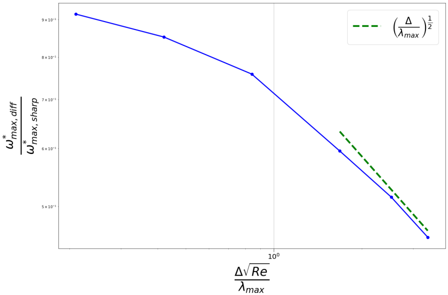


Theoretical Results With Surface Tension:

$$l_\nu = \frac{1}{\sqrt{Re}}$$

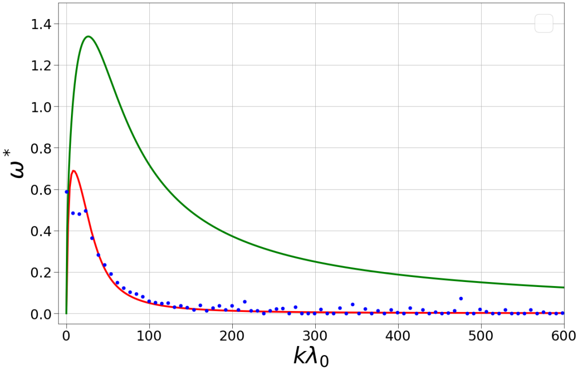
$\frac{\Delta}{l_\nu} \leq 1$ to achieve the DNS Regime



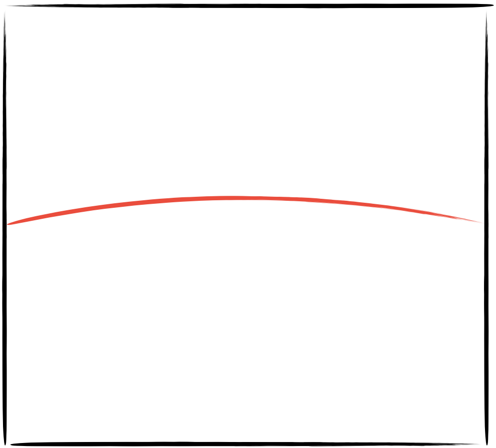


Numerical Validation for the Viscous Case :

- $\frac{\Delta}{\lambda_0} = 0.6$
- R=0.7
- Re=1000
- $l_c = 0$



To summarize:



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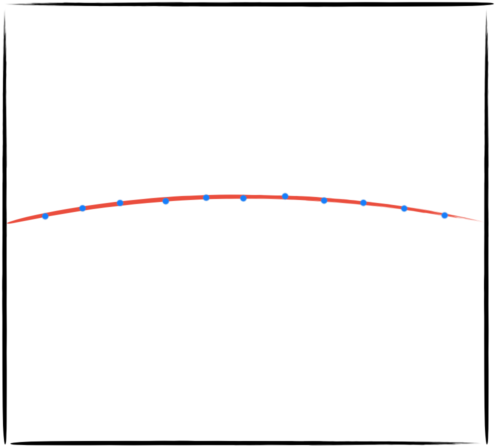

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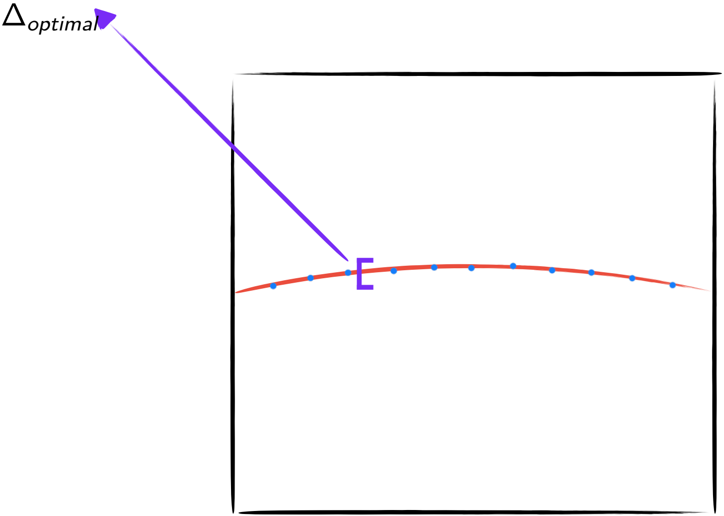

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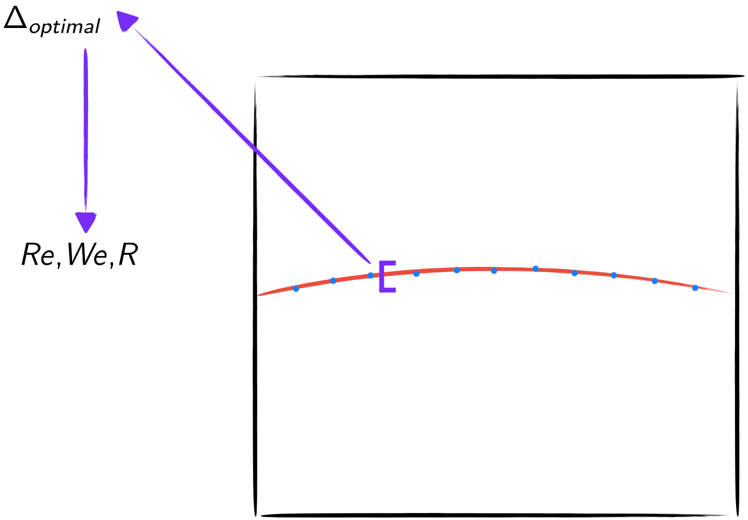

To Conclude

- Two important regimes:
- One governed by the regularization errors: $\frac{l_c}{\Delta} < 1$ & $\frac{l_\nu}{\Delta} < 1$
- One physical: $\frac{l_c}{\Delta} > 1 \vee \frac{l_\nu}{\Delta} > 1$

Conclusion: When we satisfy the last criterion VoF Works!

Thank you!

Questions?