Breaking Wave Statistics

with the multi-layer model

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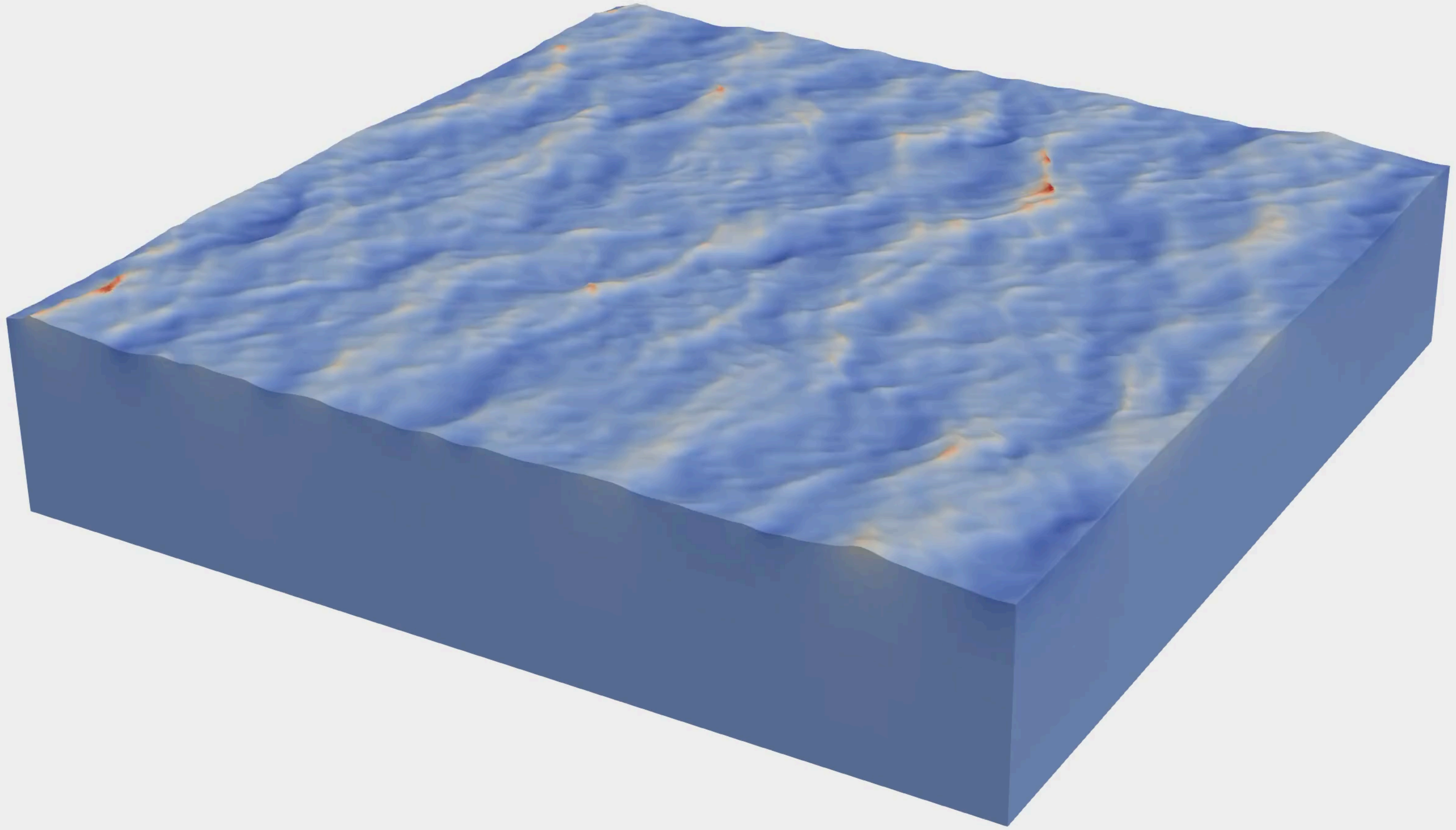
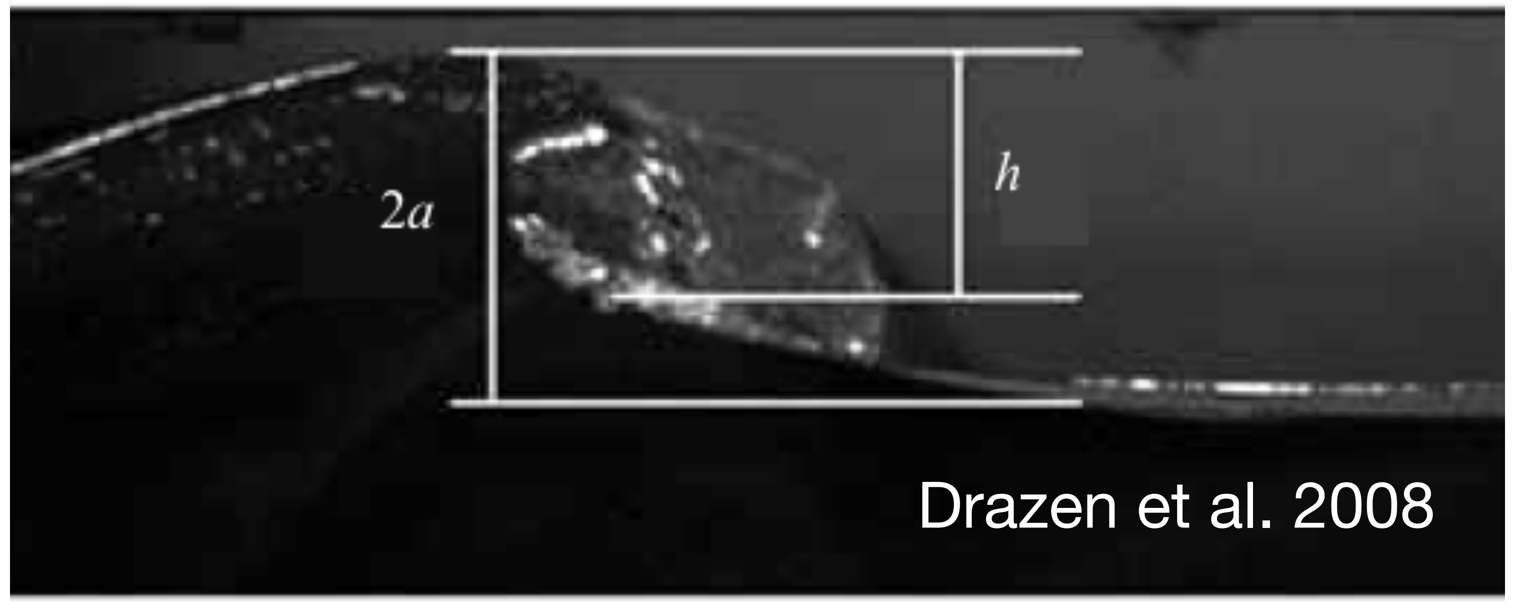


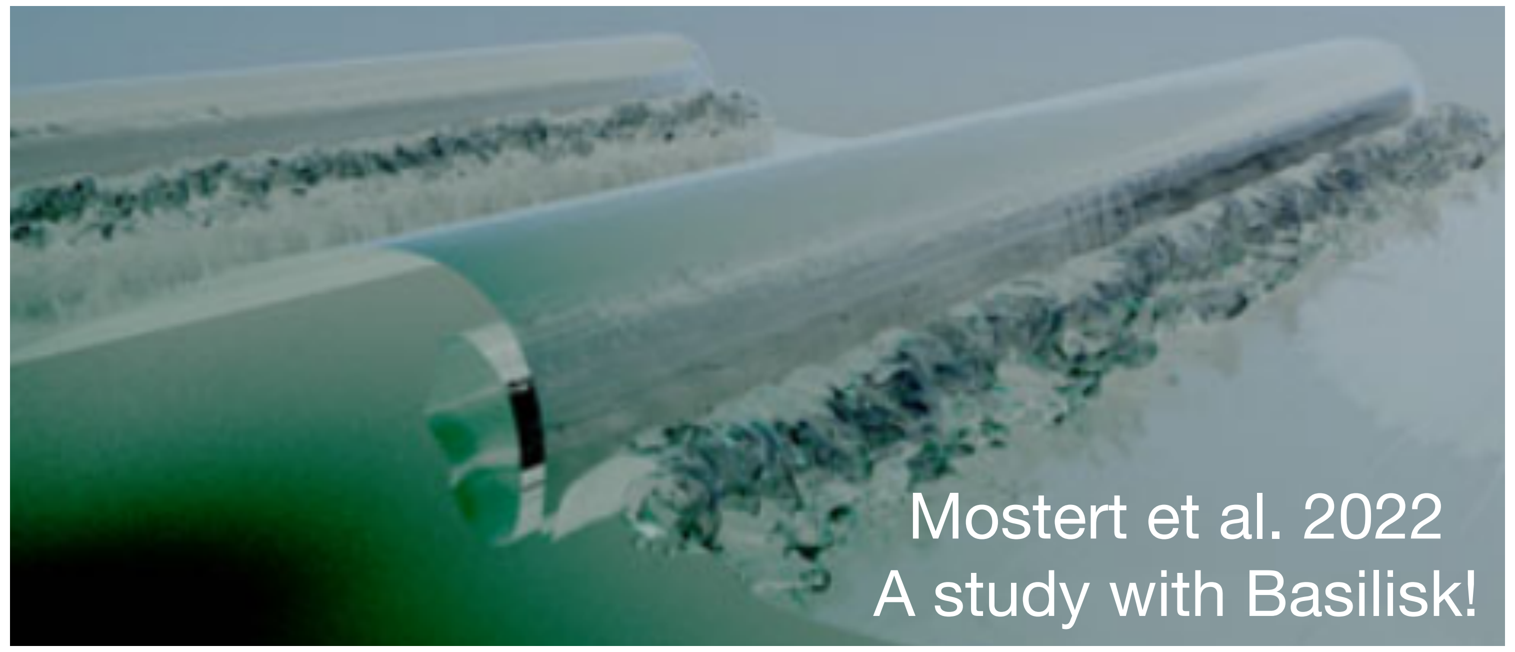
2023/07/05 Basilisk User Meeting

PRINCETON
School of Engineering and Applied Science

Wave breaking - from single breakers to broadband wave field



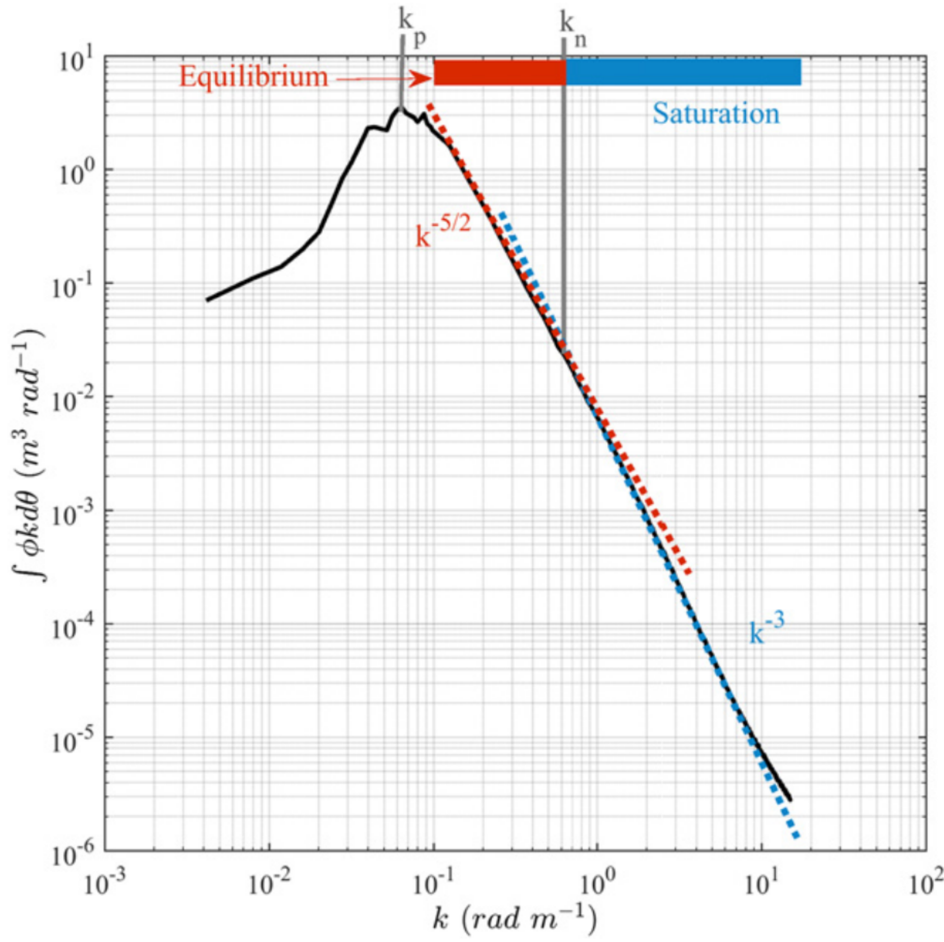
Drazen et al. 2008

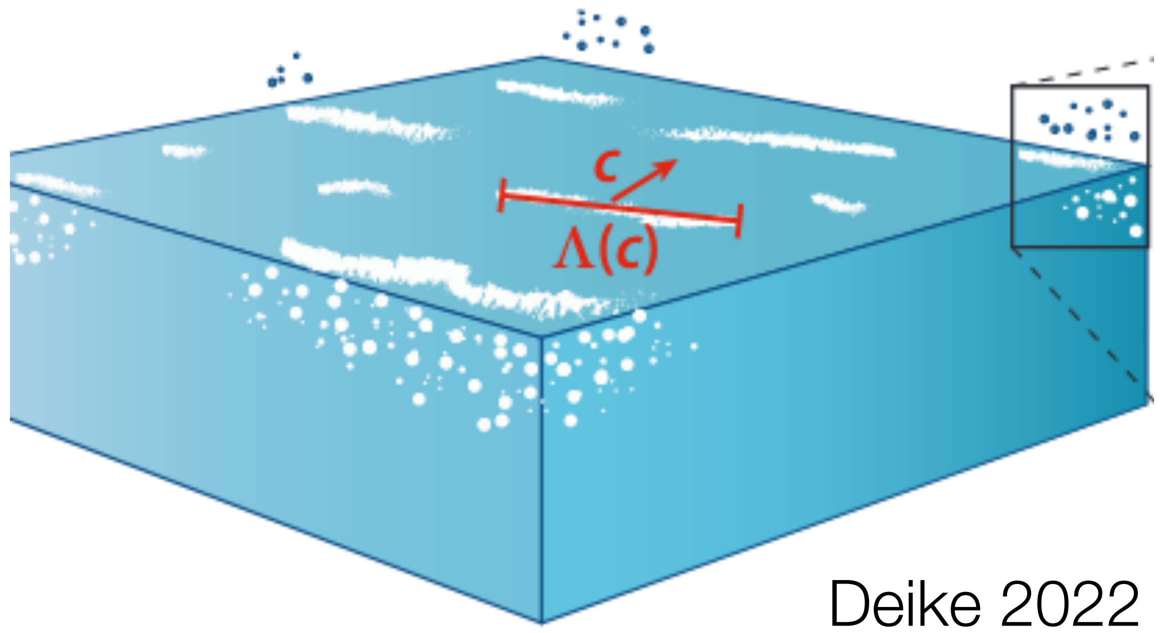


Mostert et al. 2022
A study with Basilisk!

Knowledge of dissipation, bubbles and droplet production…

Wave breaking: enhanced mass transfer; contribute to upper ocean current and mixing.

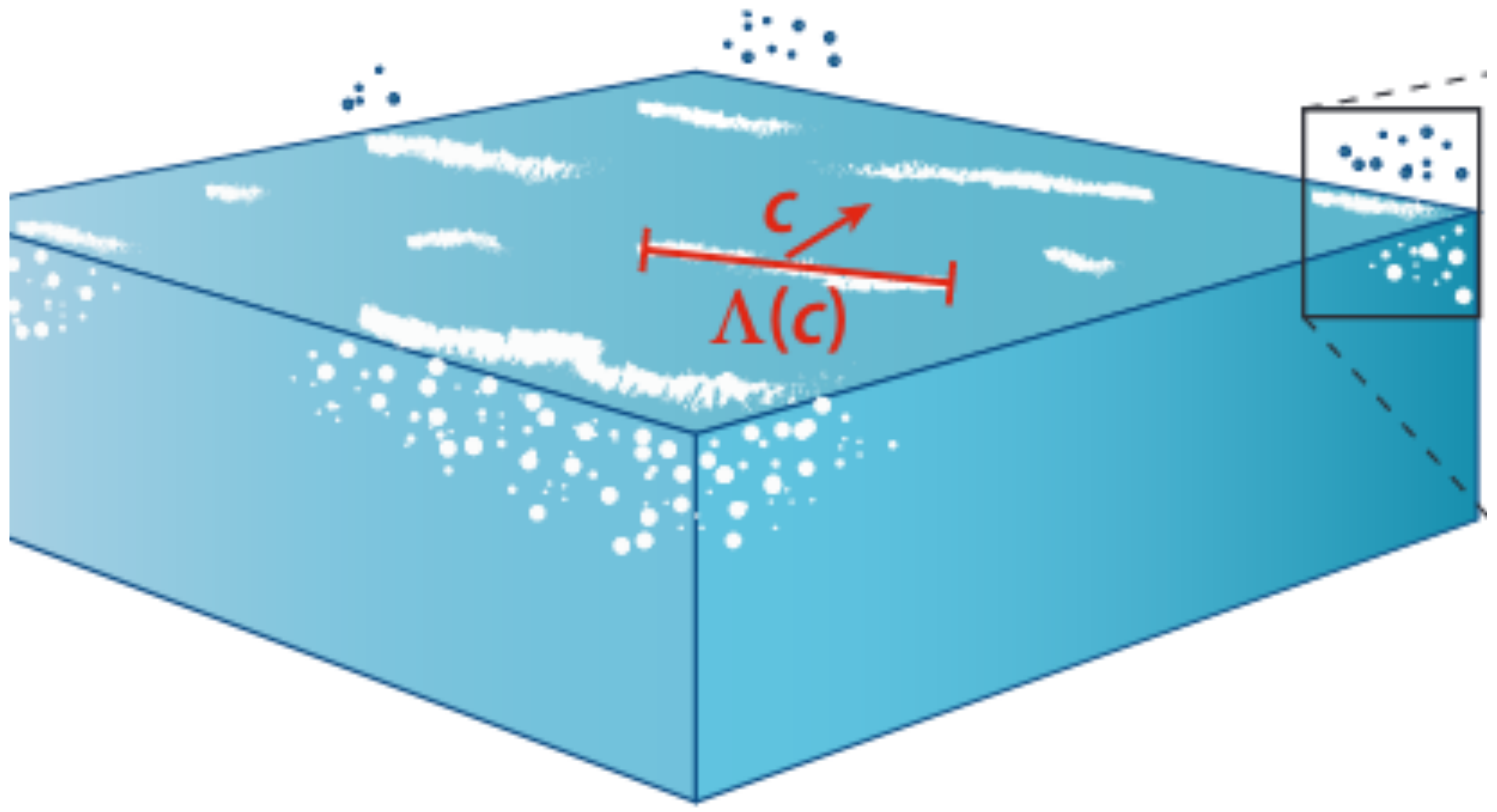




Deike 2022

Goal: study an ensemble of breaking waves and their statistics.
Breaking statistics: how frequently waves break and at what scales.

Concept of breaking distribution



Total length of breaking fronts per unit surface area: $L = \int \Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Larger c - smaller underlying k - stronger breaker

Fraction of total surface area turned over per unit time: $R = \int c\Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Fractional whitecap coverage: $W \propto \int c^2\Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Rate of air entrainment per unit surface area: $V_a \propto \int c^3\Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Momentum flux per unit surface area: $M \propto \int c^4\Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Energy dissipation per unit surface area: $E \propto \int c^5\Lambda(\mathbf{c})\mathrm{d}\mathbf{c}$

Romero 2019

Q: How do we predict breaking distribution $\Lambda(c)$ from a particular wave spectrum $\phi(k)$?

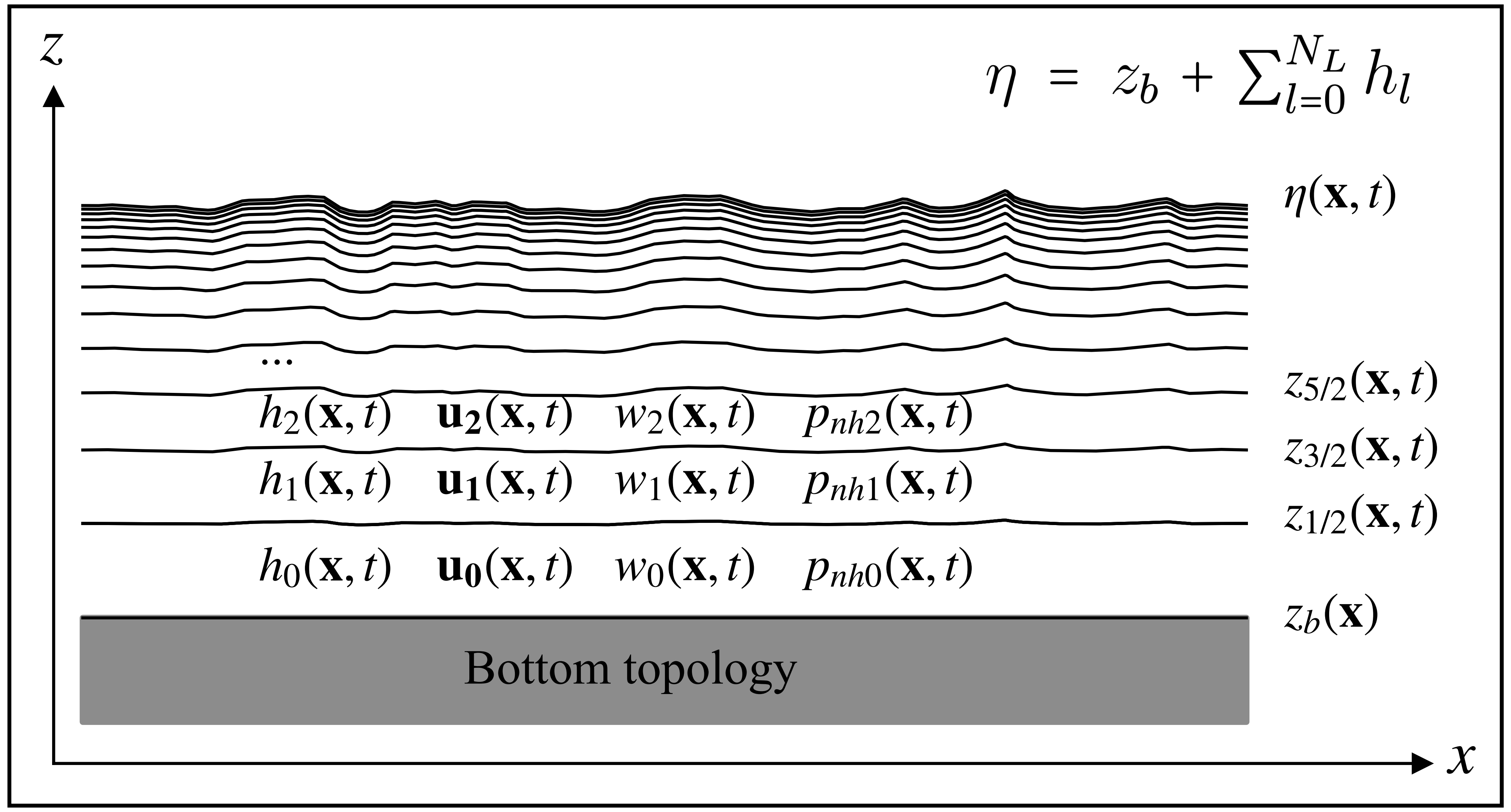
- Theories (Phillips 1985): for a typical wind wave spectrum
$$\Lambda(c) \propto u_*^3 g c^{-6}$$
- Limited field campaigns (Gemmrich 2008, Kleiss & Melville 2010, Sutherland & Melville 2013, Deike 2017, etc.) proposed **empirical** corrections
- The breaking distribution and the associated dissipation in operational wave models are either **empirical** or **heavily tuned**
- **Numerical challenges**: highly non-linear, multi-scale, intermittent in time and space (need to integrate for a relatively long time).

Our approach: phase-resolved broadband wave field simulations in the physical space that permit strong non-linearity (with reasonable computational cost.)

The multi-layer model

- Multi-layer model http://basilisk.fr/src/layered/README (Popinet 2020);
- Discretization: horizontally Eulerian and vertically Lagrangian, with generalized vertical coordinate.

Layer structure illustration



# of grid points: 1024*1024*15

(Depth-integrated) NS equation

$$\frac{\partial h_l}{\partial t} + \nabla_H \cdot (h\boldsymbol{u})_l = 0$$

$$\frac{\partial (h\boldsymbol{u})_l}{\partial t} + \nabla_H \cdot (h\boldsymbol{u}\boldsymbol{u})_l = \boxed{-g h_l \nabla_H \eta} - \boxed{\nabla_H (h\phi)_l + [\phi \nabla_H z]_l}$$

$$+ \boxed{[\nu_1 \partial_z \boldsymbol{u}]_l + \nu_2 \nabla_H^2 \boldsymbol{u}}$$

$$\frac{\partial (hw)_l}{\partial t} + \nabla_H \cdot (hw\boldsymbol{u})_l = \boxed{-[\phi]_l} + \boxed{[\nu_1 \partial_z w]_l + \nu_2 \nabla_H^2 w}$$

$$\nabla_H \cdot (h\boldsymbol{u})_l + [w - \boldsymbol{u} \cdot \nabla_H z]_l = 0$$

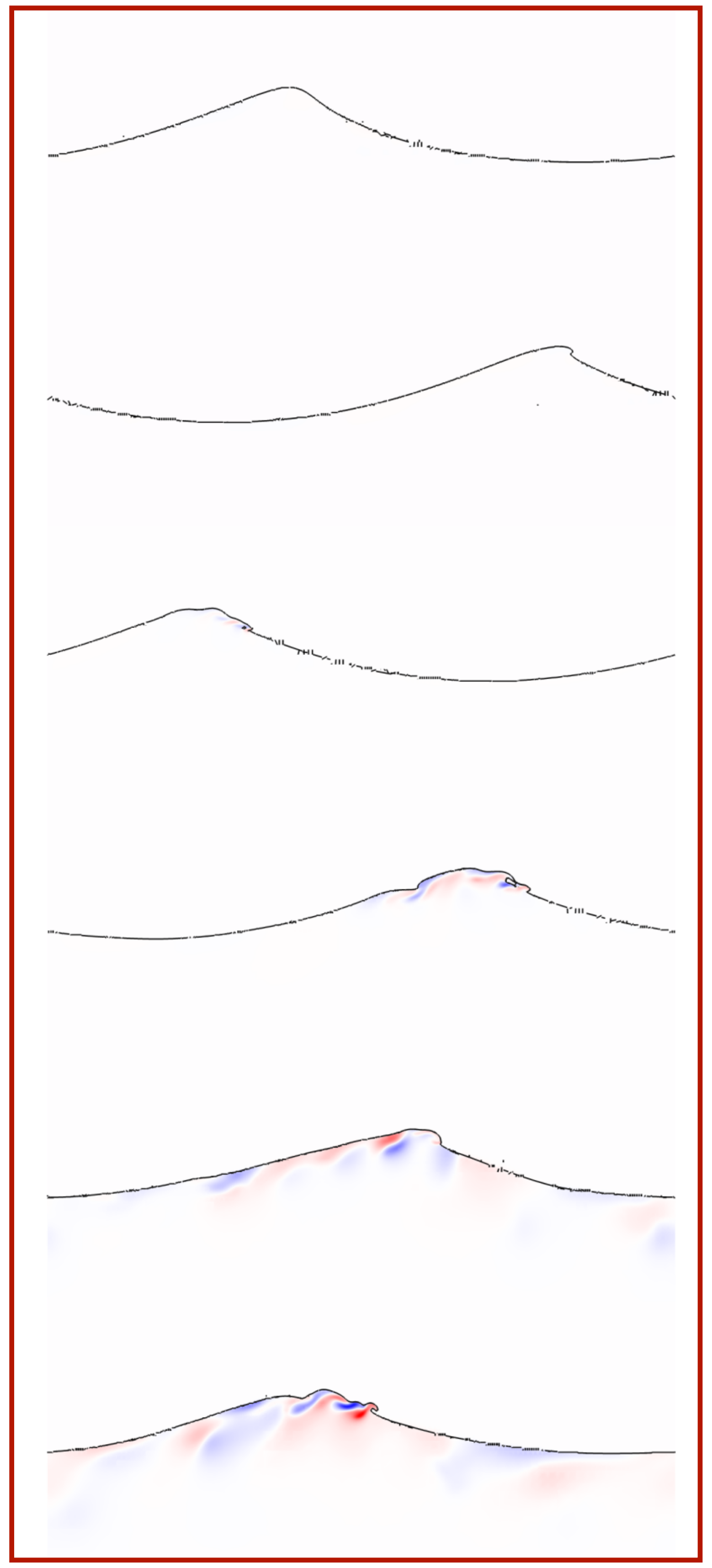
ϕ: non-hydrostatic pressure

Combined with a gradient-limiter for breaking

$$\partial z/\partial x = \begin{cases} \partial z/\partial x, & |\partial z/\partial x| \leqslant s_{\max} \\ \operatorname{sign}(\partial z/\partial x) s_{\max}, & |\partial z/\partial x| > s_{\max}. \end{cases} \qquad s_{max} = 0.577$$

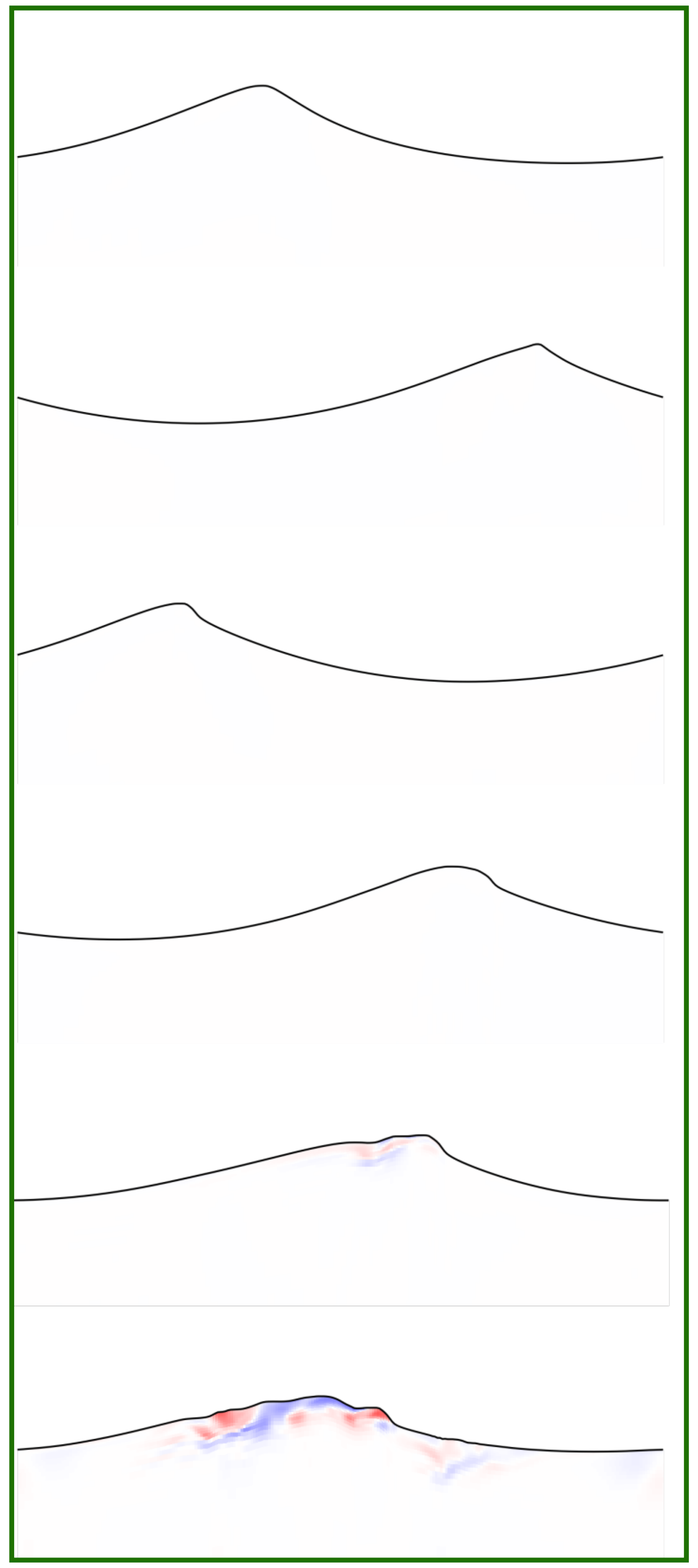
Numerical scheme: advection - projection - remapping (Popinet 2020)

Multil-layer v.s. NS for a single breaker



Two-phase DNS

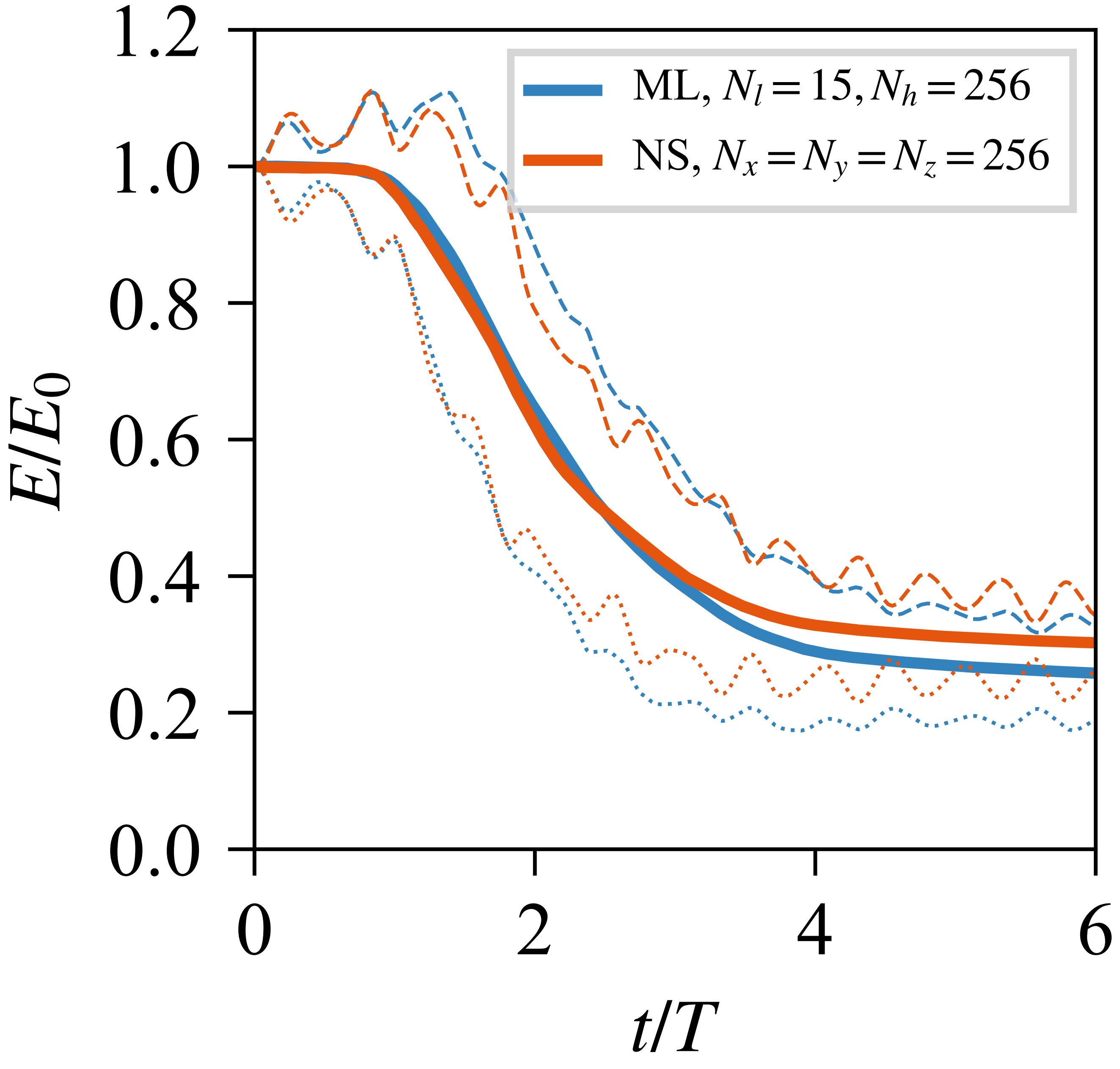
Deike et al. 2015,
Mostert et al. 2022, etc.



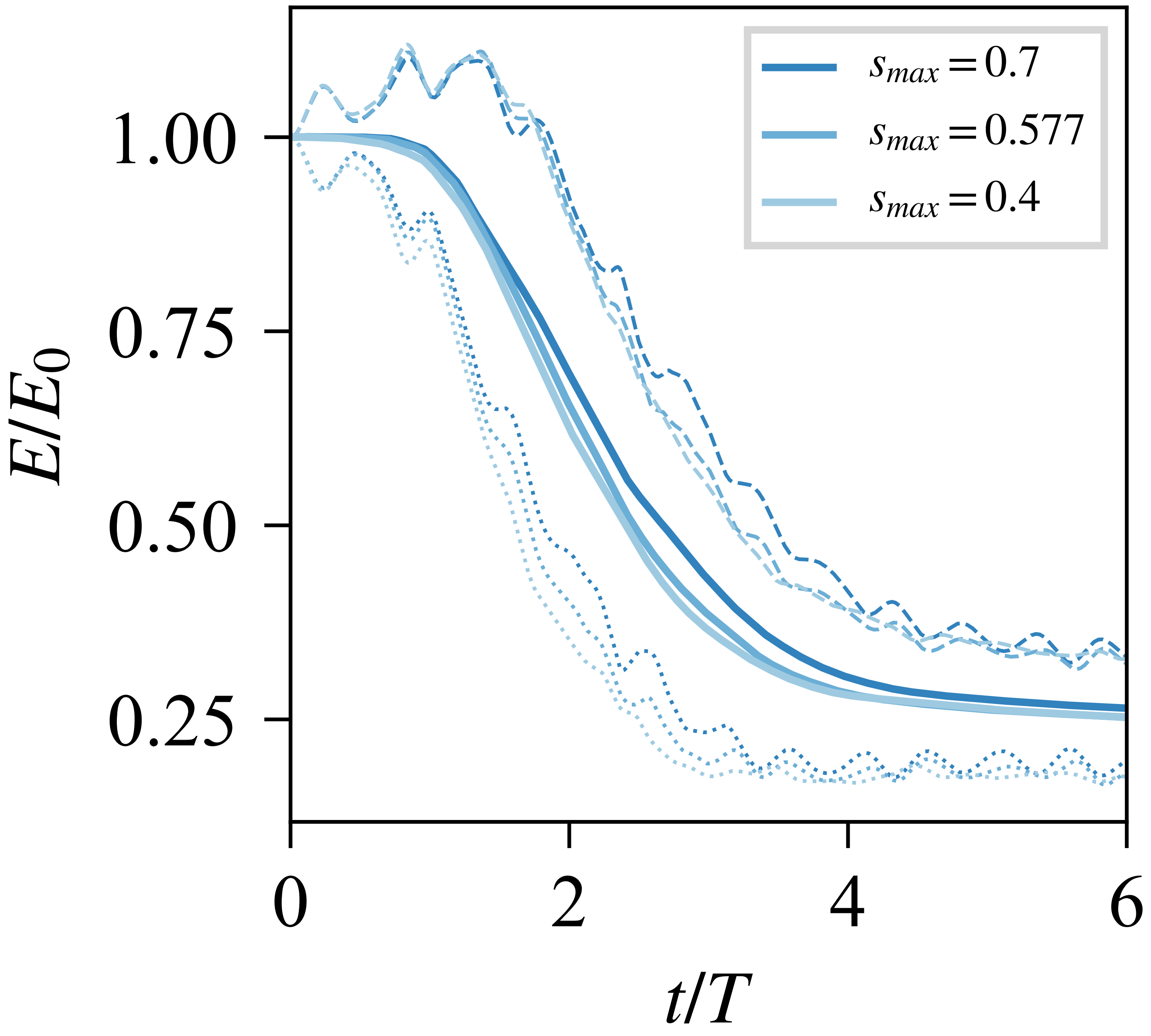
Multi-layer model

About 40 times faster.

Given enough horizontal and vertical resolution, the multi-layer solution converges to the full two-phase Navier-Stokes solution despite no explicit surface overturning.

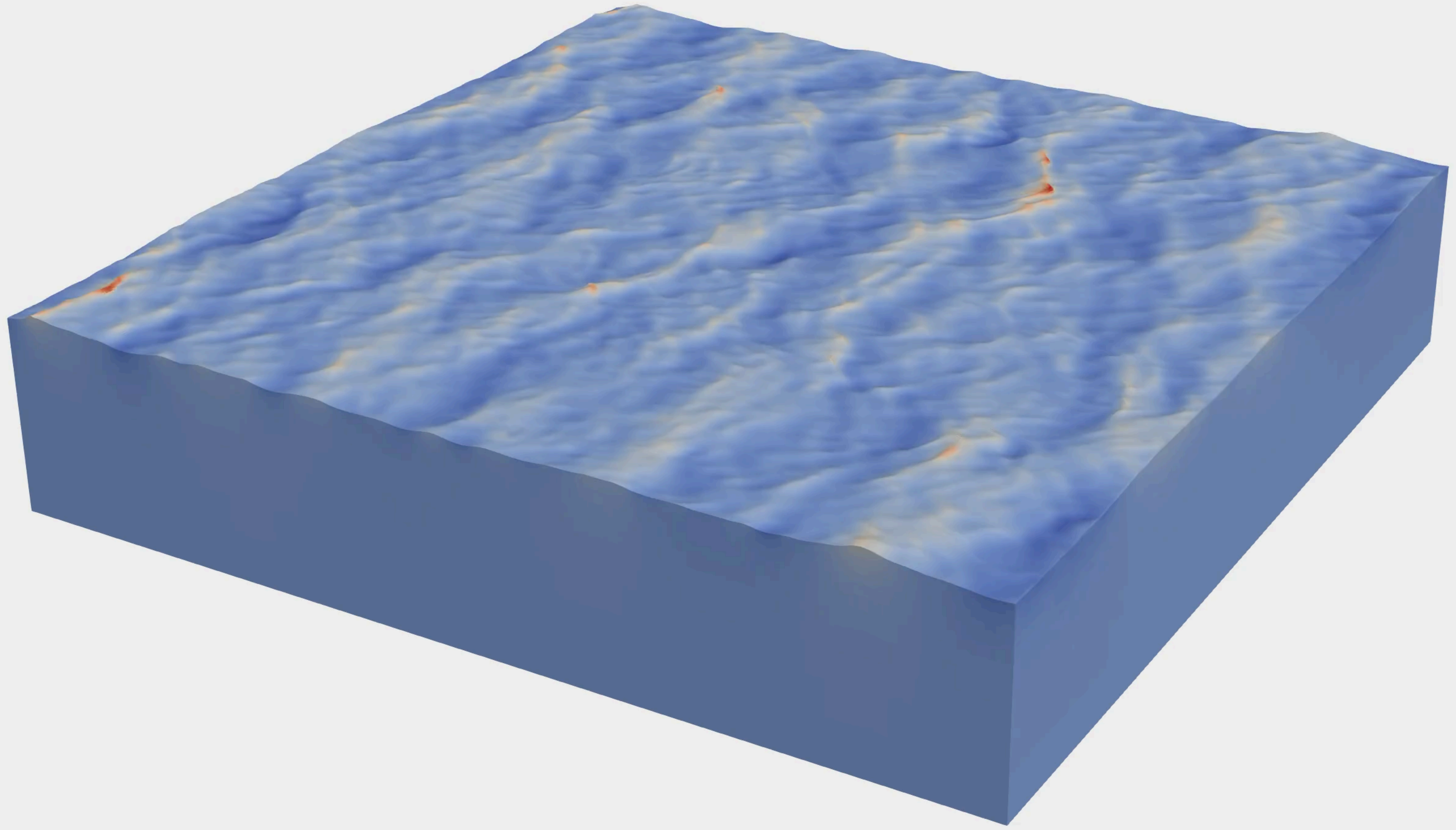


Energy dissipation



Insensitive to s_{max}

Simulation configuration



- Choose a typical wind wave energy spectrum:

$$\phi(k) = P g^{-1/2} k^{-2.5} \exp[-1.25(k_p/k)^2]$$

- Directional spreading:

$$F(k,\theta) = (\phi(k)/k)\cos^N(\theta) / \int_{-\pi/2}^{\pi/2} \cos^N(\theta) d\theta$$

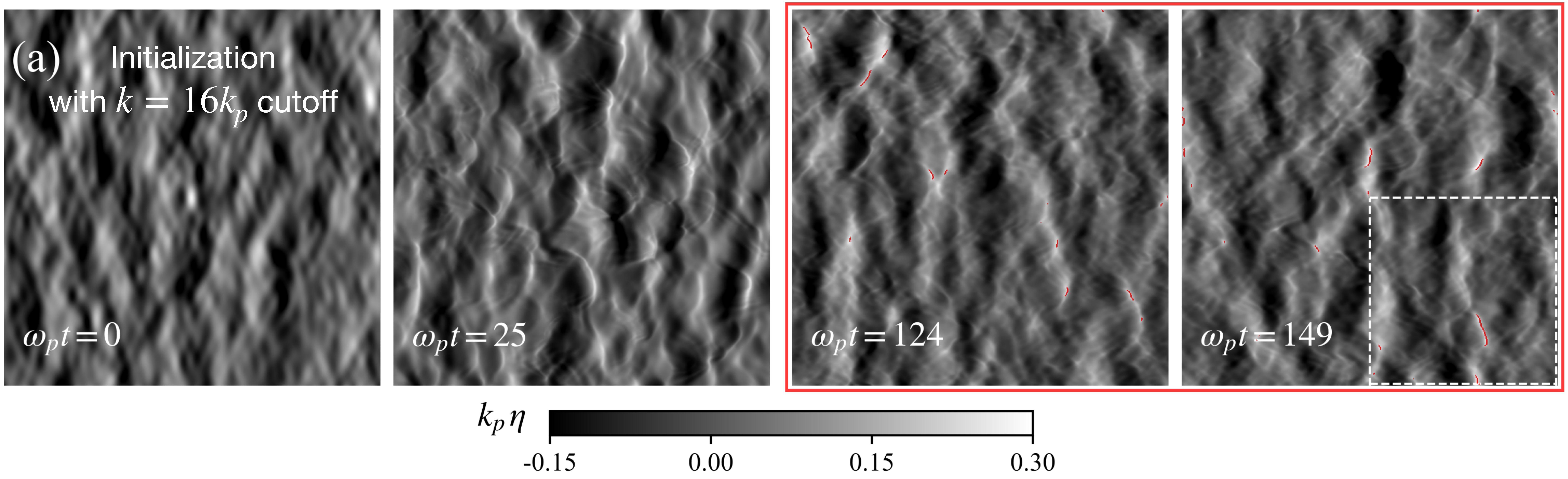
- Create one random realization of the wave spectrum: superposition of linear waves with random phase as initial conditions:

$$\eta = \sum_{i,j} a_{ij}\cos(\psi_{ij}), \qquad a_{ij} = [2F(k_{xi}, k_{yj}) dk_x dk_y]^{1/2}, \quad \psi_{ij} = k_x x + k_y y + \psi_{\text{rand}}$$

- Breaking modeled with a gradient-limiter (no surface overturning). No wind forcing.

- 8 hrs on 256 cores for ~ 40 peak wave period.

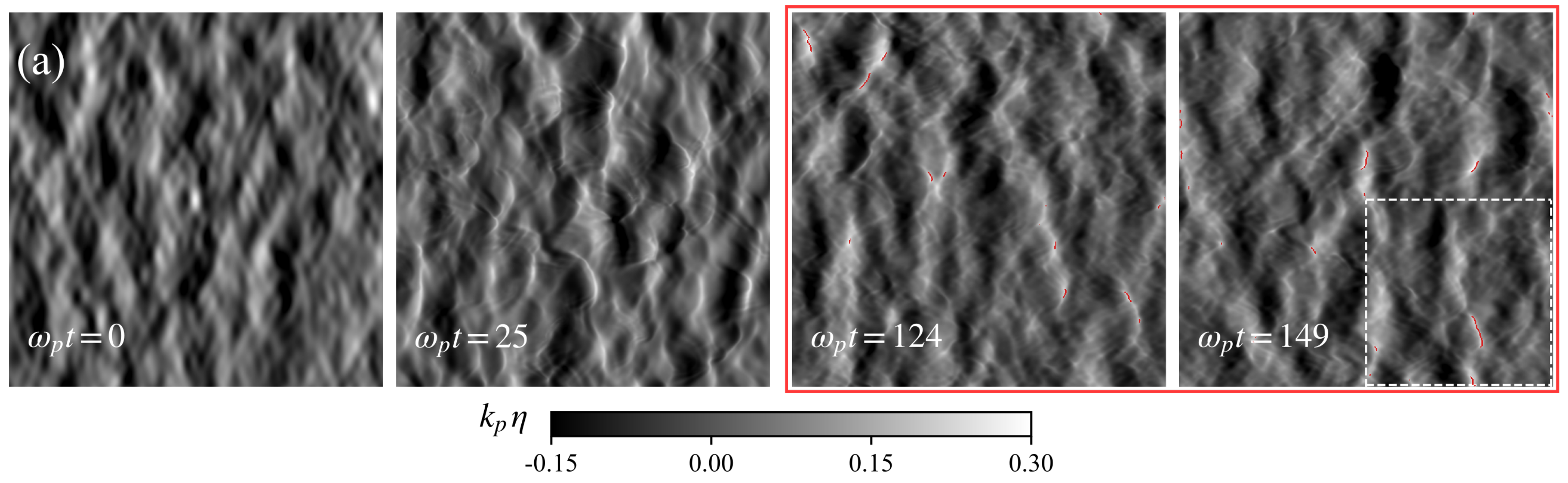
Broadband wave field evolution



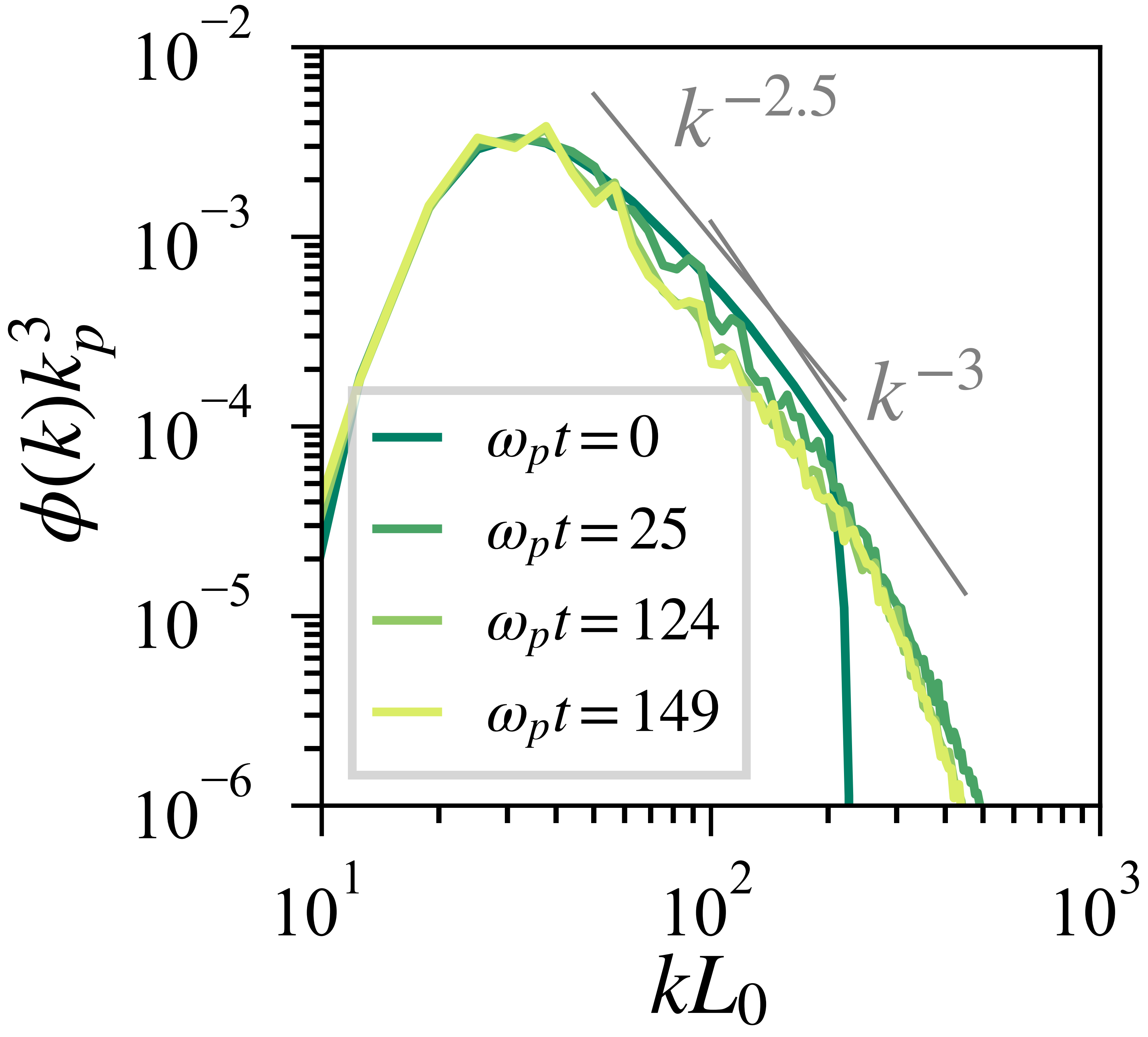
Two global steepness (slope) parameter:

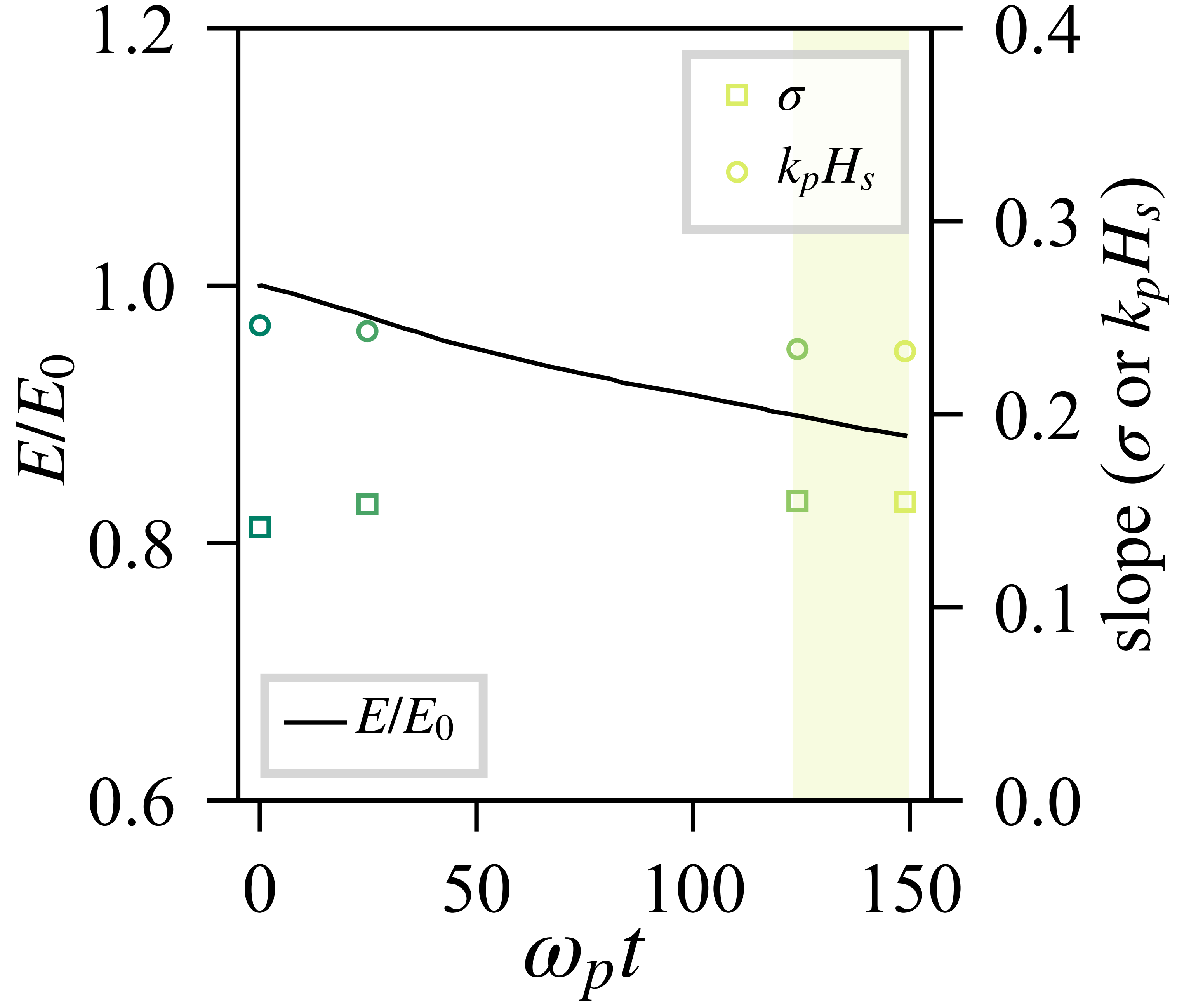
- Effective slope (energy): $k_p H_s = 4k_p\langle\eta^2\rangle^{1/2} = 4k_p(\int_0^{k_{max}}\phi(k)dk)^{1/2}$
- Root mean square slope (roughness): $\sigma = \langle m_x\rangle^{1/2} + \langle m_y\rangle^{1/2} = (\int_0^{k_{max}}k^2\phi(k)dk)^{1/2}$, $m_x = \partial\eta/\partial x$

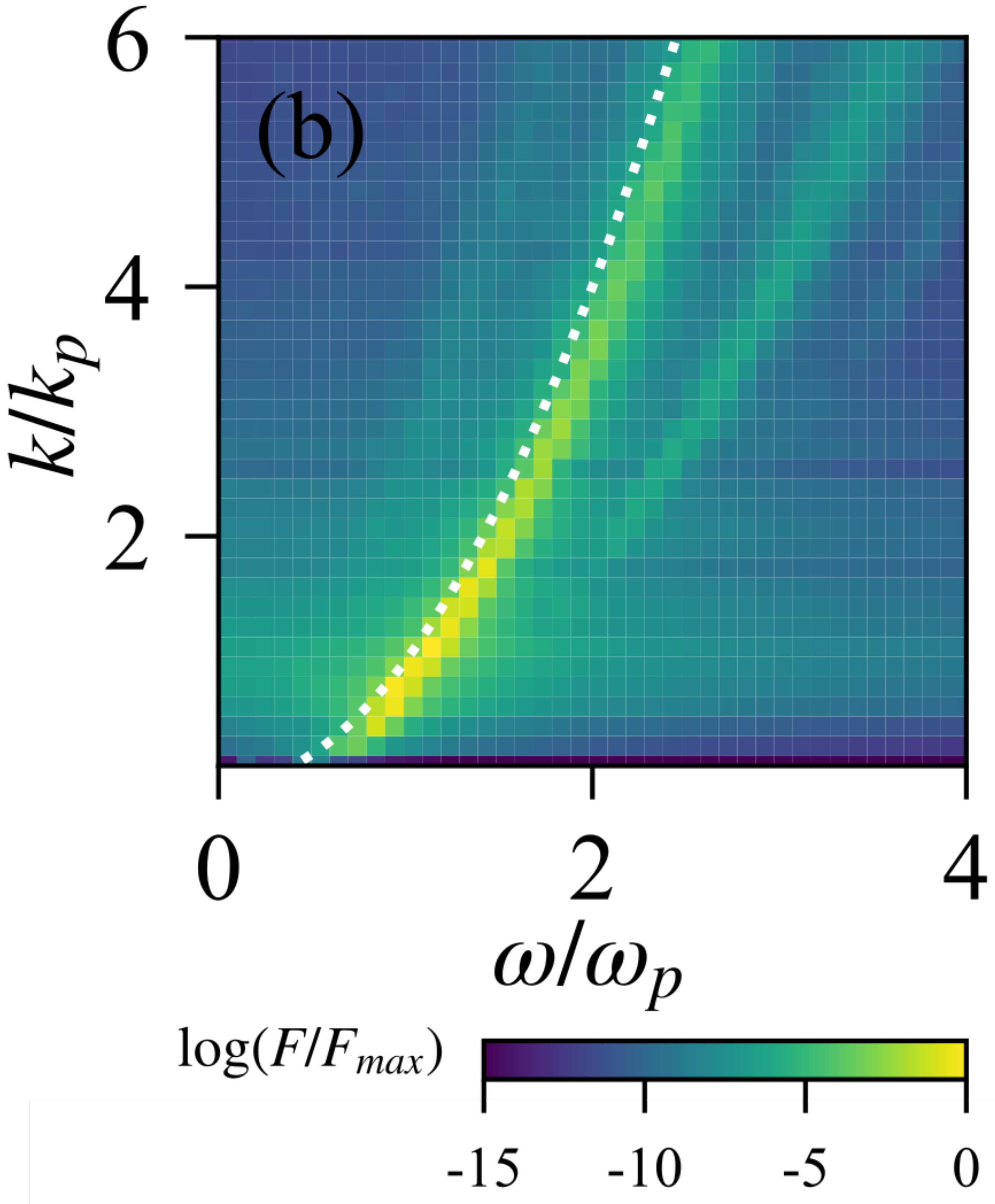
Broadband wave field evolution



- Saturation: k^{-3} spectral shape.
- Energy transfer to small scale by breaking.
- Constant energy dissipation, but the spectrum is stationary and the steepness parameters almost constant.
- Captures the dispersion relation well.

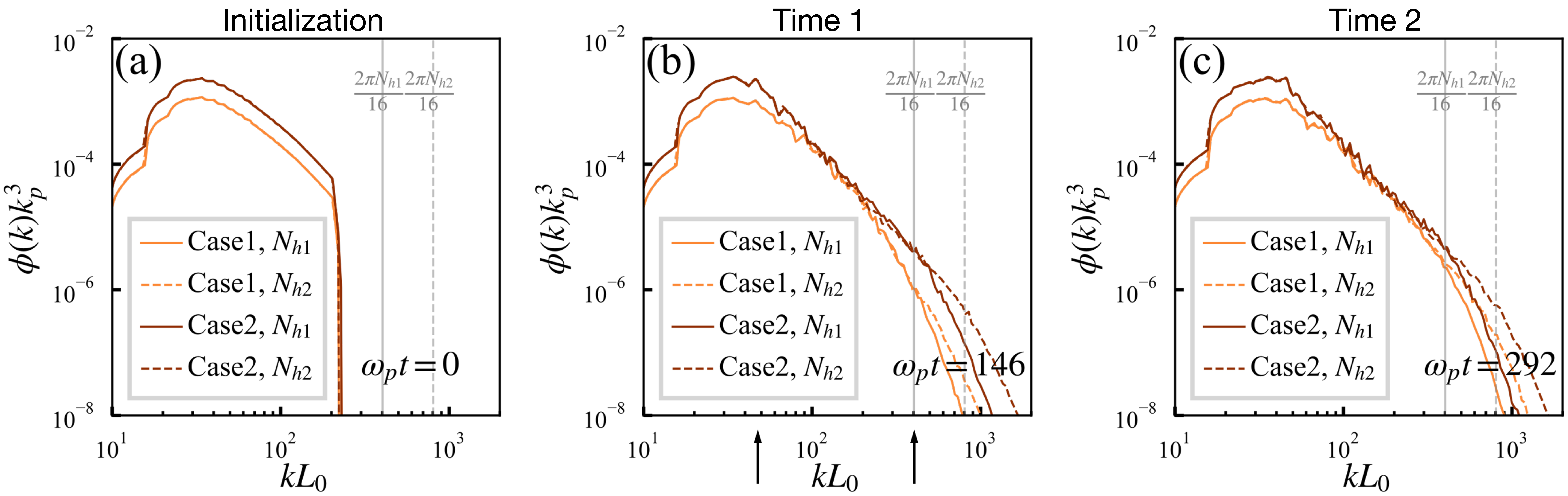






Side note: $\sigma \ll 0.1$ is an underlying assumption for weak turbulence theory.

Resolution and length scales



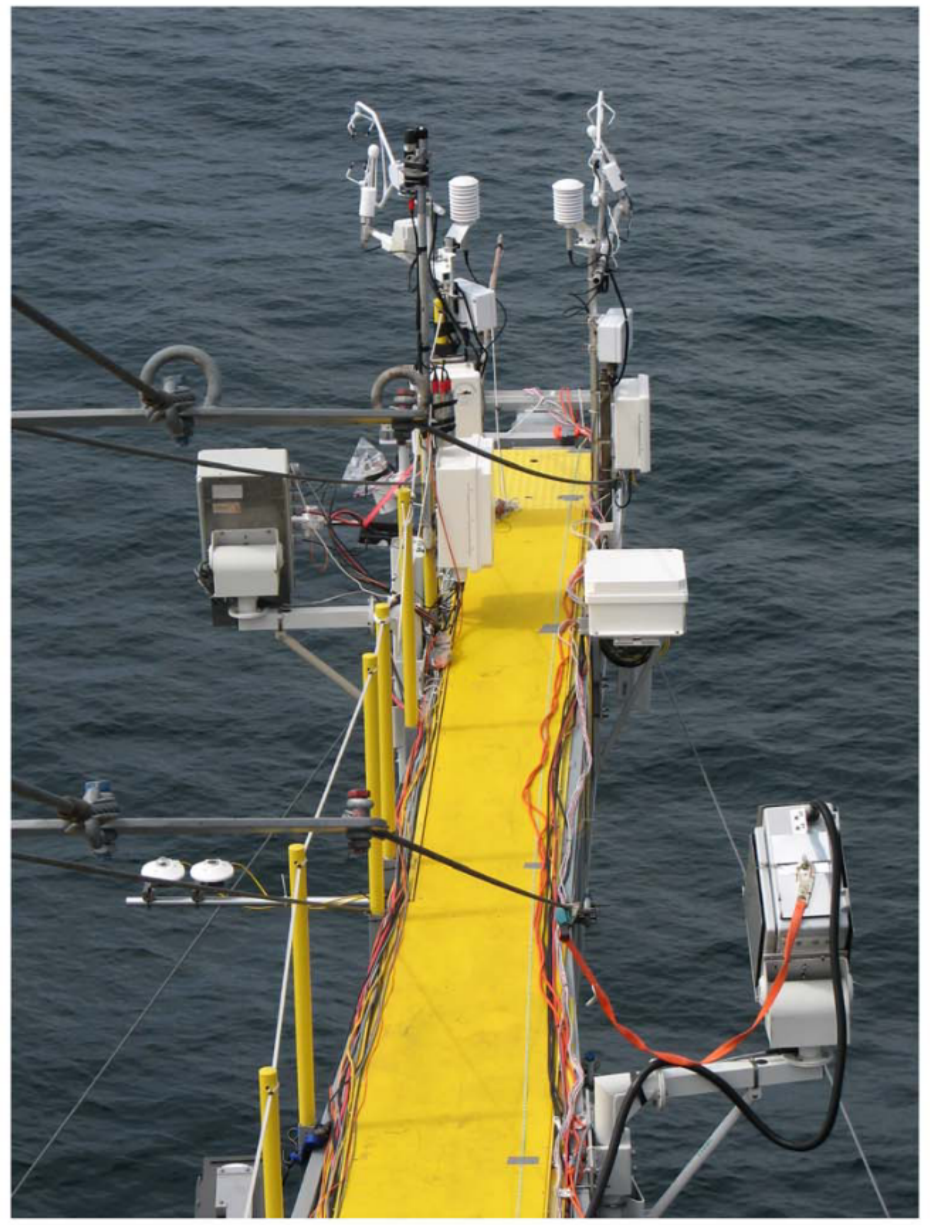
Nh1 = 1024, Nh2 = 2048

40 m 3.1m L0 = 200 m
100 m 7.8m L0 = 500 m

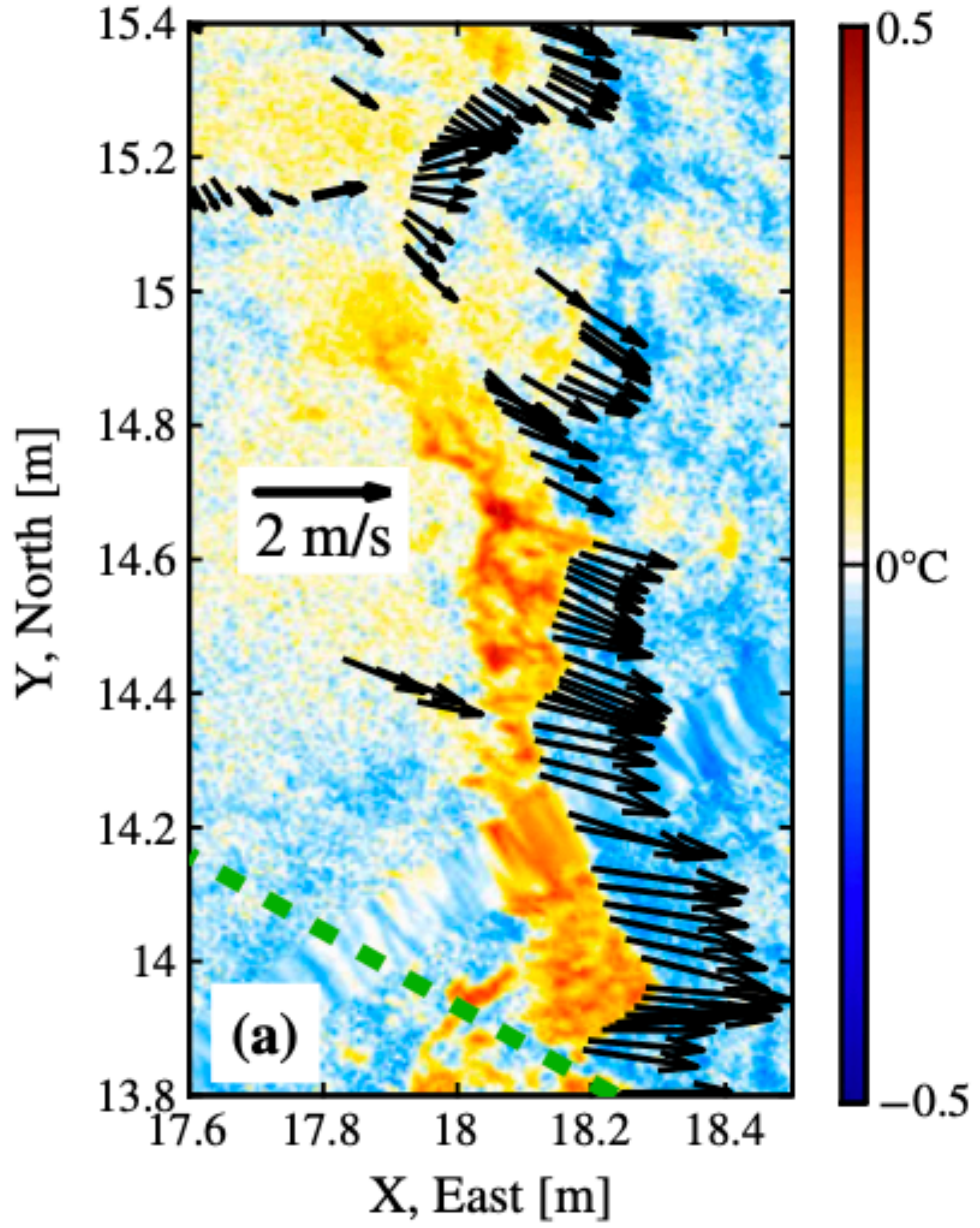
- Smallest scale is set by numerical resolution. Ratio of peak to smallest length scale O(10).
- Less steep wave field has slower transfer of energy into small scales.
- Eventually k^{-3} shape is established for numerically resolved range.

Postprocessing for $\Lambda(c)$ distribution

Field observations



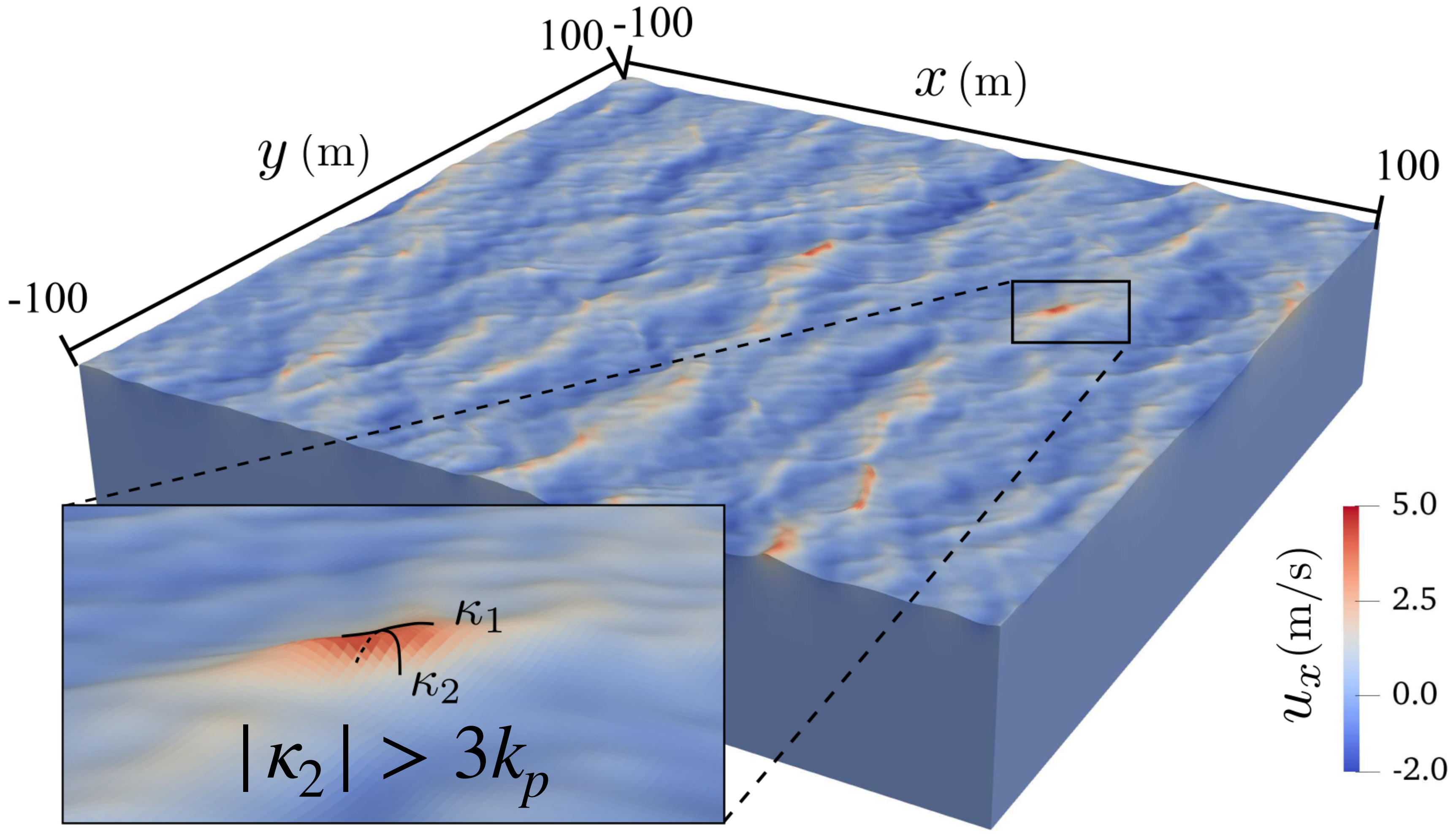
Zappa et al. 2012

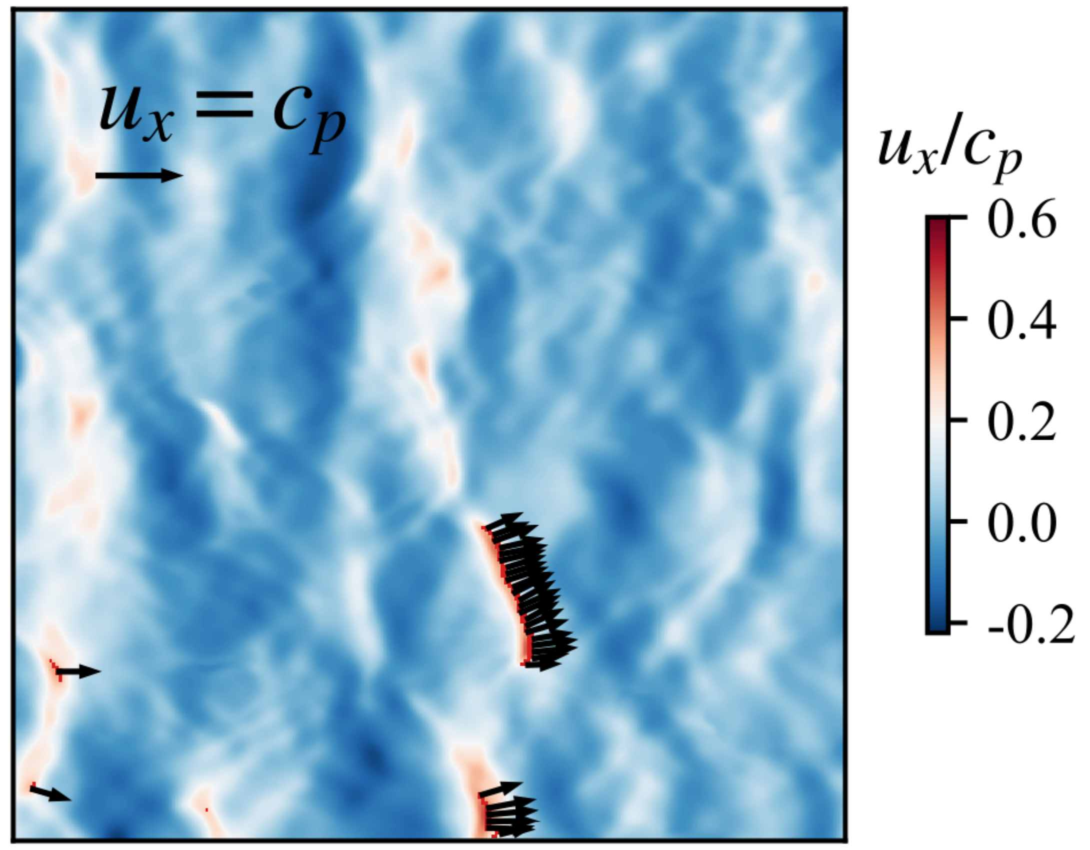


Sutherland & Melville 2013

Numerical experiments

Curvature-based breaking detection + Velocity mapping





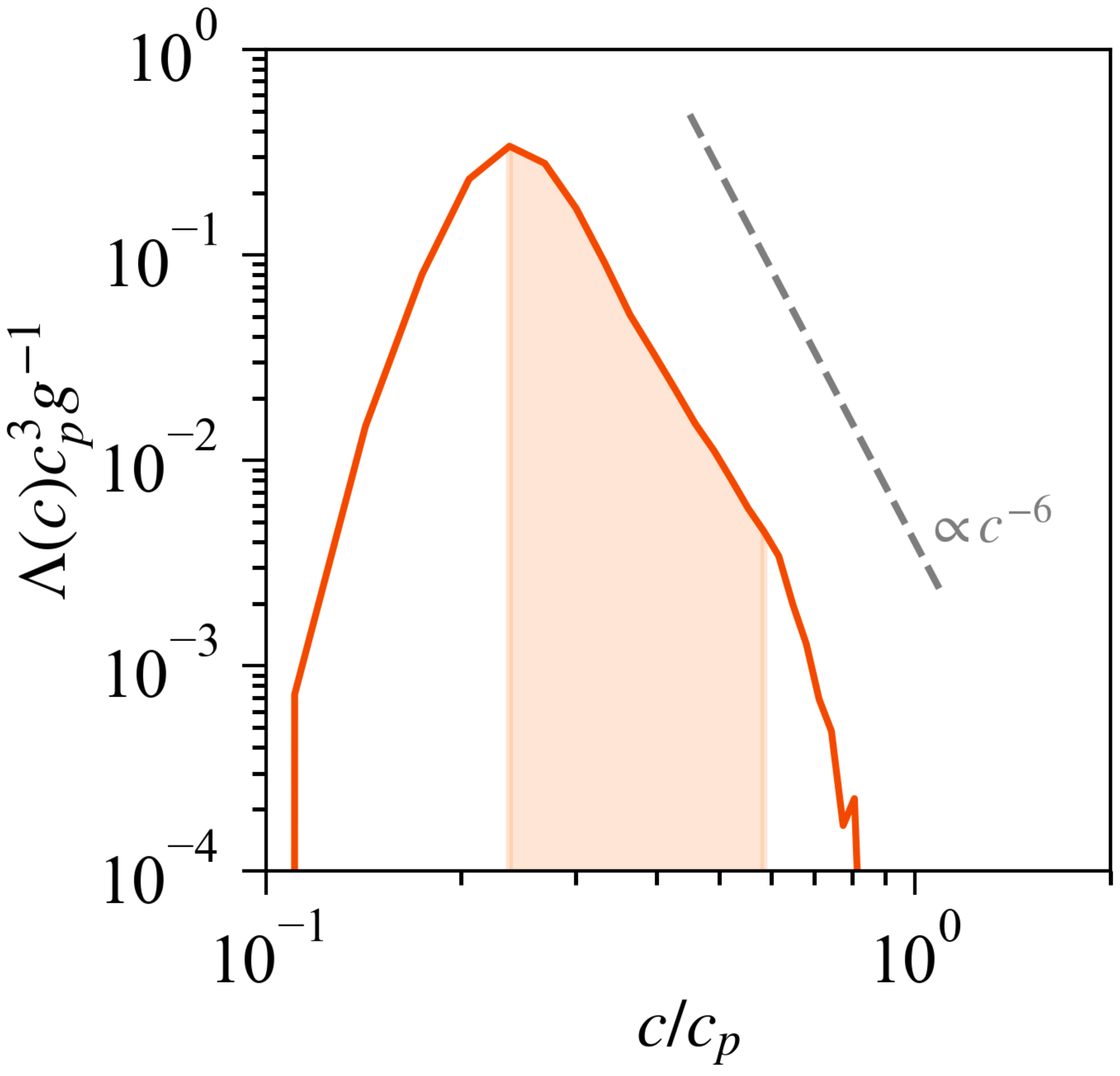
$\Lambda(c)$ **distribution**

$\Lambda(c)$: the expected length of breaking fronts (per unit area) with speed $(c, c + dc)$.

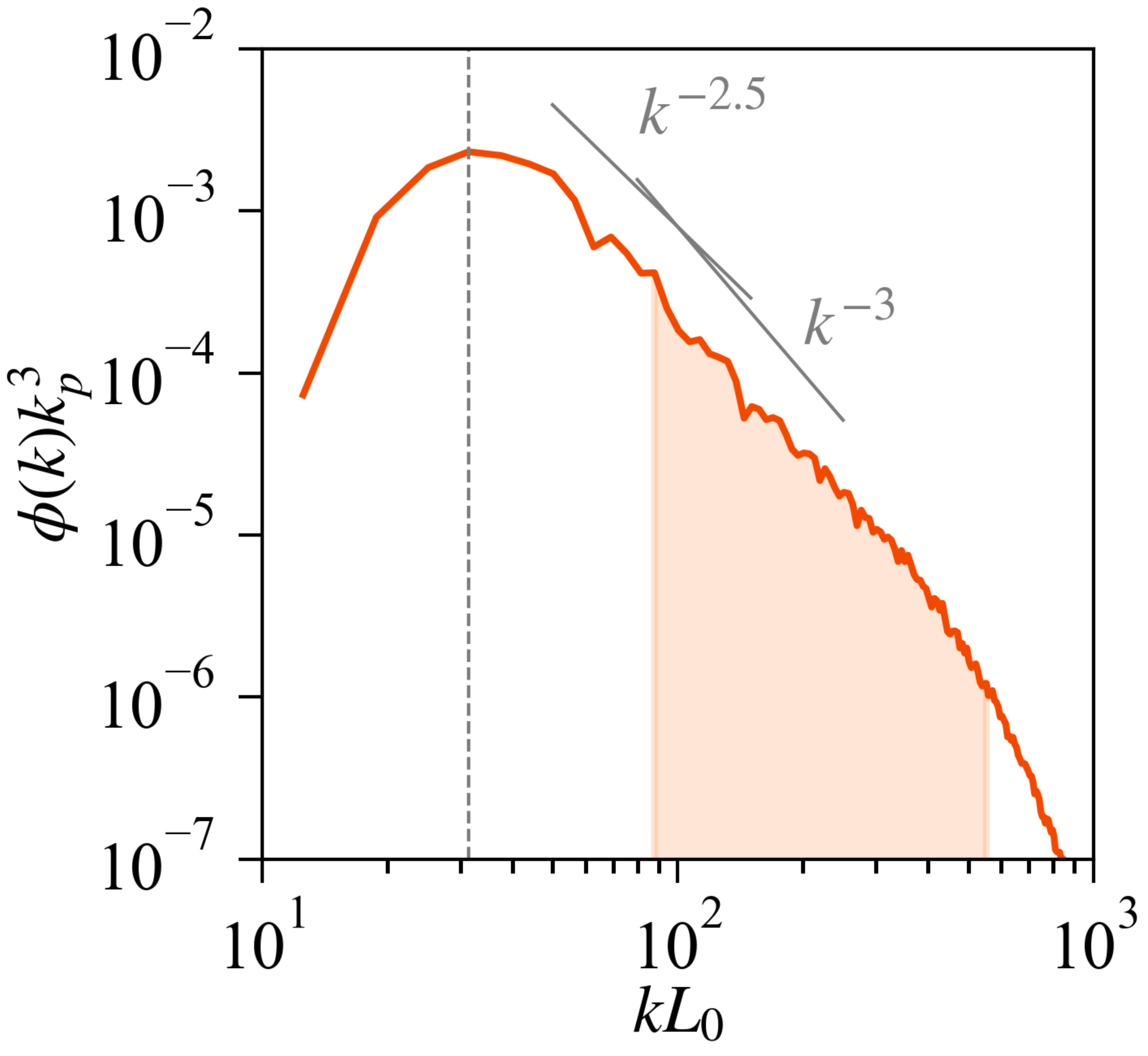
The variable c indicates the scale of the wave, and quantifies the strength of the breakers (affects dissipation etc.).

Breaking statistics - $\Lambda(c) \propto c^{-6}$ power law

Breaking distribution



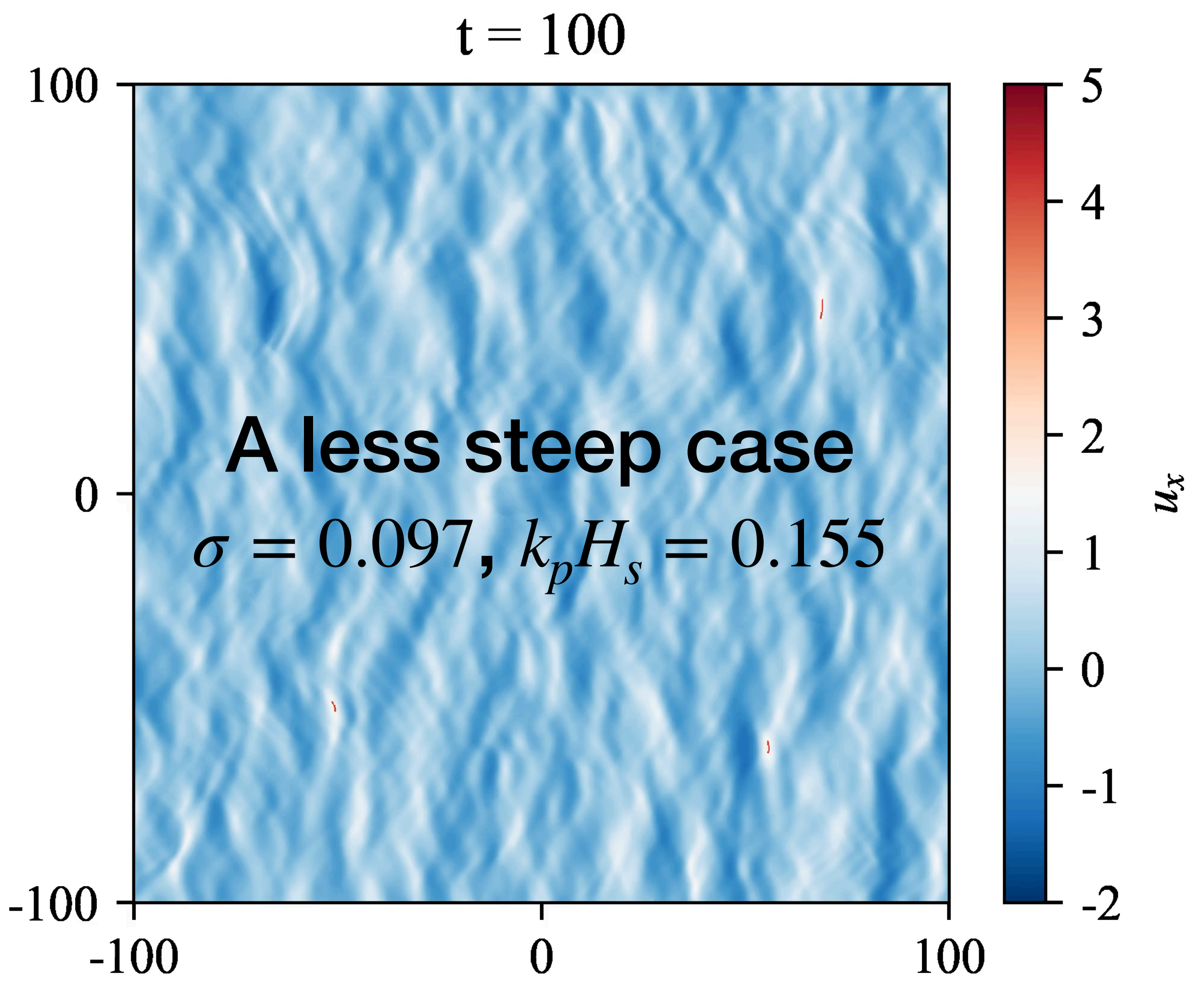
The corresponding wave spectrum



Observe $\Lambda(c) \propto c^{-6}$ power-law, which agrees with Phillips' theoretical prediction.

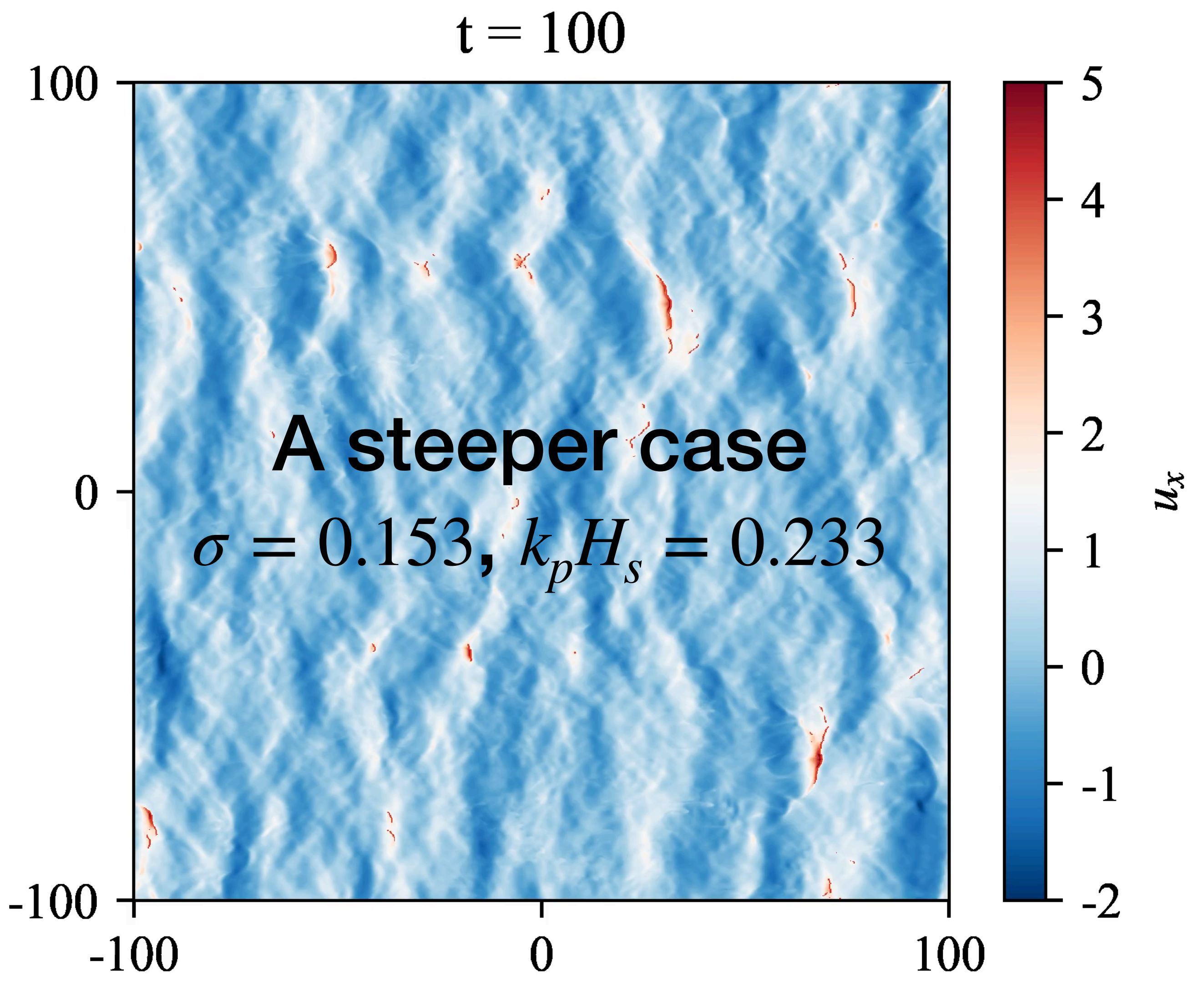
Identify the scale of the breakers ($k = g/c^2$).

Breaking statistics - steepness scaling



A less steep case

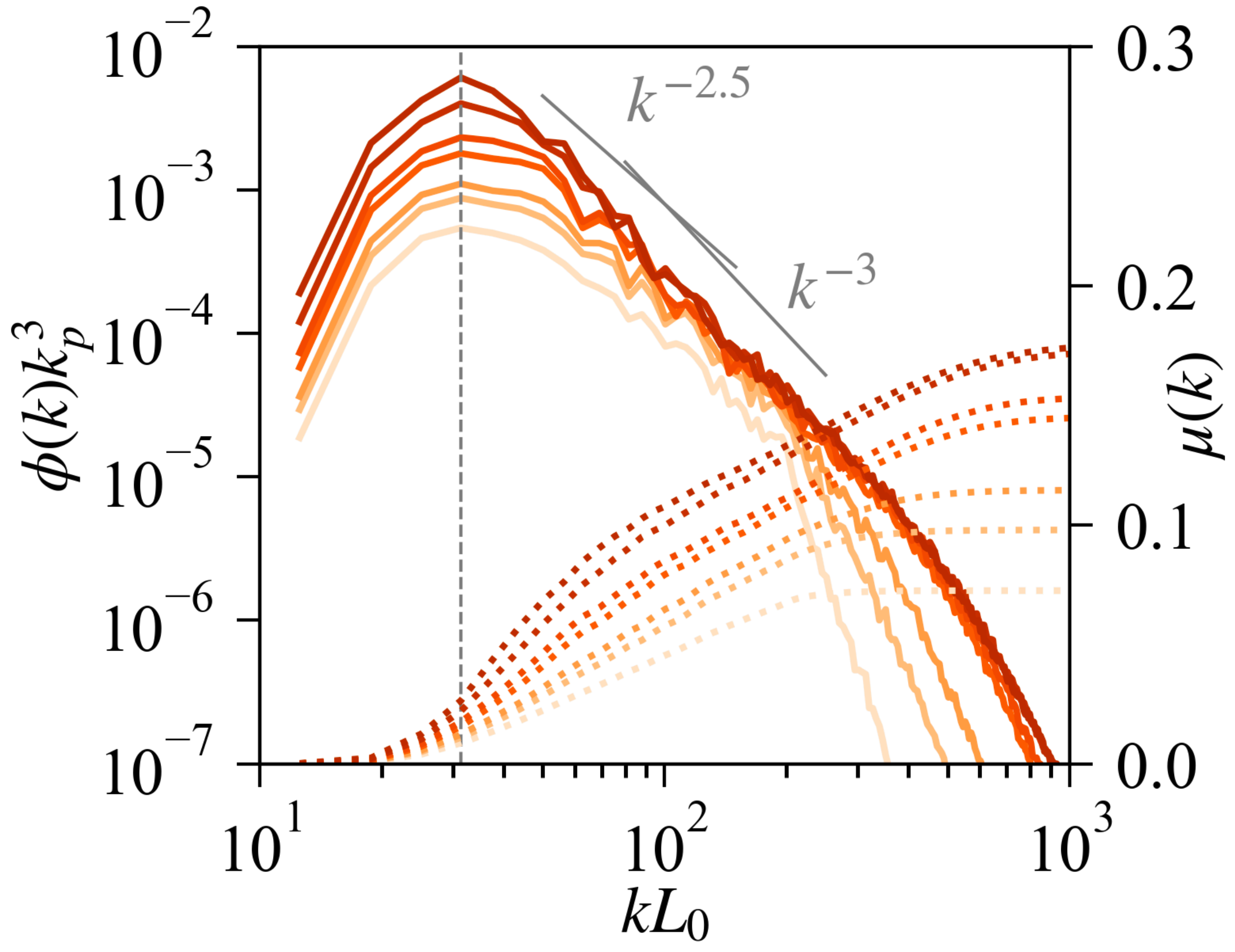
$\sigma = 0.097$, $k_p H_s = 0.155$



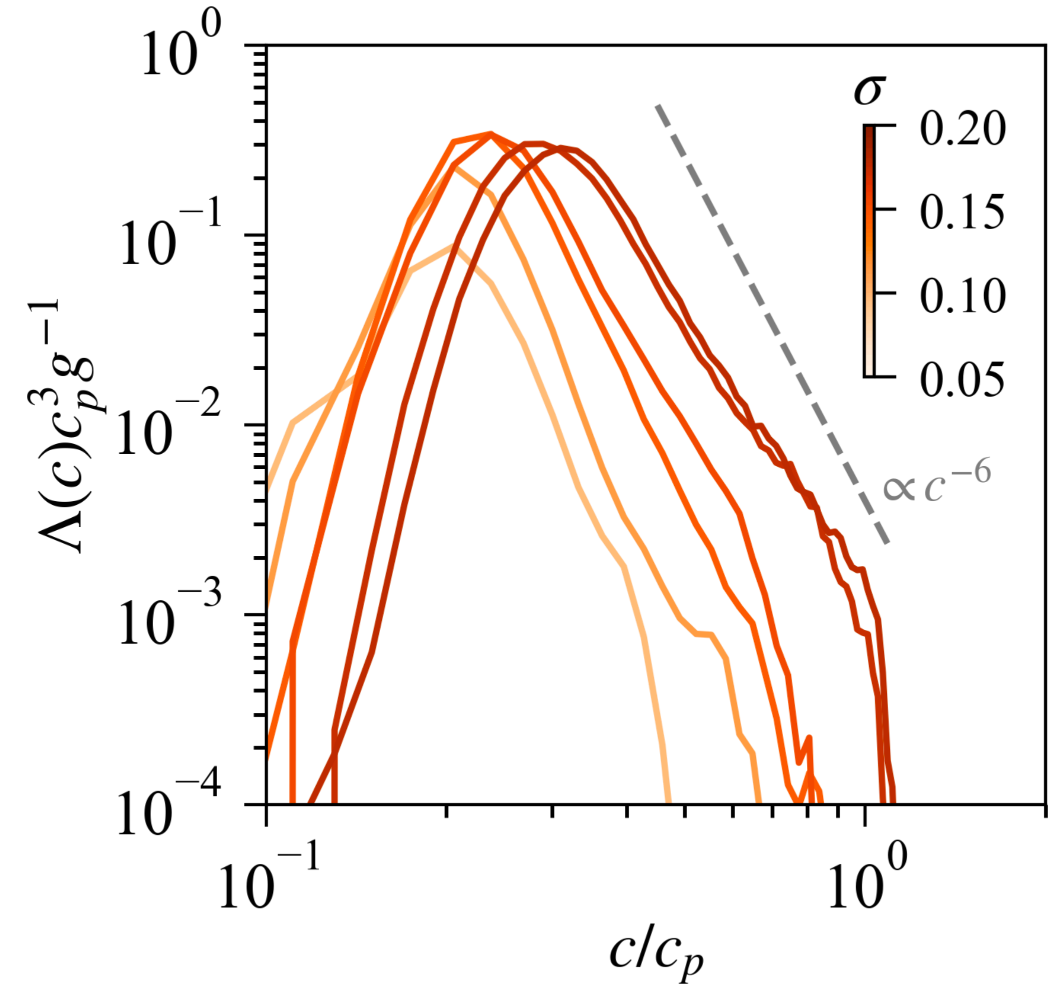
A steeper case

$\sigma = 0.153$, $k_p H_s = 0.233$

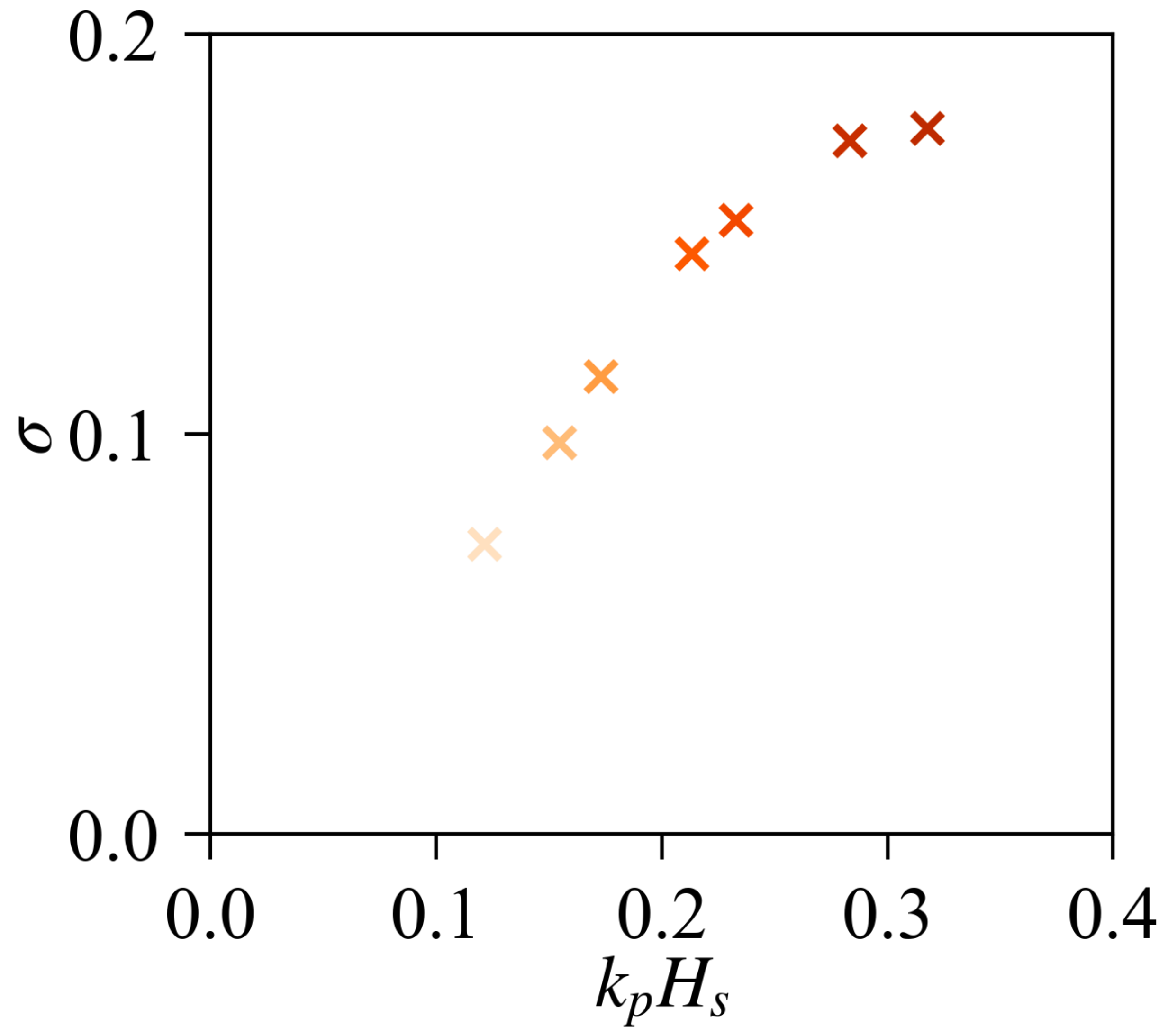
Spectra with varying steepness



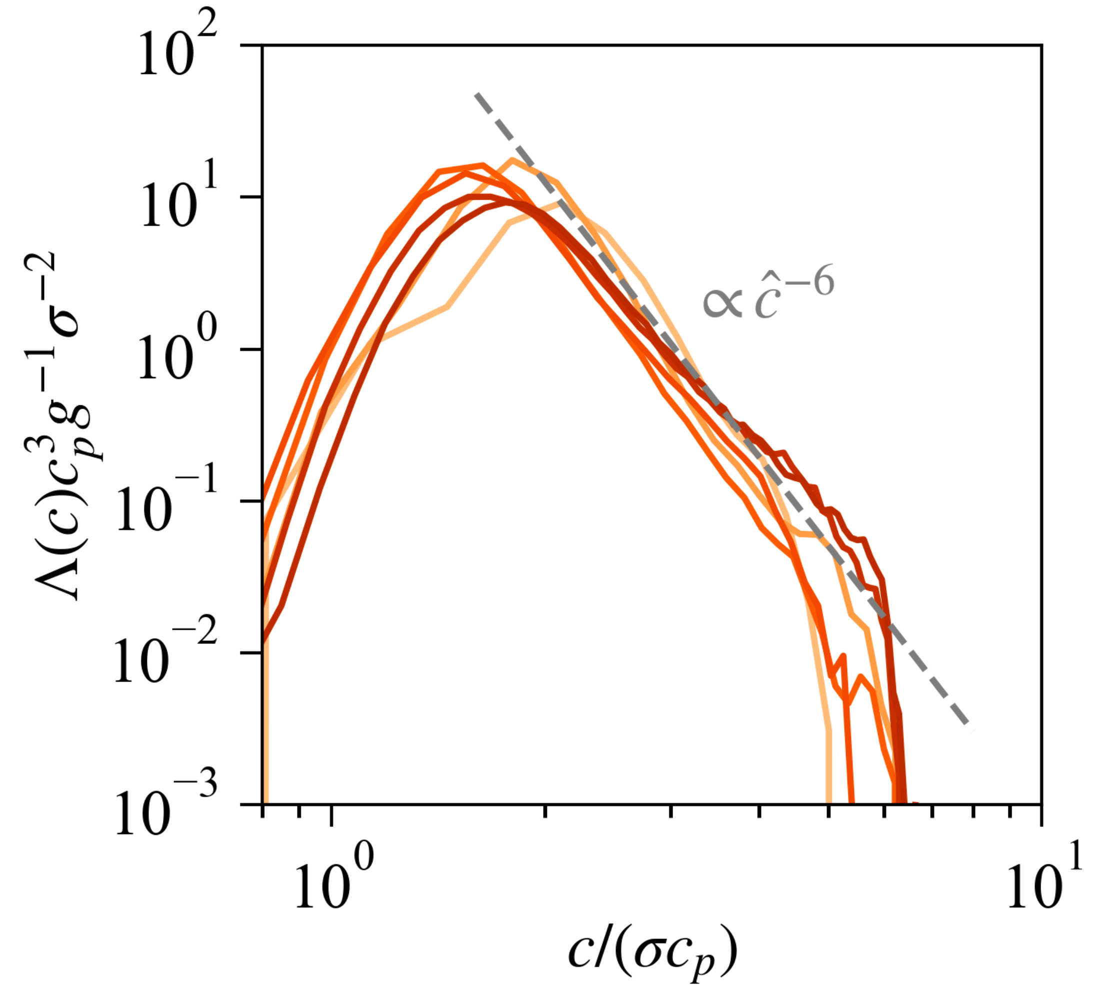
$\Lambda(c)$



Global steepness parameters

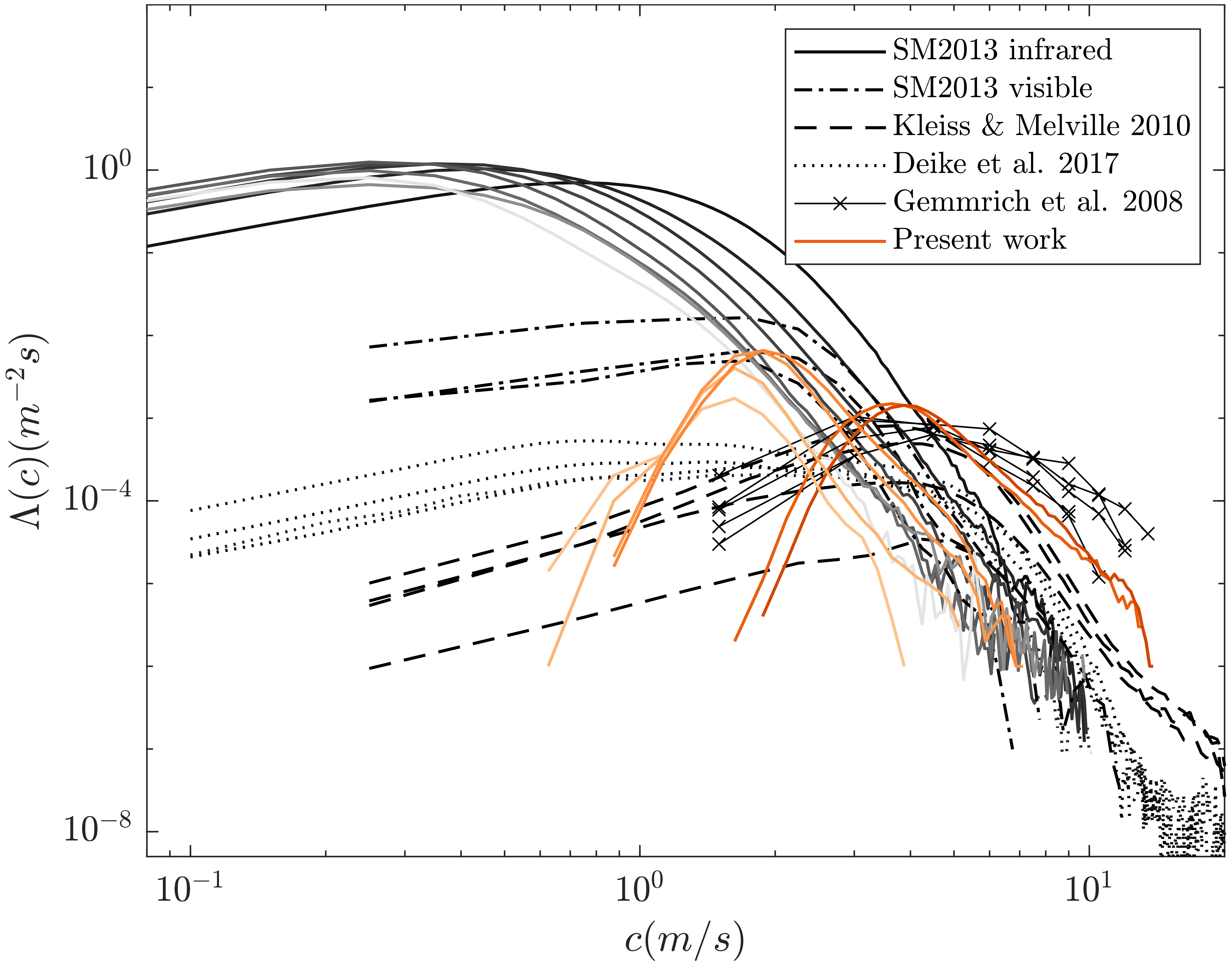


$\Lambda(c)$ with normalization



Comparison with field data

Unscaled data in physical units



Different scalings in literature

1. Phillips 1985 wind-only scaling (does not match data well):

$$\Lambda(c) \propto u_*{}^3 g c^{-6}$$

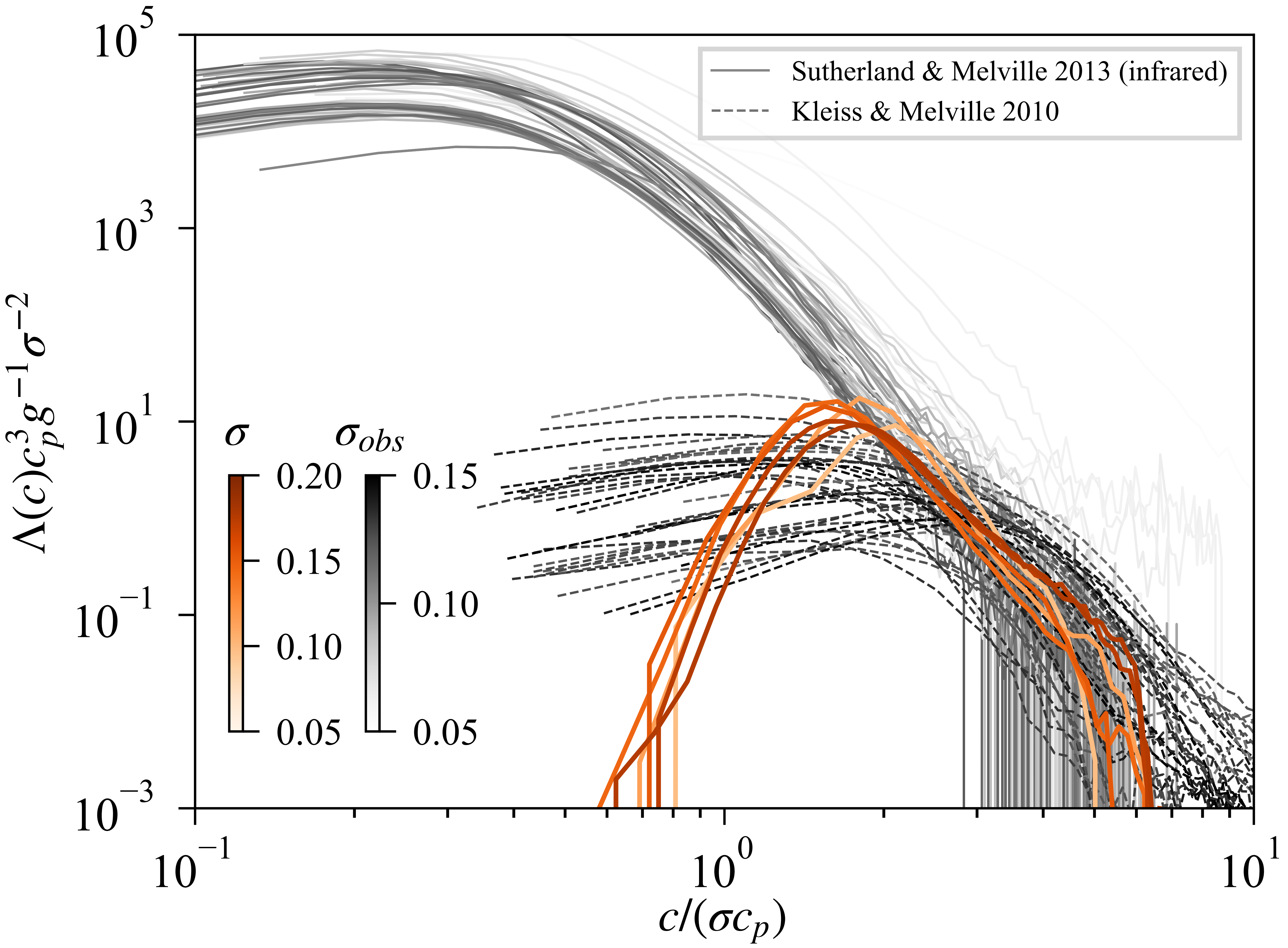
2. Empirical wind-wave-mixed scaling (Sutherland & Melville 2013 etc):

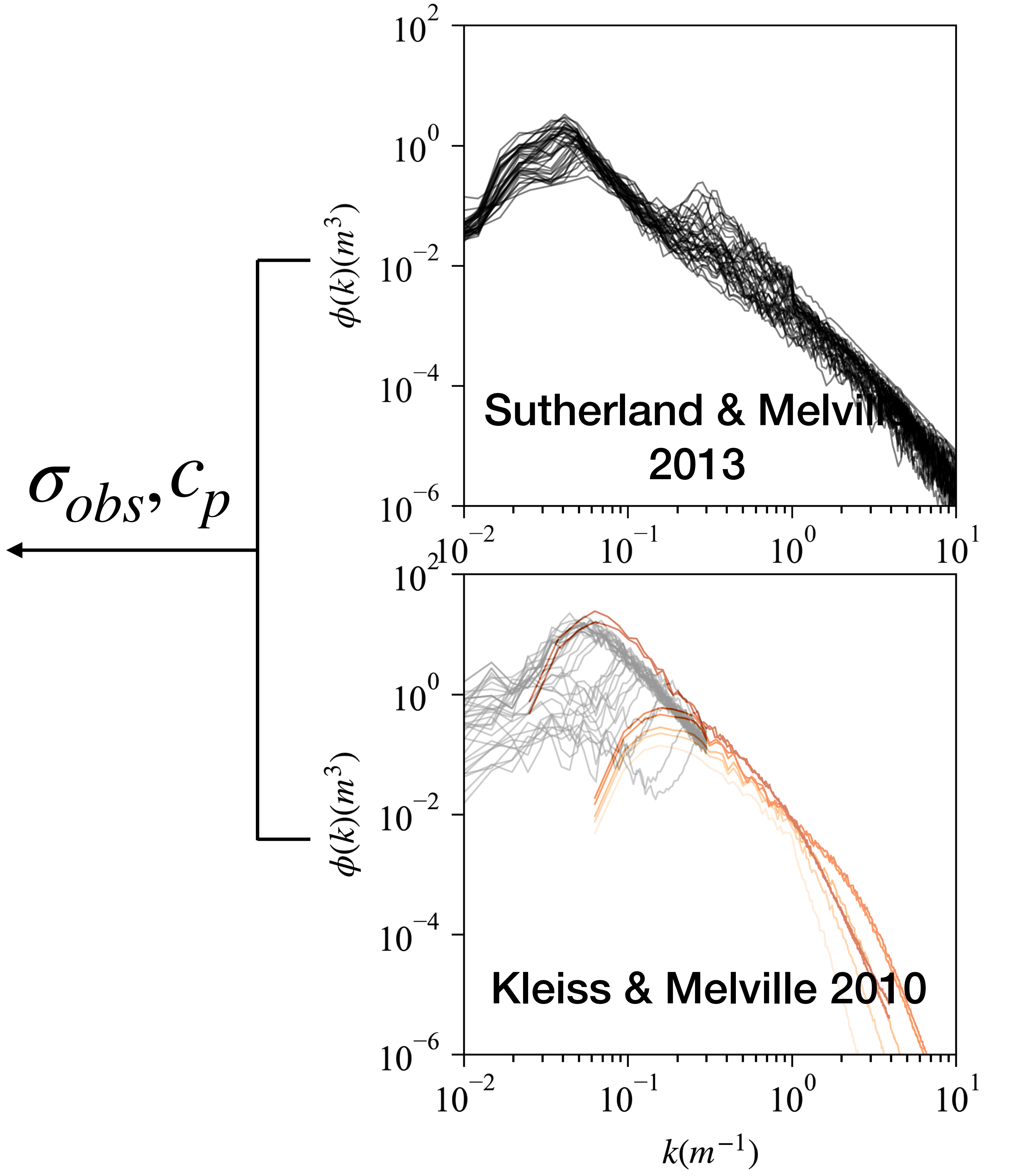
$$\Lambda(c) c_p^3 g^{-1} (c_p/u_*)^{1/2} \propto (c/\sqrt{gH_s})^{-6}$$

3. Present work wave-only scaling (without wind forcing):

$$\Lambda(c) c_p^3 g^{-1} \sigma^2 \propto (c/(\sigma c_p))^{-6}$$

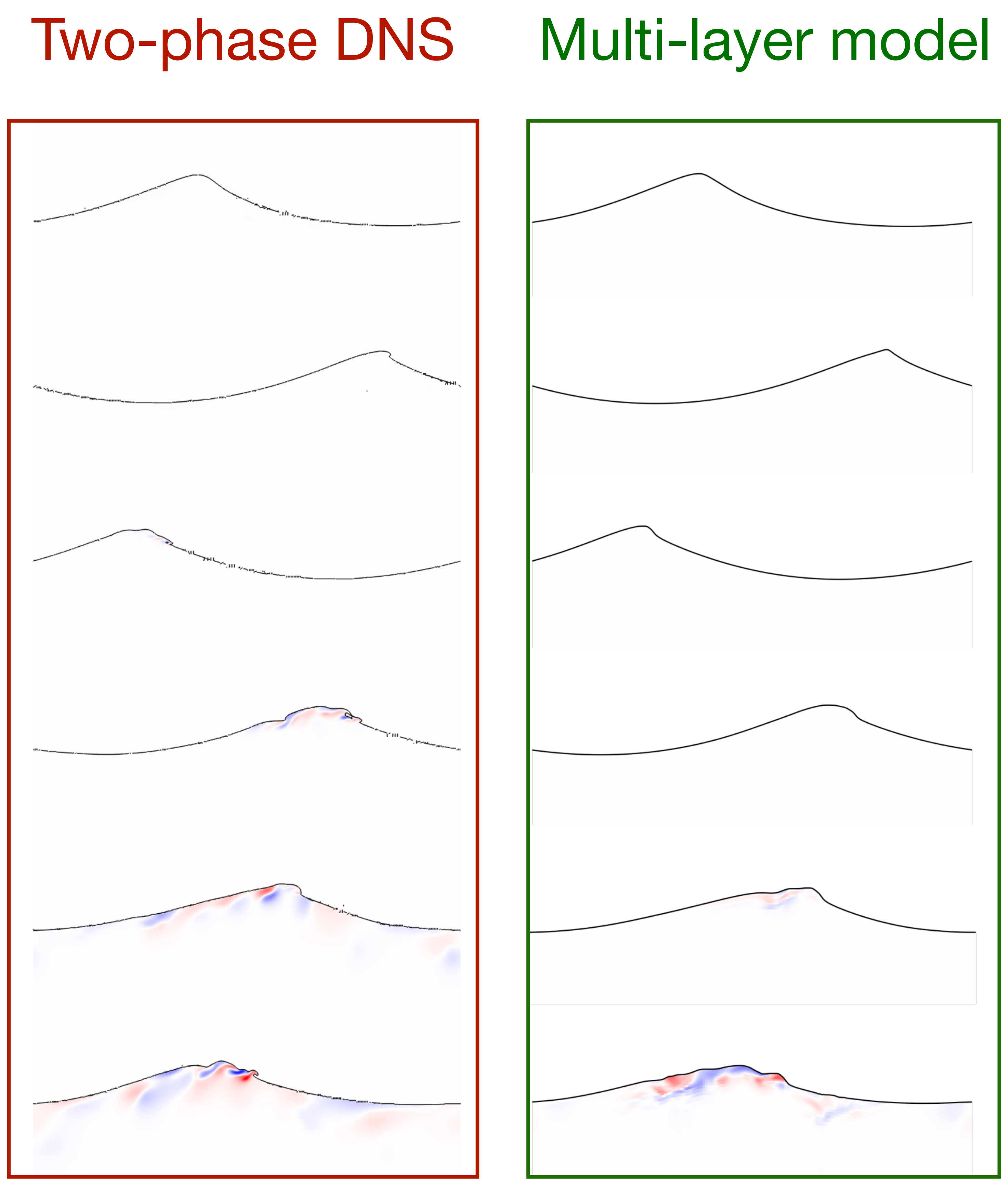
Scaling tested on field data


Wave slope based scaling

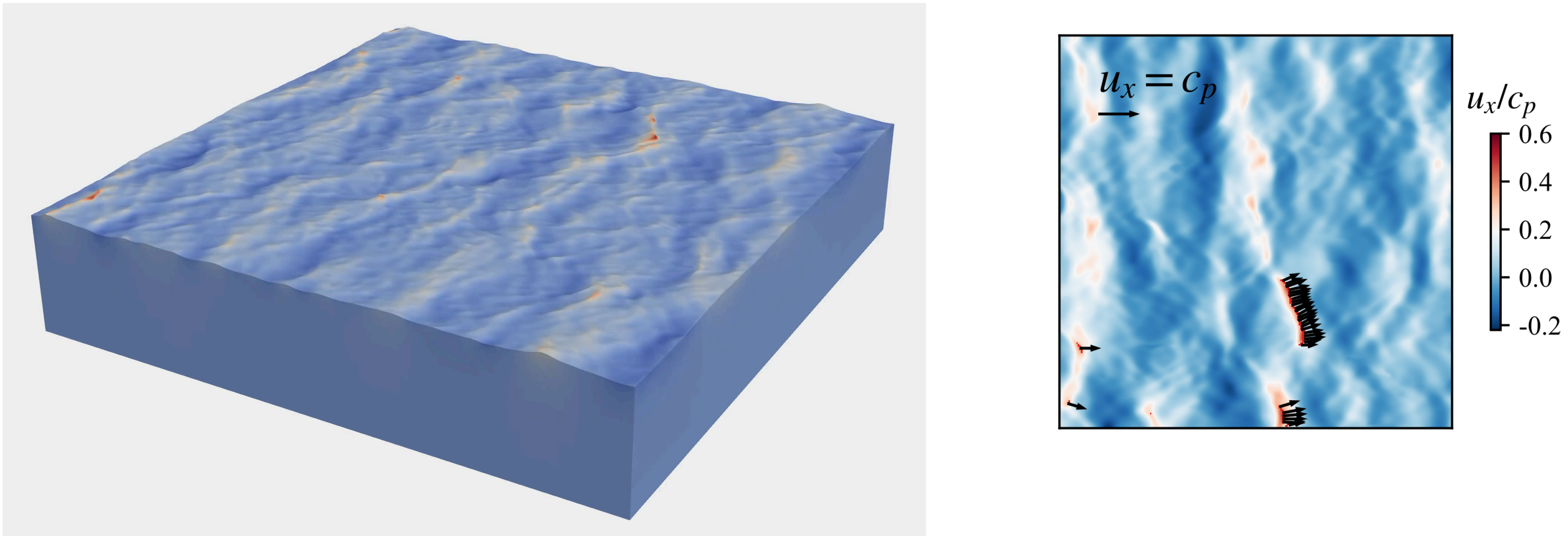

Wave energy spectrum

Preprint: https://arxiv.org/abs/2211.03238

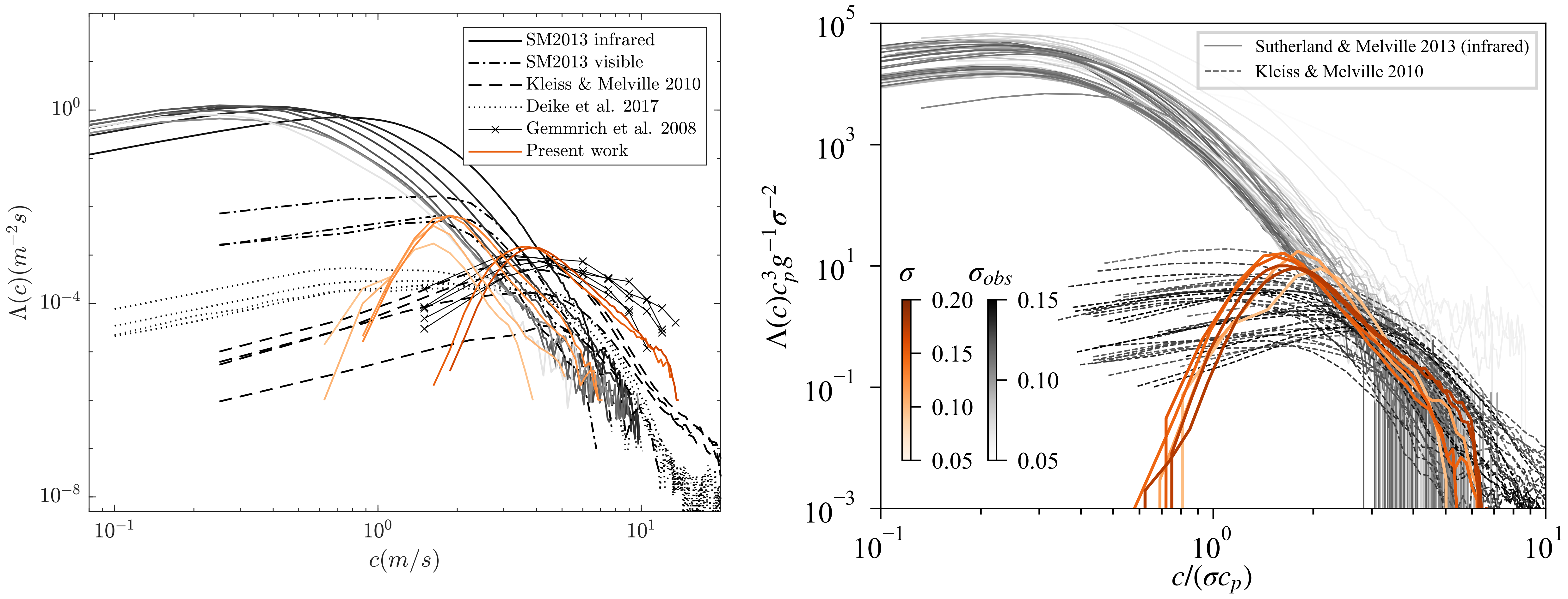
Summary



Broadband wave field



Interesting wave field evolution. Collect breaking statistics.

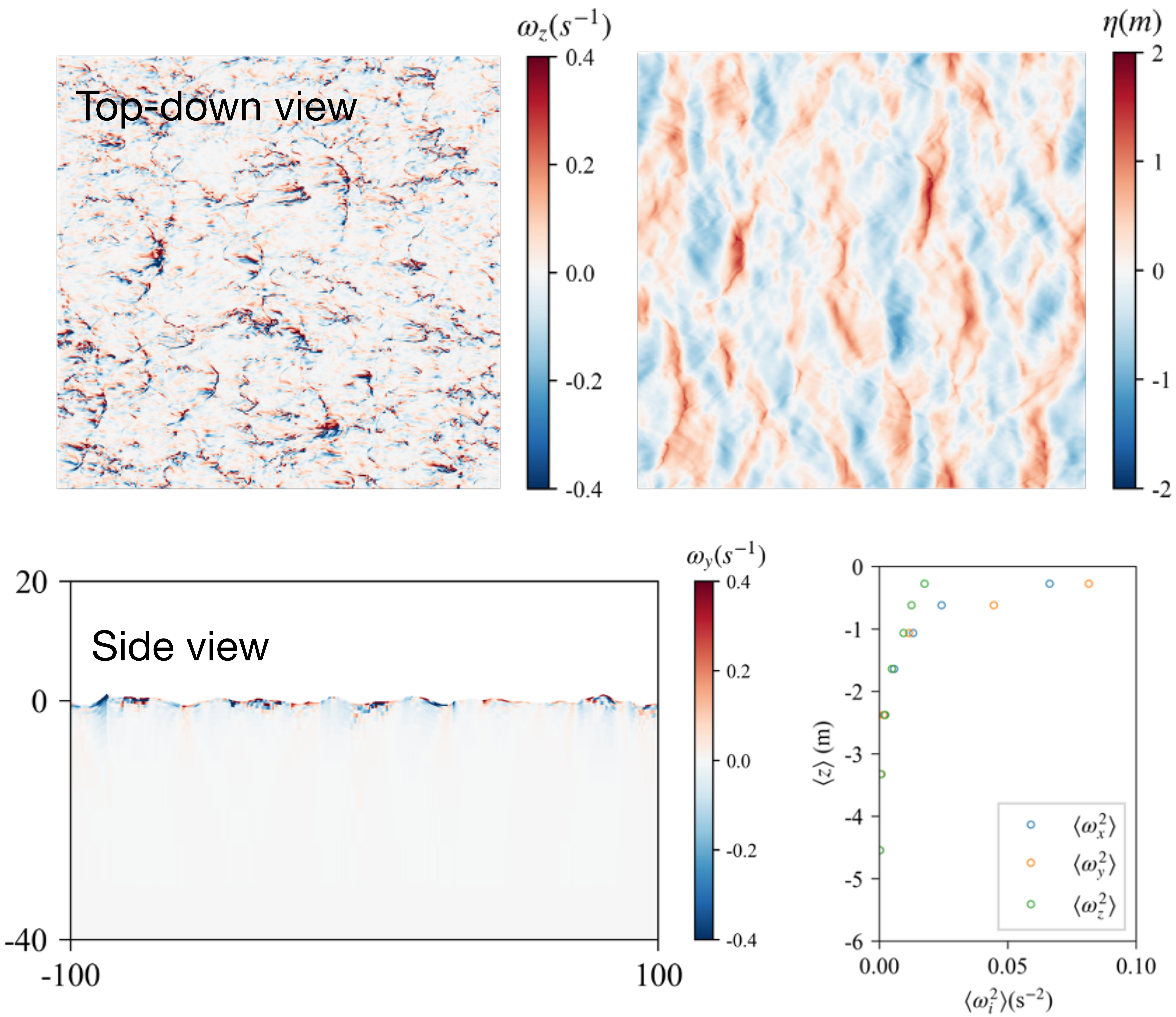


Similar level of breaking frequency to field observations.
Proposed wave-slope only scaling.

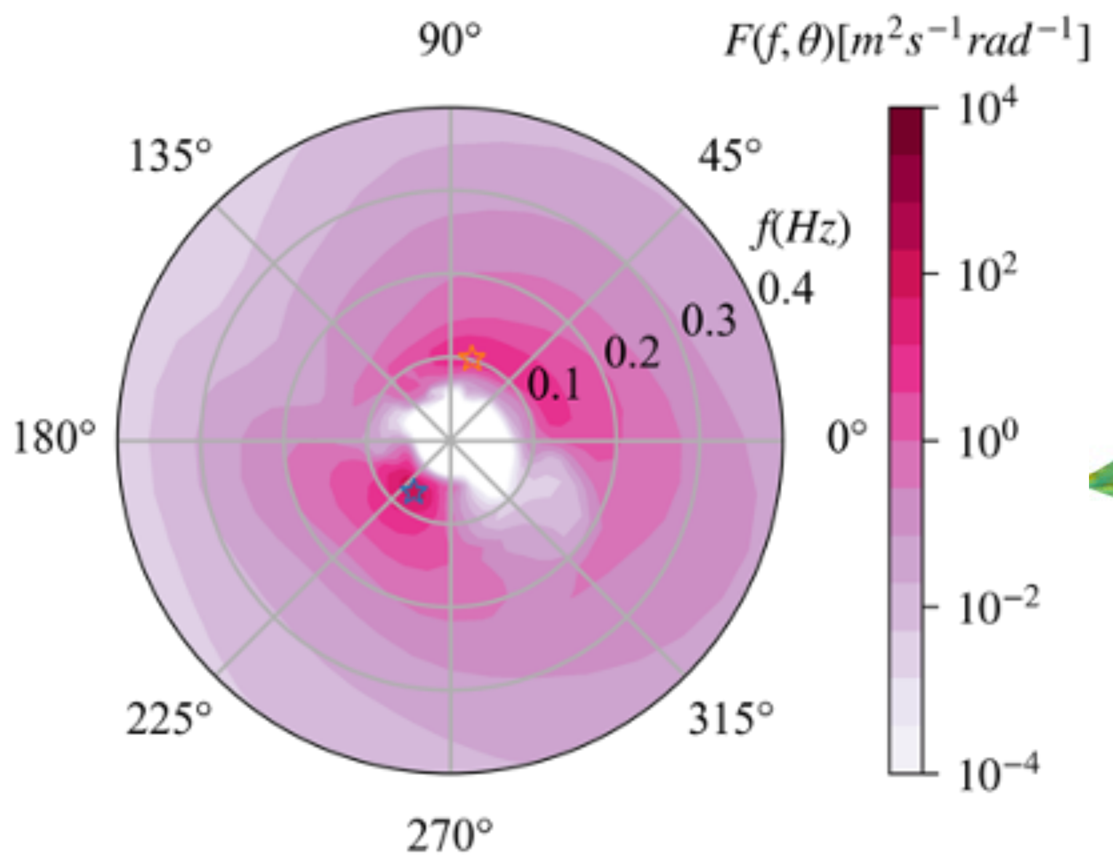
Phase-resolved wave simulations in the physical space that permit strong non-linearity (without surface overturning) with reasonable computational cost.

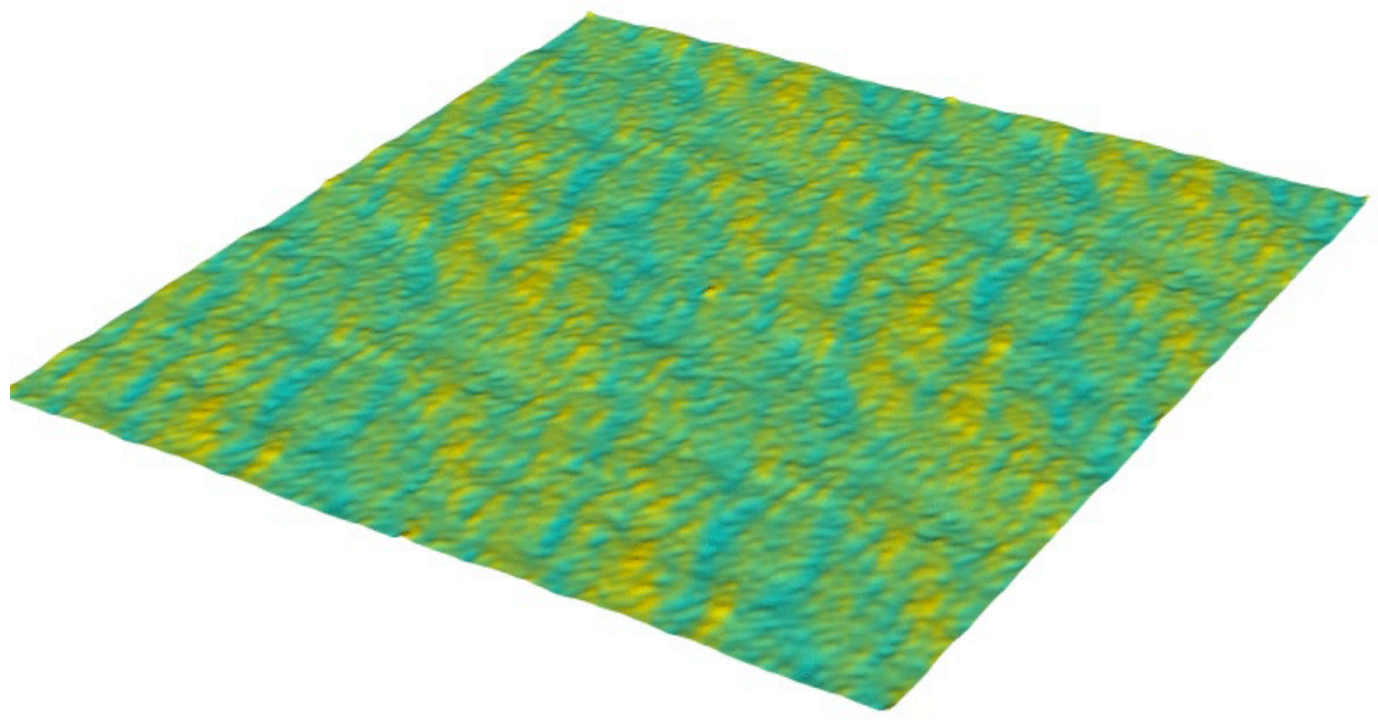
Future work

- **Underwater vorticity and dissipation**



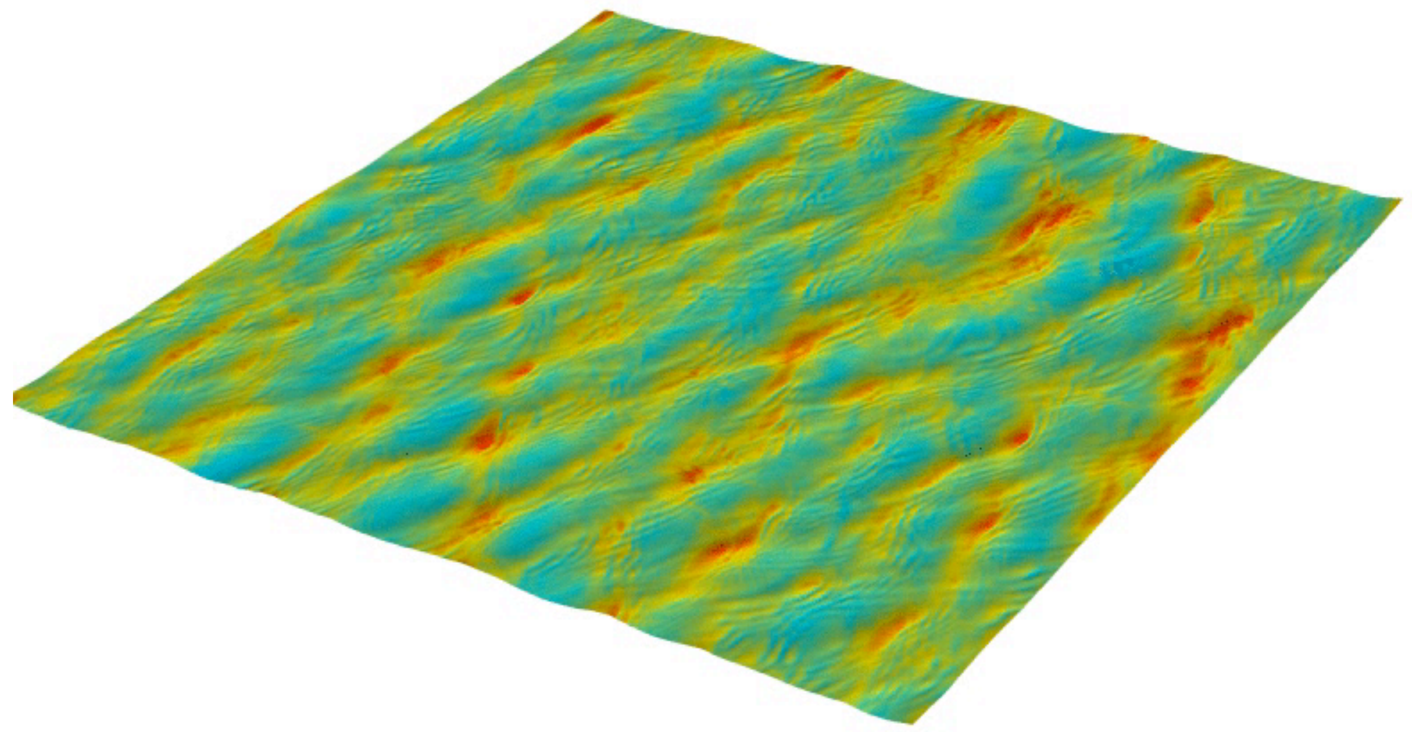
- **Mixed swell-wind-wave sea state**



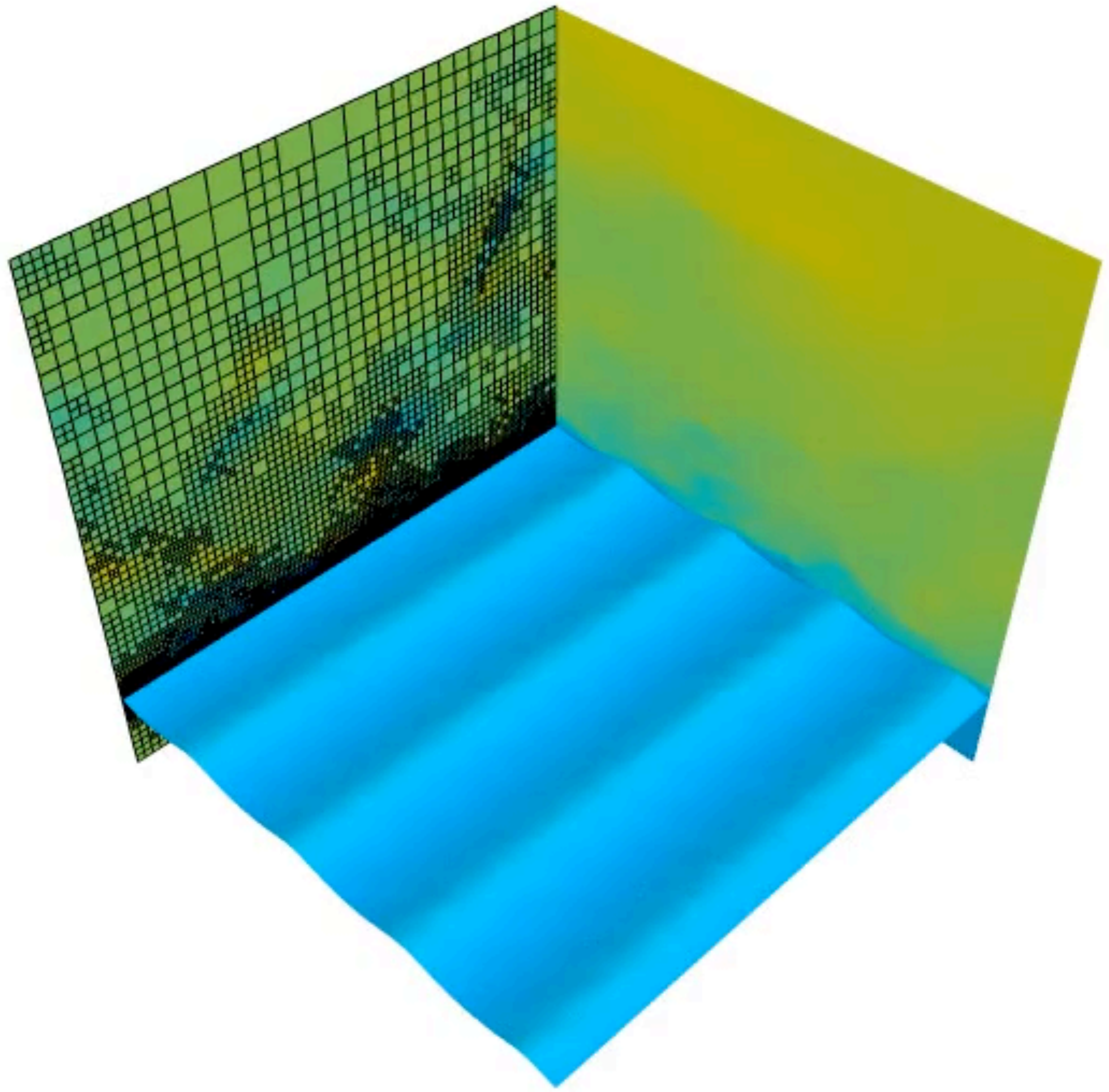


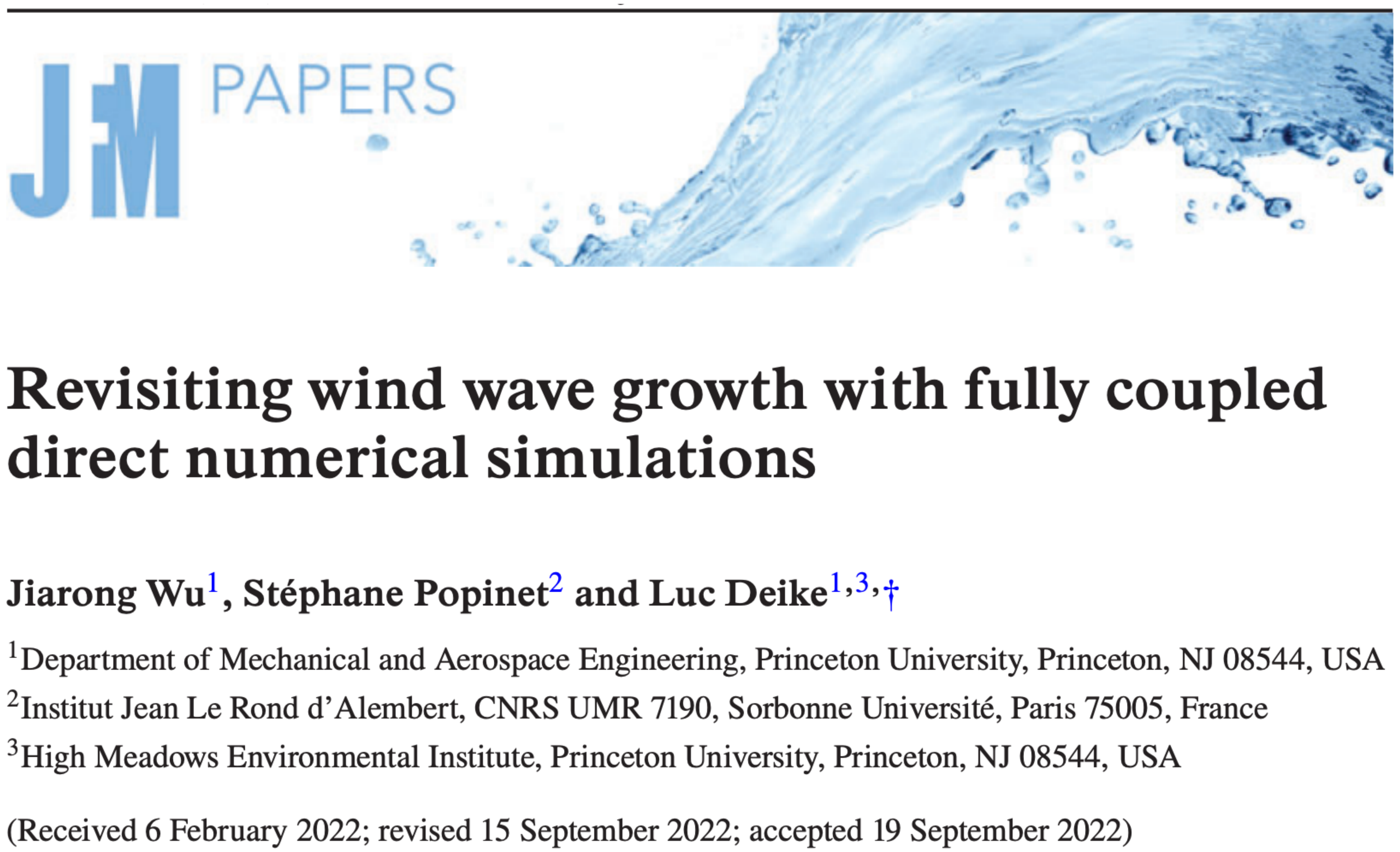
- **Surface tension for gravity capillary waves**

http://basilisk.fr/sandbox/crobert/ credit to Clément Robert



Wind wave growth with VOF





JFM PAPERS

Revisiting wind wave growth with fully coupled direct numerical simulations

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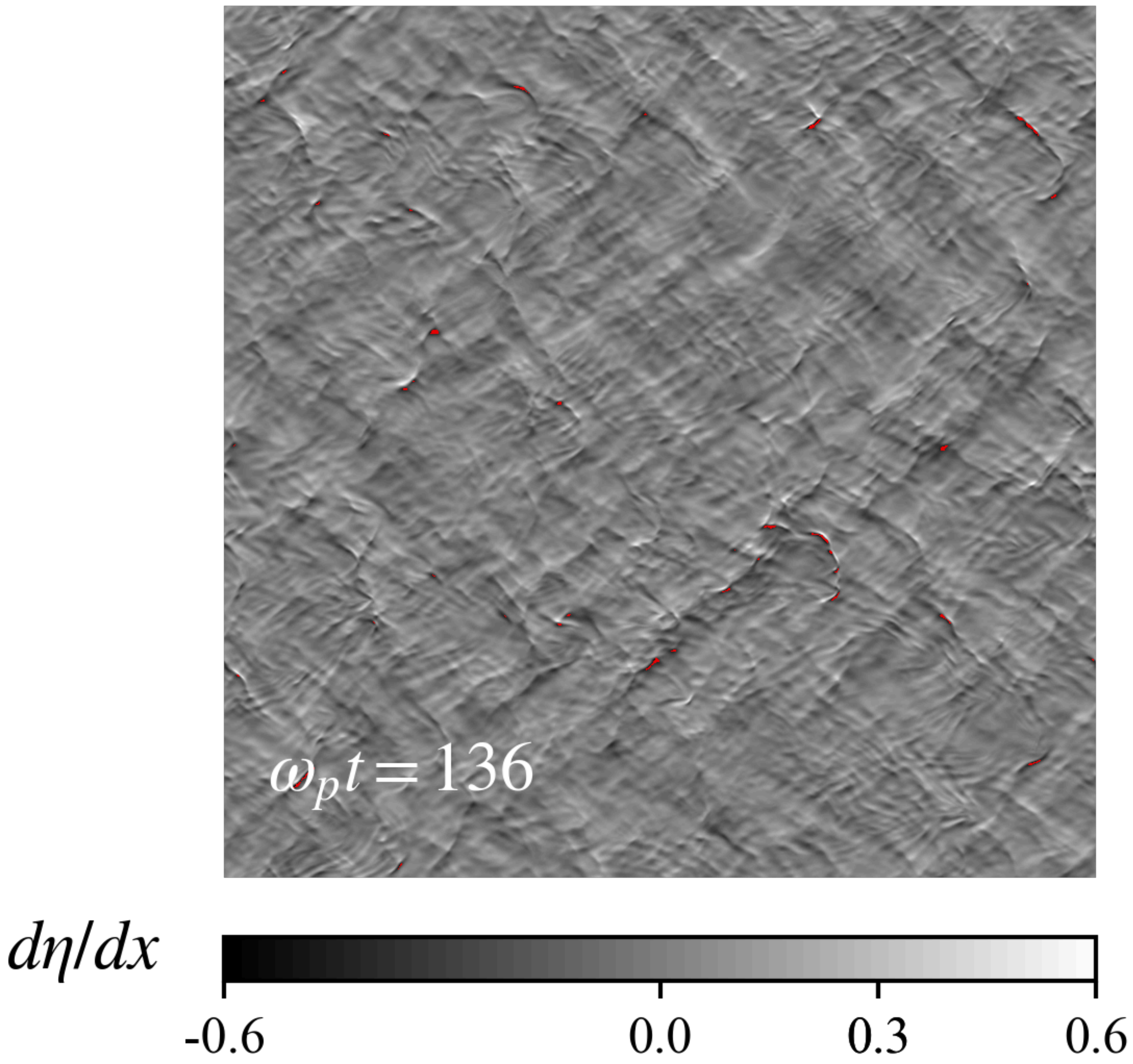
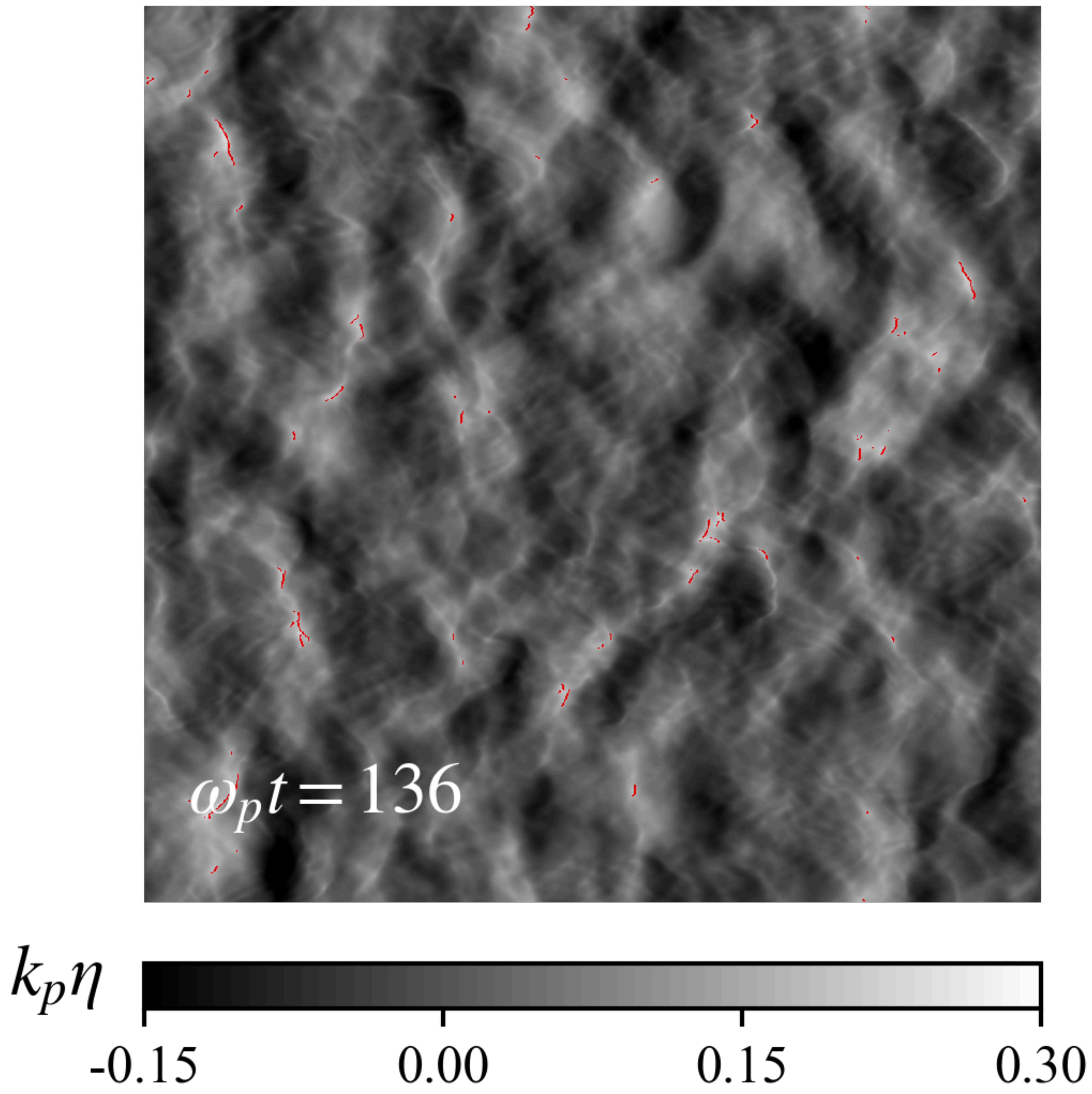

$\omega_p t = 136$
$d\eta/dx$
-0.6
0.0
0.3
0.6


$\omega_p t = 136$
$k_p\eta$
-0.15
0.00
0.15
0.30