

# Viscous Free-Surface Flows

Vatsal Sanjay



European Research Council  
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Physics of Fluids

**UNIVERSITY  
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Link: <https://youtu.be/3FZPkWxprDM>

# Collaborators & Acknowledgements

- **Detlef Lohse**
- Pierre Chantelot (Institut Langevin)
- Mazi Jalaal (UvA, Amsterdam)
- Uddalok Sen (WUR, Wageningen)
- **Jacco Snoeijer**
- Cunjing Lv (Tsinghua Univ.)
- **Alexandros Oratis**
- **Vincent Bertin**
- Youssef Saade (Canon)\*
- Mandeep Saini\*
- **Pallav Kant (Univ. of Cambridge)**
- Jonathan Pham (Univ. of Kentucky)
- Doris Vollmer (MPI-Mainz)

**Canon**

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**Physics of Fluids**



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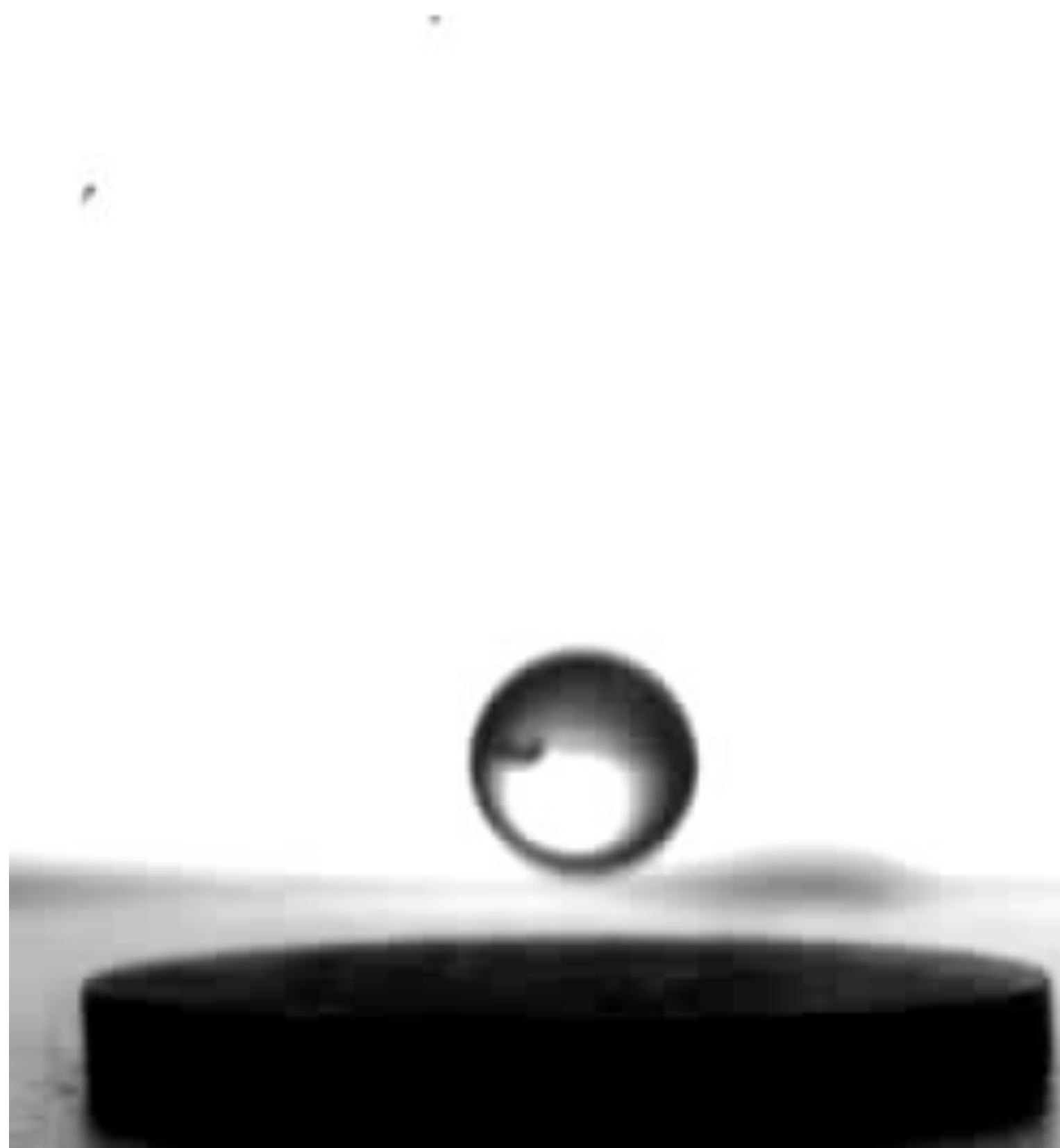
Physics of Fluids

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# Drop impact on a superhydrophobic surface: What are the forces exerted on the substrate?



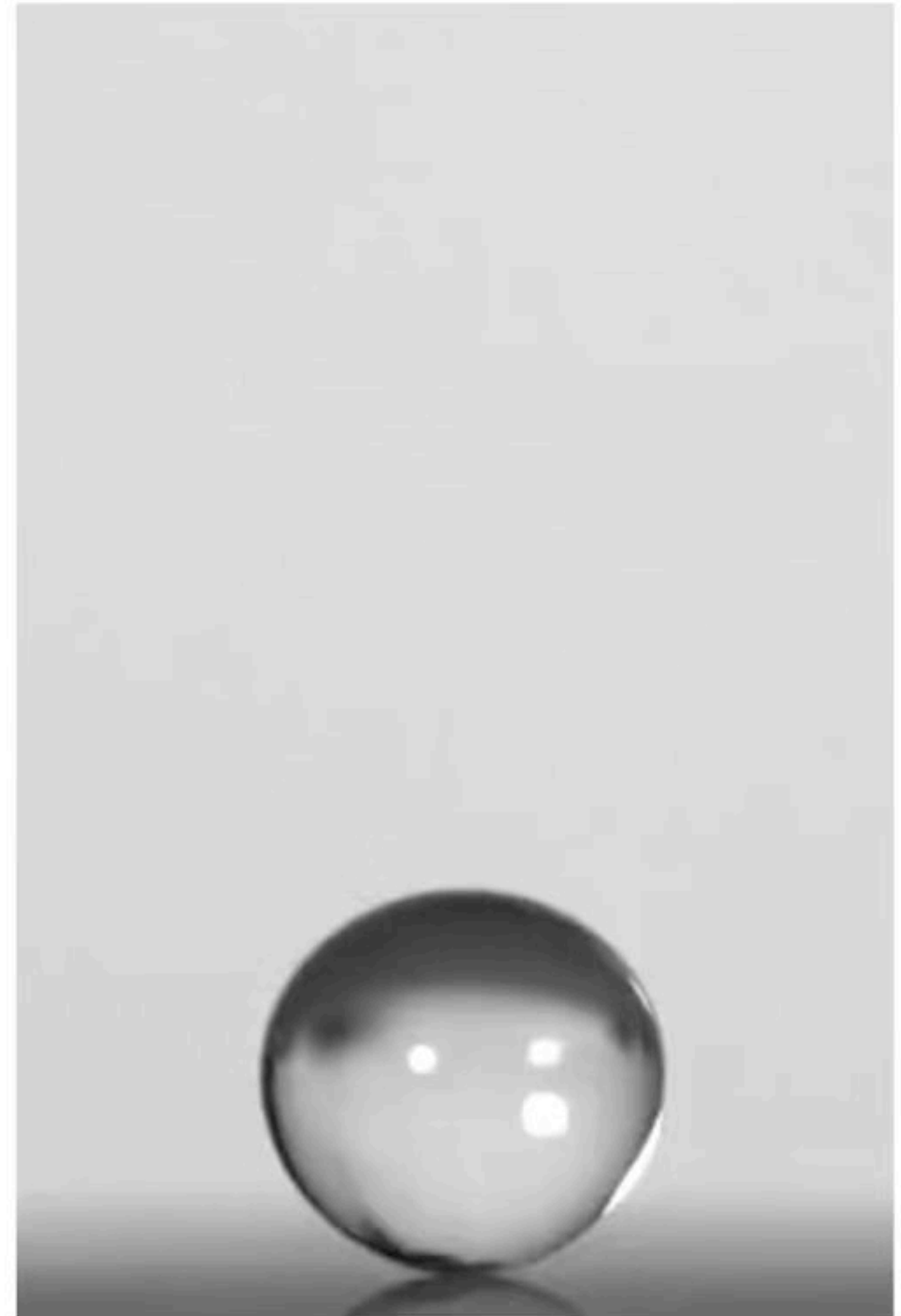
low velocity



high velocity

# Parameter space of drop impact

- Velocity
- Diameter
- Viscosity
- Density
- Surface tension
- Surface roughness
- Temperatures
- Ambient air pressure
- ...



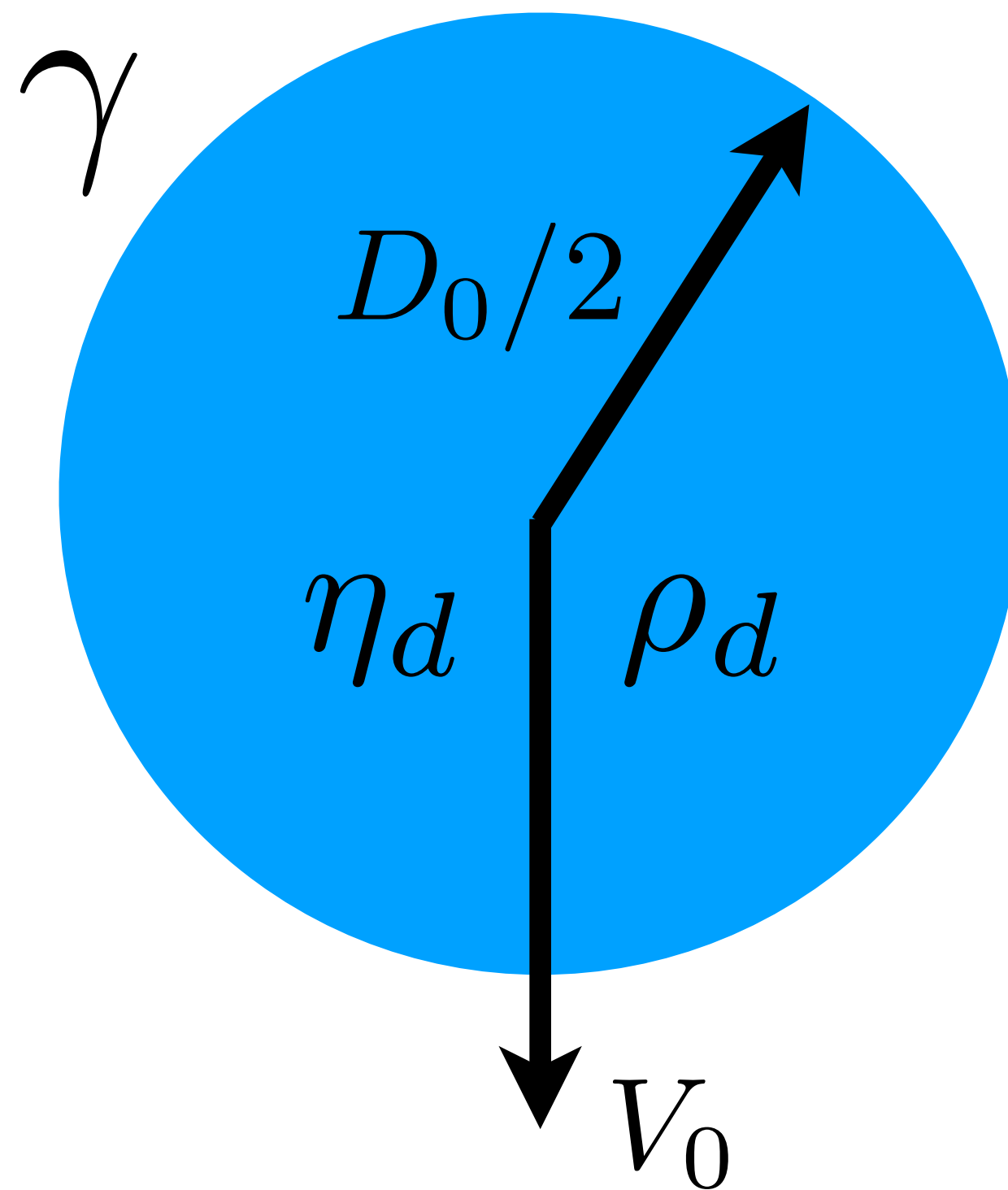
# Drop Impact onto a solid surface



Viscous Free-Surface Flows

$$We = \frac{\rho_d V_0^2 D_0}{\gamma}$$

$$Oh = \frac{\eta_d}{\sqrt{\rho_d \gamma D_0}}$$

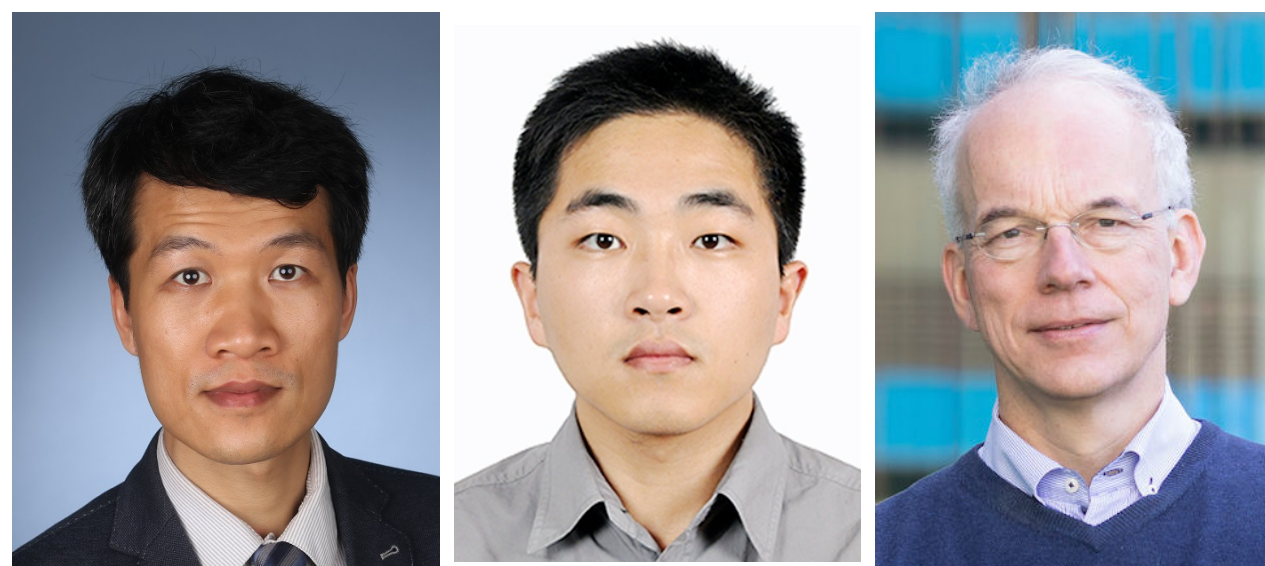


$$Oh_s = \frac{\eta_s}{\sqrt{\rho_d \gamma D_0}} = 10^{-5}$$

$$\rho_s / \rho_d = 10^{-3}$$

C. Lv B. Zhang D. Lohse

Superhydrophobic substrate



# How does it look like?

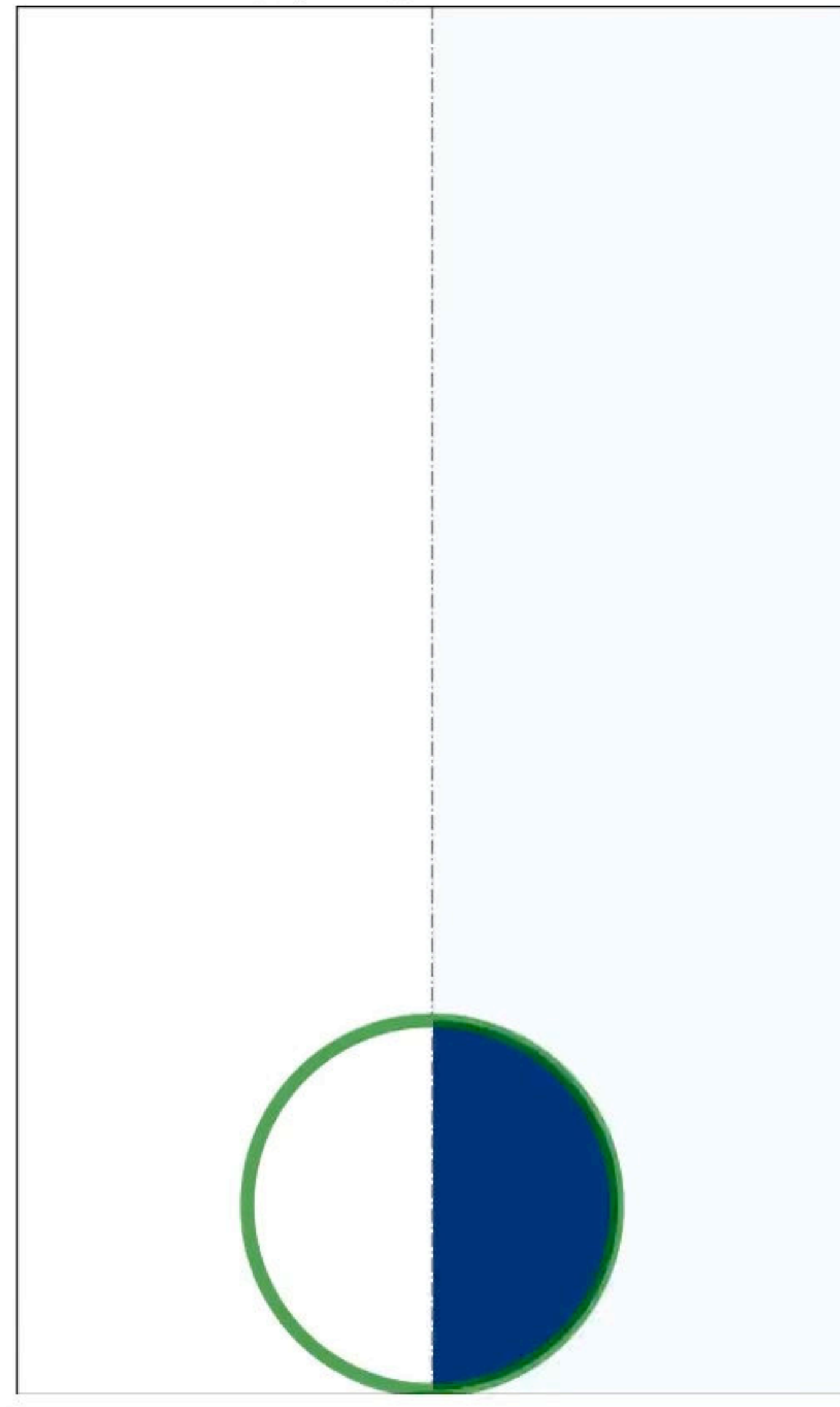
Volume of Fluid Method:  
Basilisk-Code



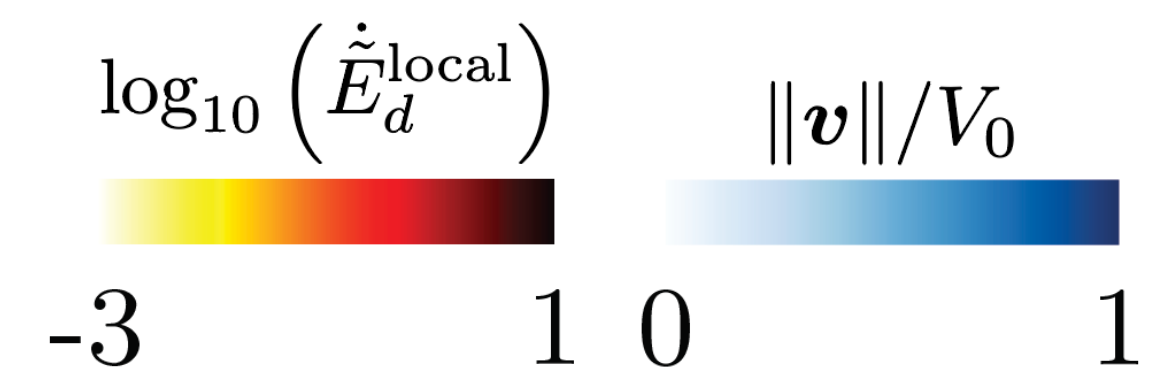
Stéphane Popinet *et al.*

$$We = 40$$

$$V_0 t / D_0 = 0.000$$



- Impact
- Spreading
- Recoil
- Take-off

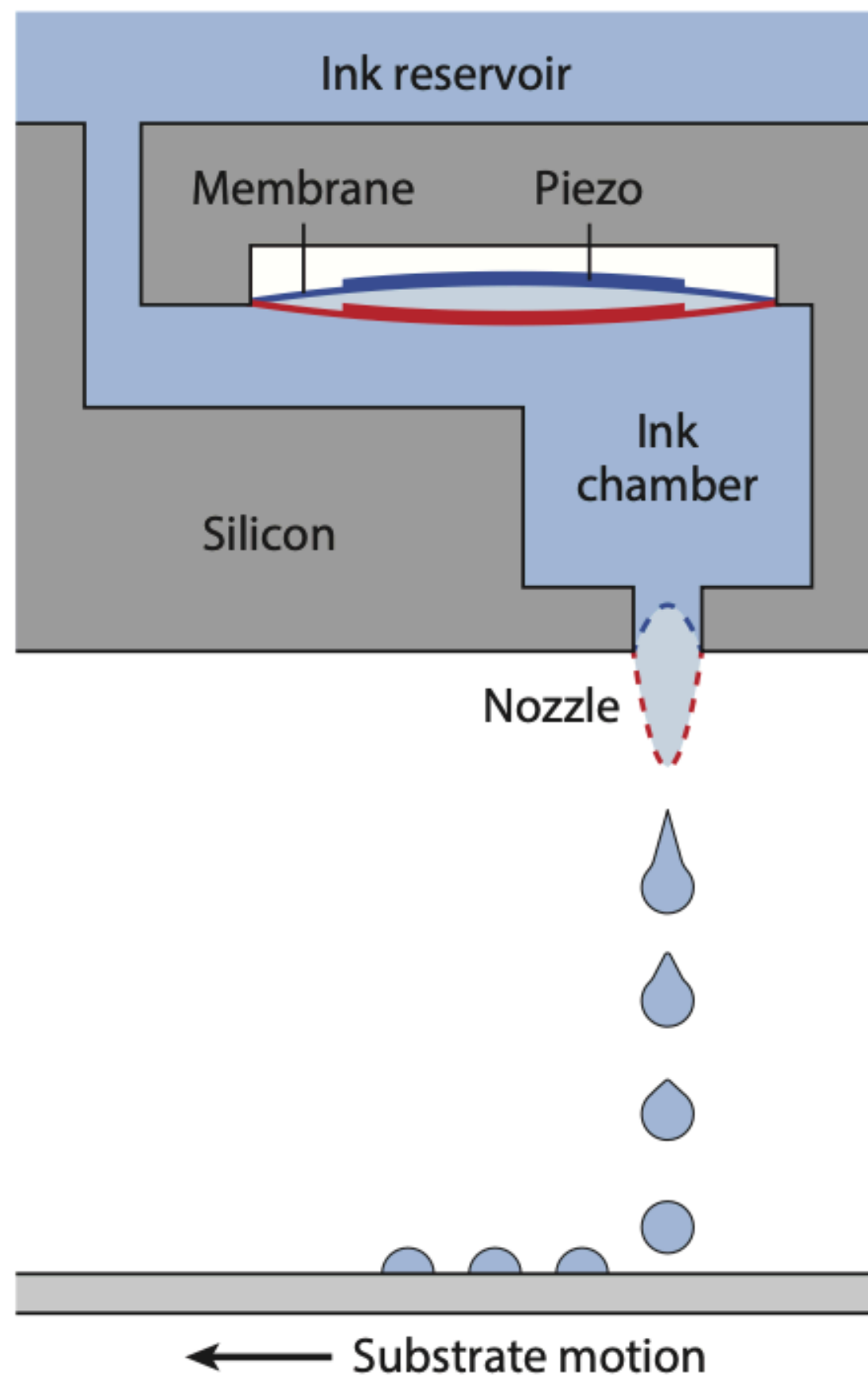


$$\dot{E}_d^{\text{local}} = 2\eta (\mathcal{D} : \mathcal{D})$$
$$\mathcal{D} = (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) / 2$$

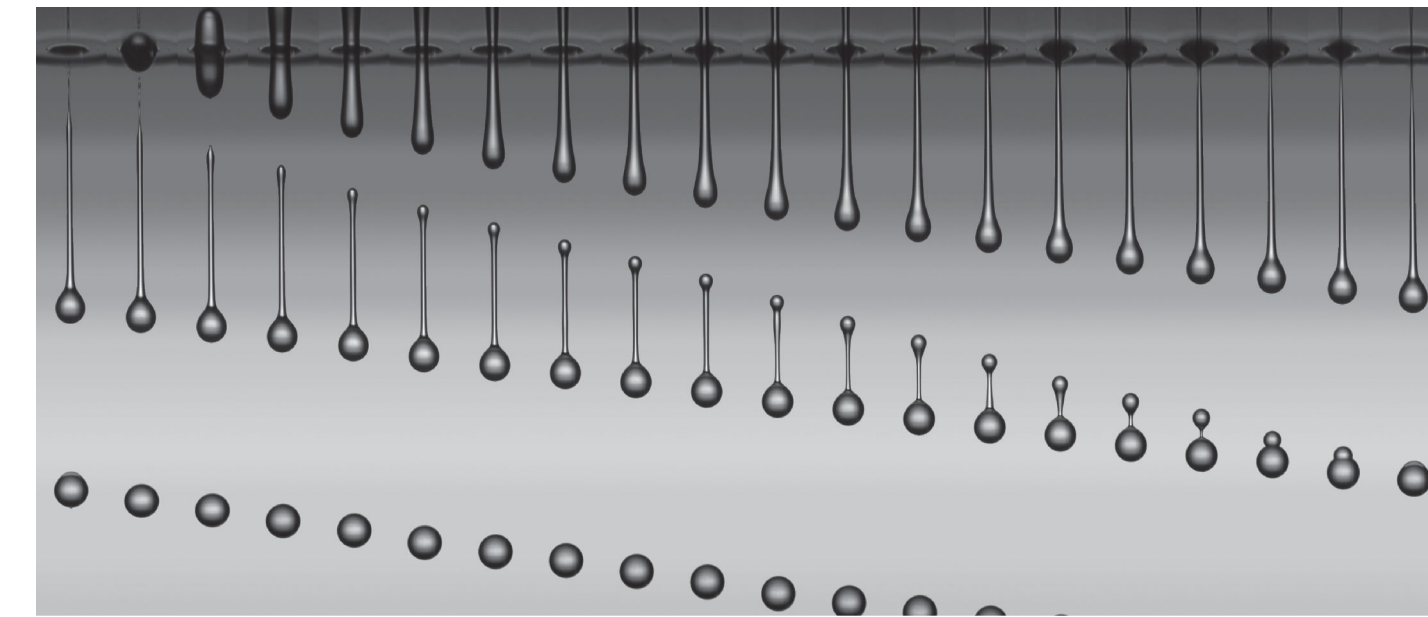
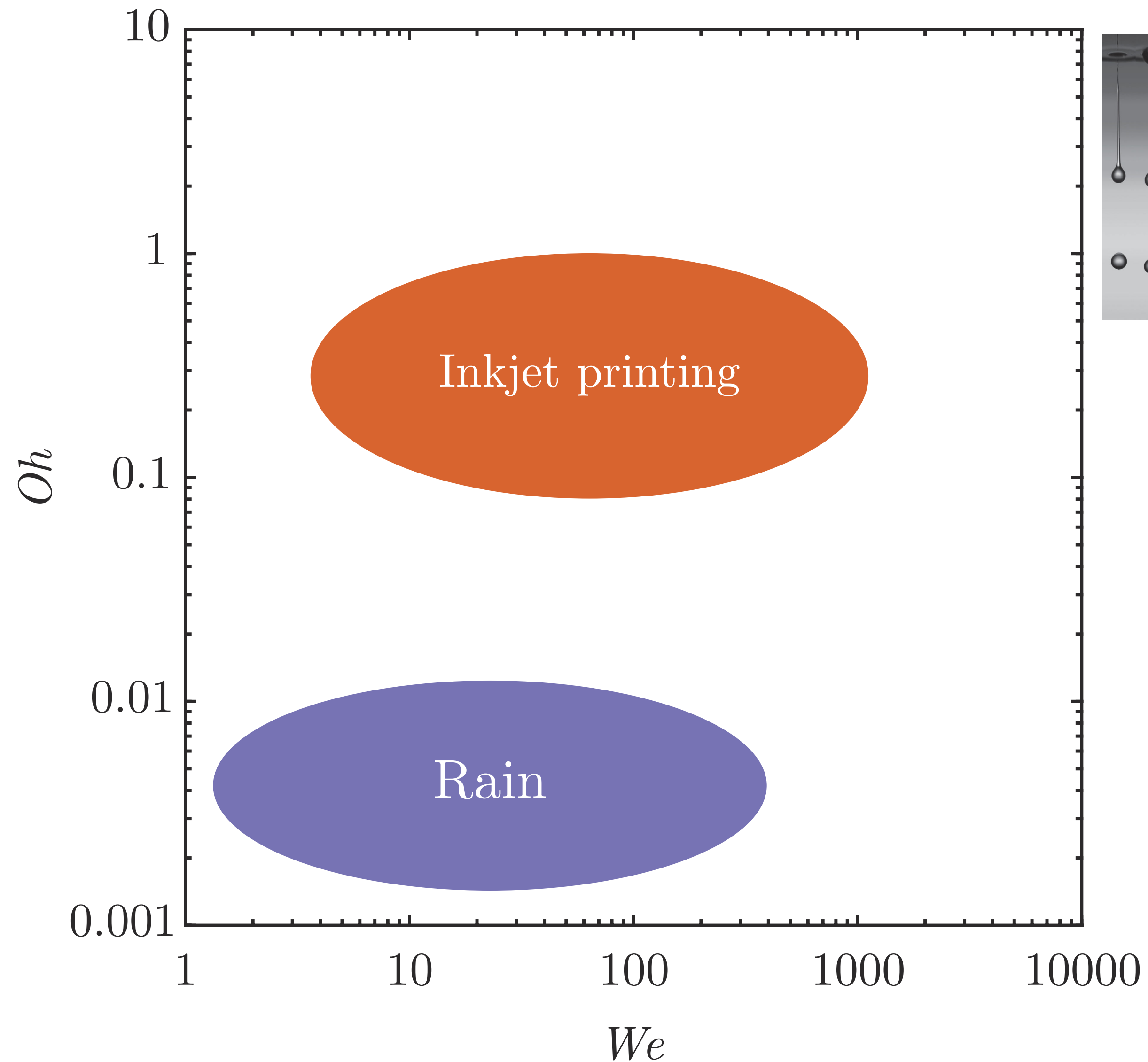
# $Oh$ vs $We$ parameter space



Viscous Free-Surface Flows



Lohse, Annu. Rev. Fluid Mech.  
2022. 54:349–82



$$Oh = \frac{\eta_d}{\sqrt{\rho_d \gamma D_0}}$$

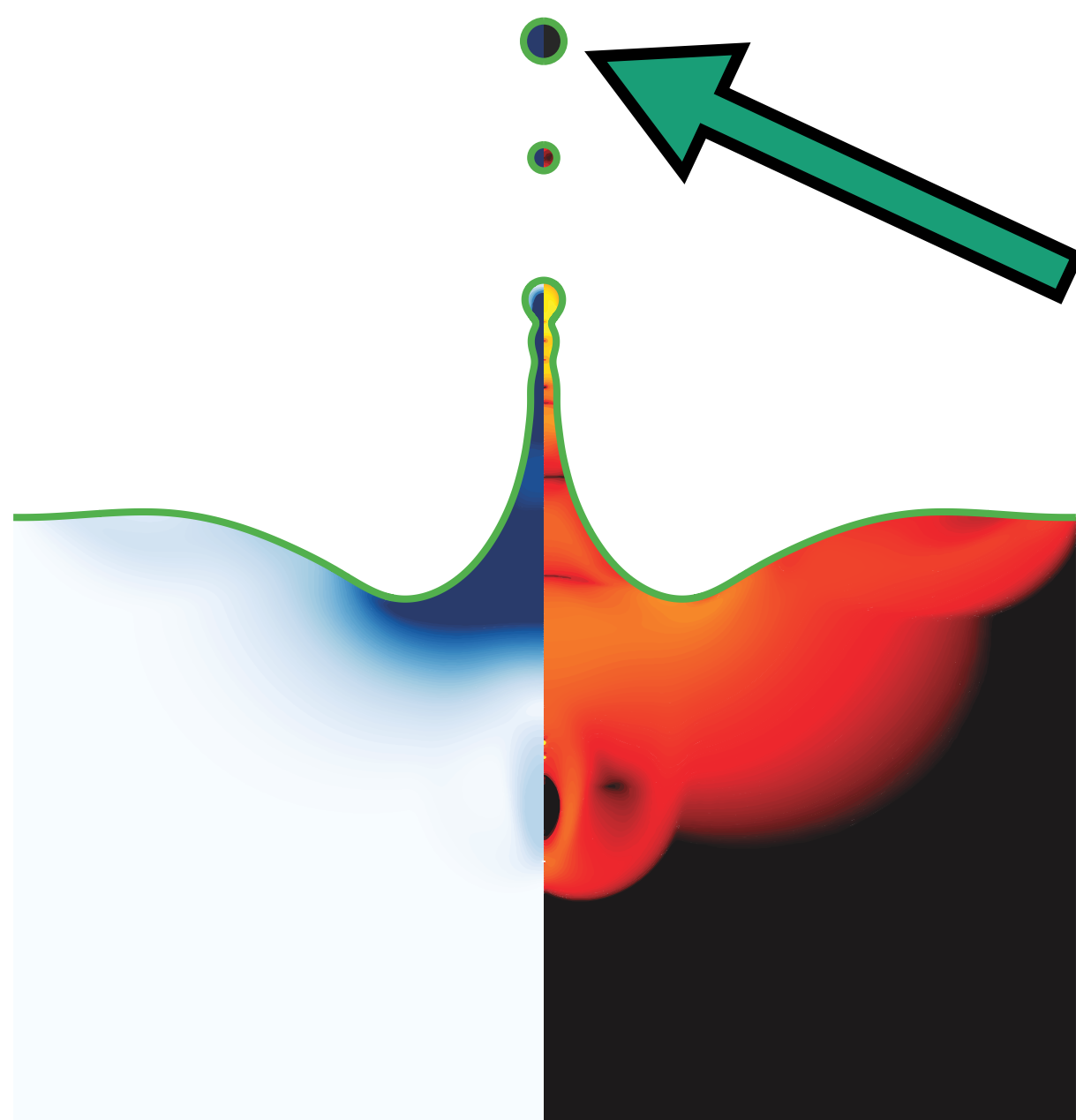
$$We = \frac{\rho_d V_0^2 D_0}{\gamma}$$



# $Oh$ vs $We$ parameter space

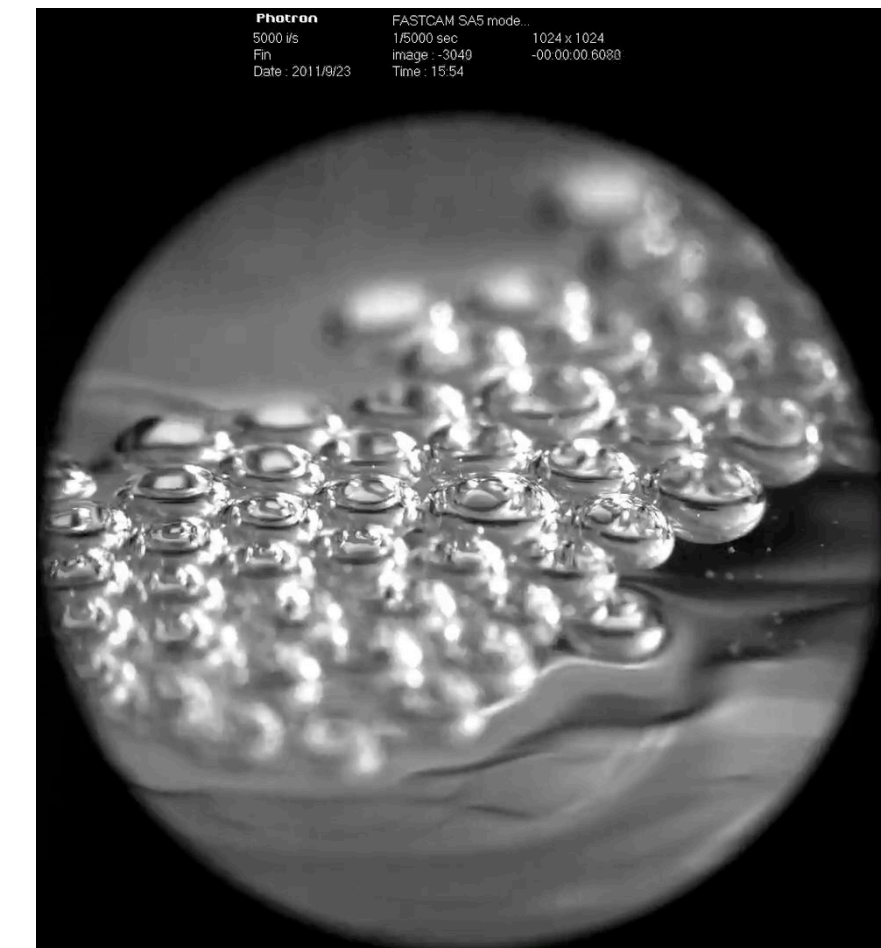
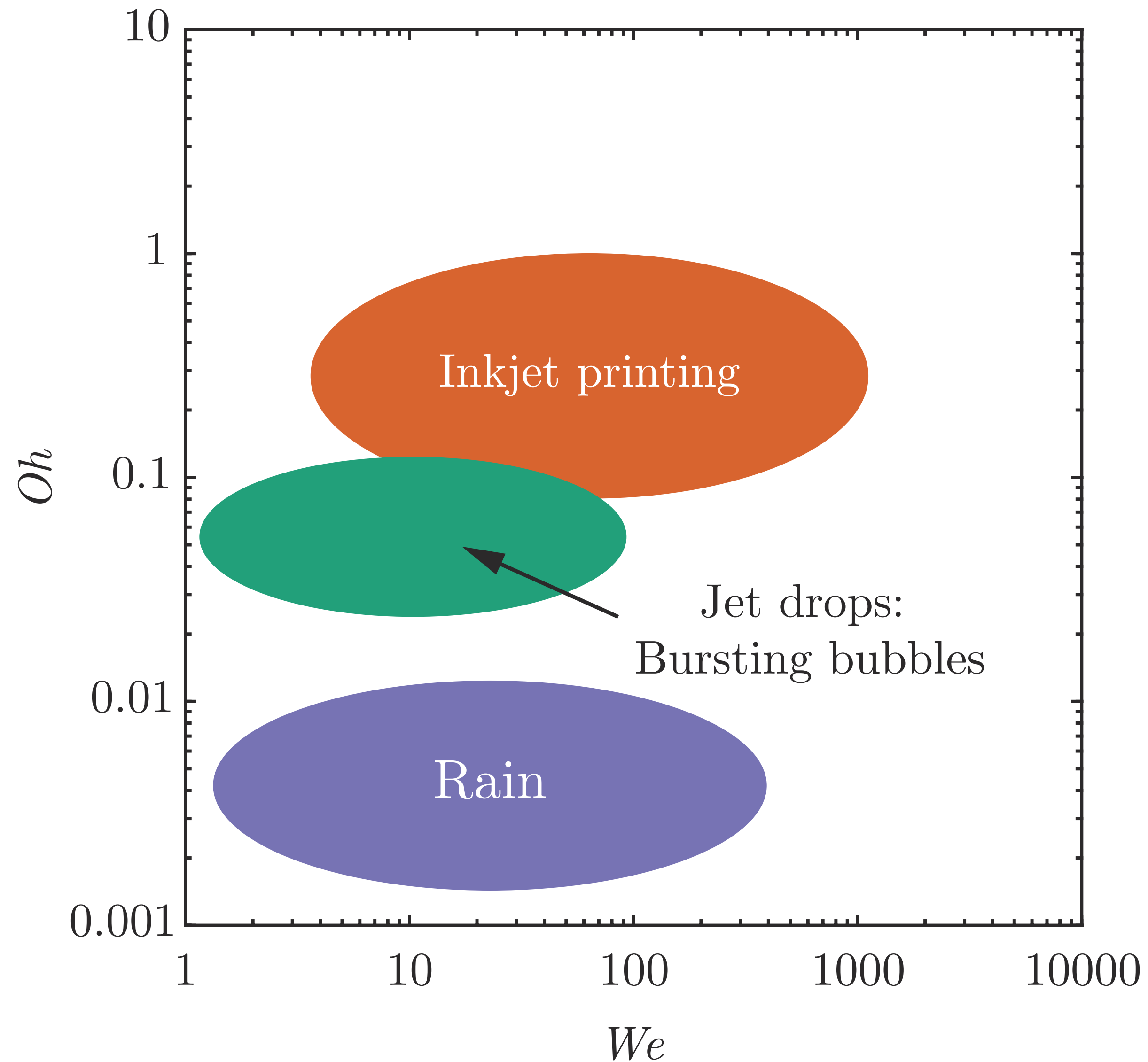


Viscous Free-Surface Flows



Sanjay, Lohse, & Jalaal,  
J. Fluid Mech. (2021), vol. 922, A2

Néel & Deike,  
Phys. Rev. Fluids **7**, 103603



Video:  
Thomas Séon & Arup Kumar Das

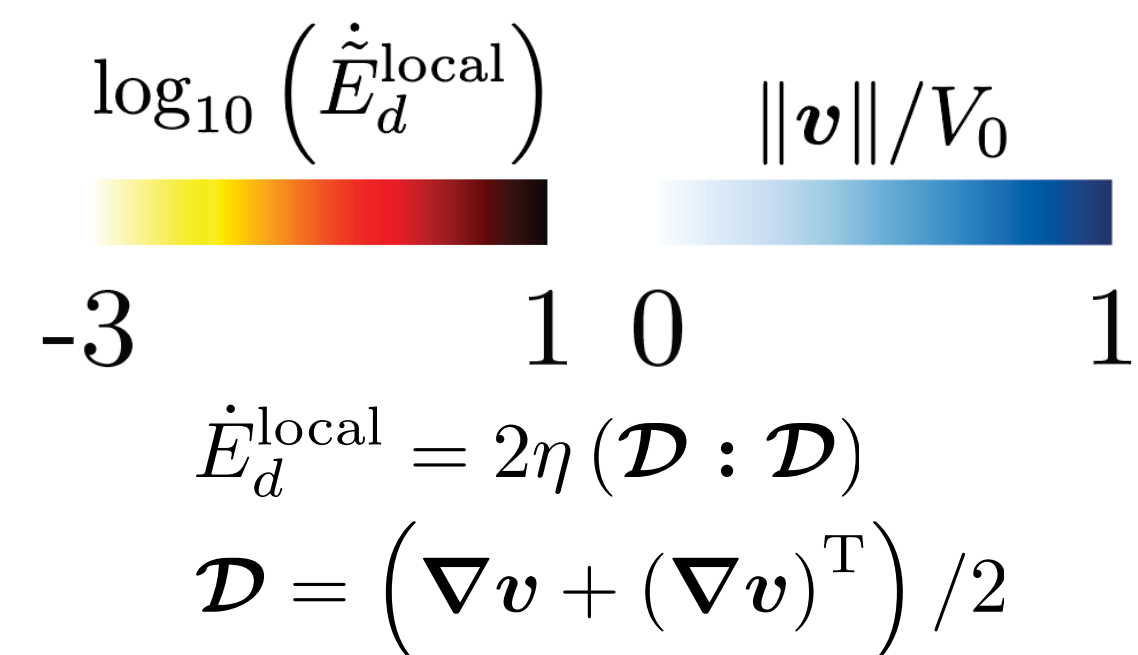
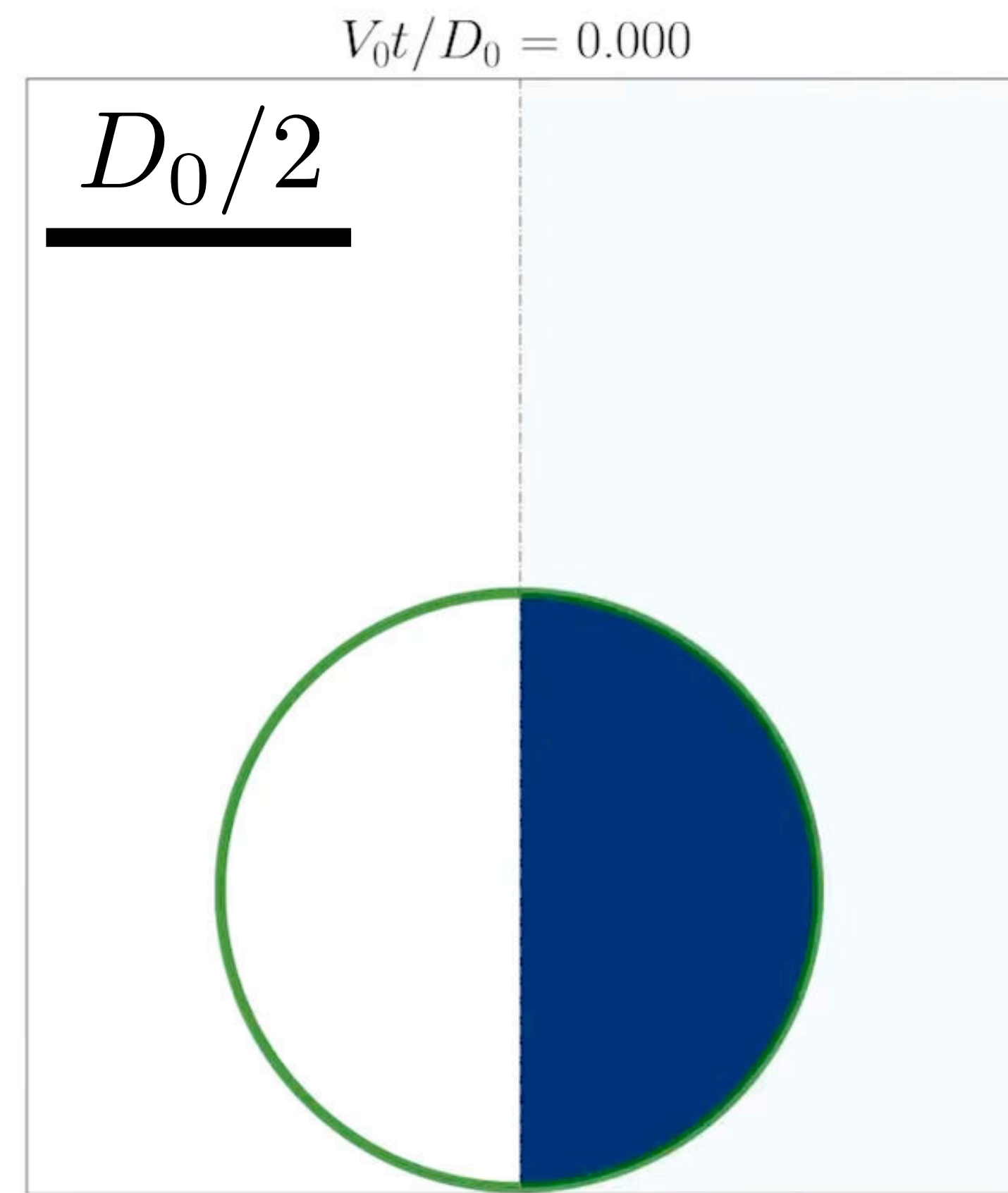
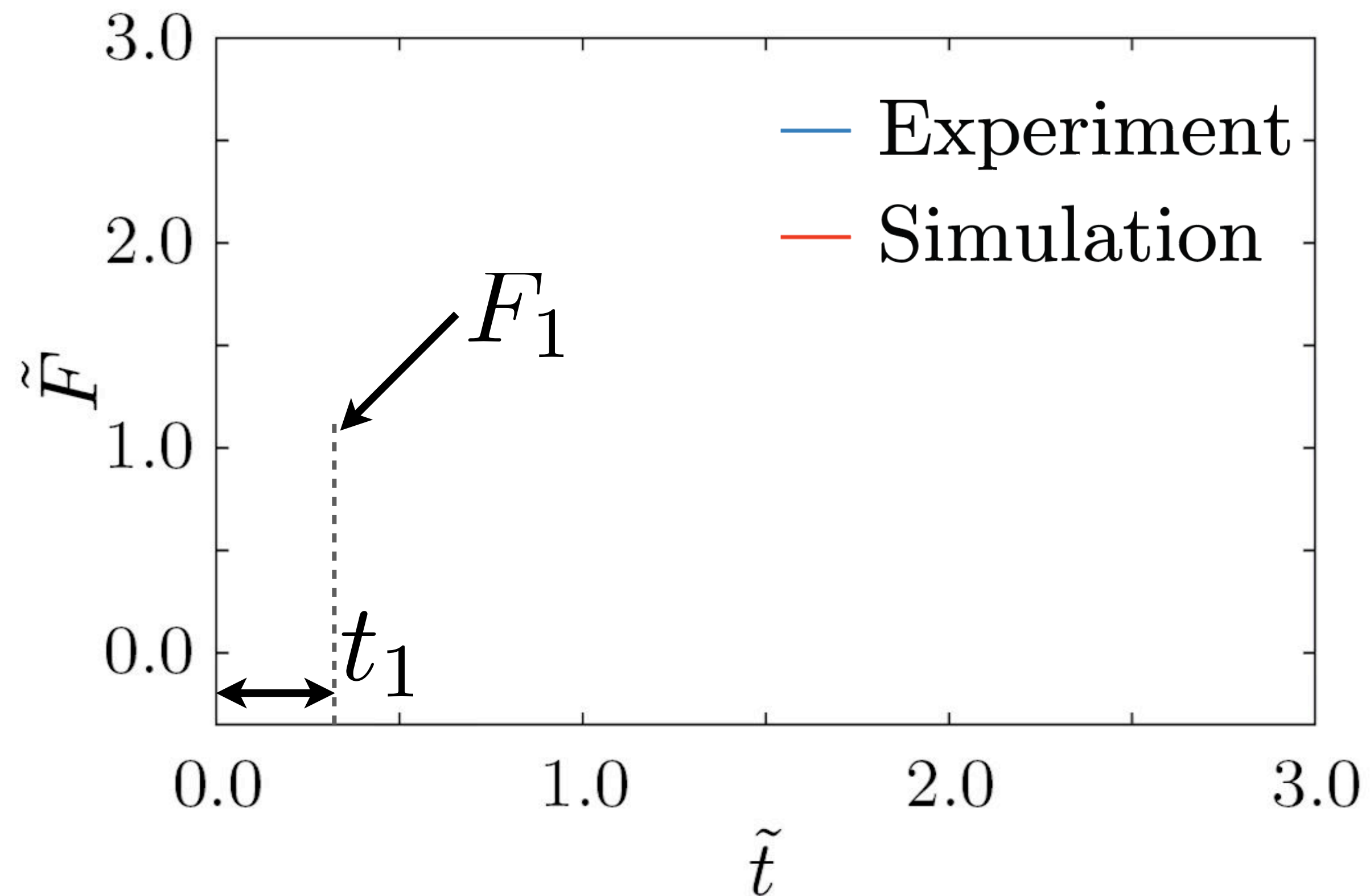
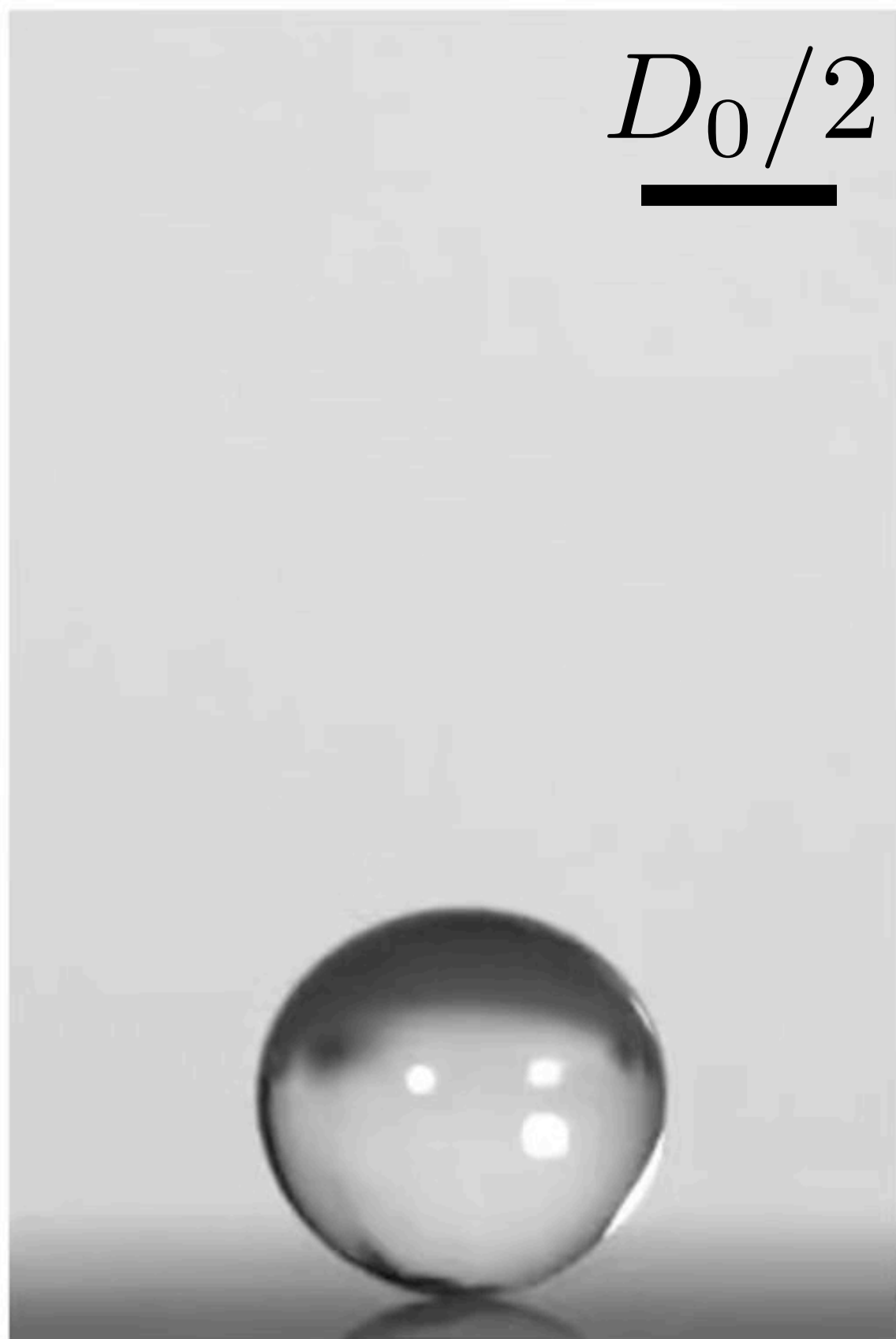
$$Oh = \frac{\eta_d}{\sqrt{\rho_d \gamma D_0}}$$

$$We = \frac{\rho_d V_0^2 D_0}{\gamma}$$



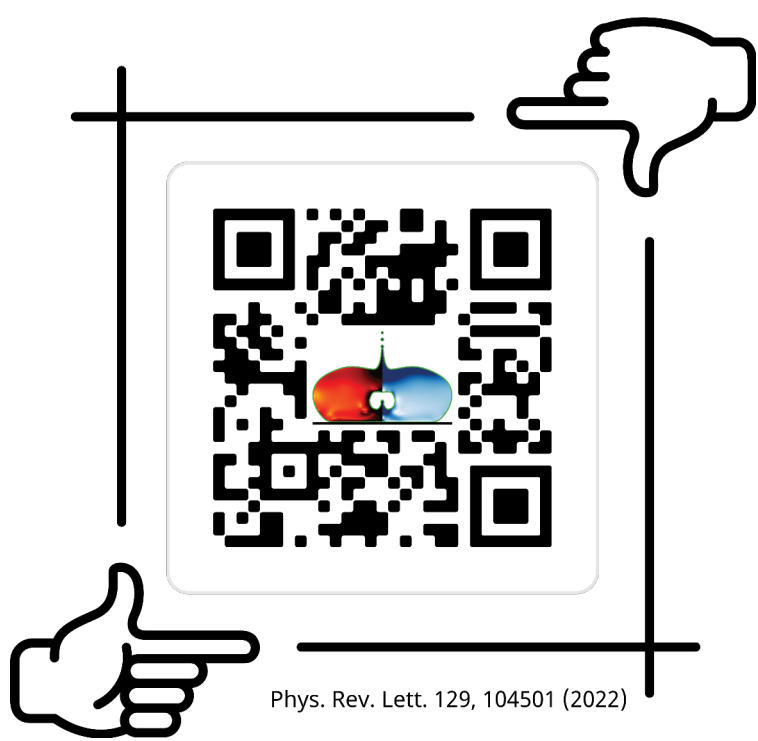
$$We = 9$$

$$Oh = 0.0025$$



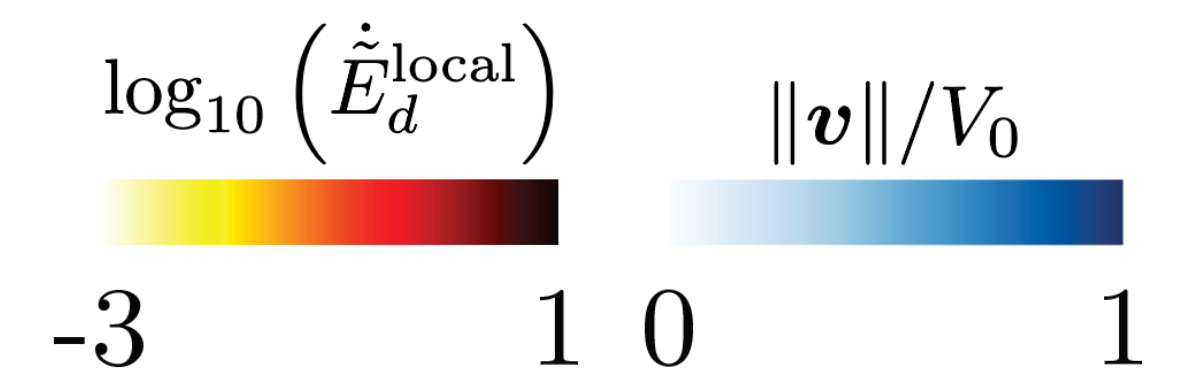
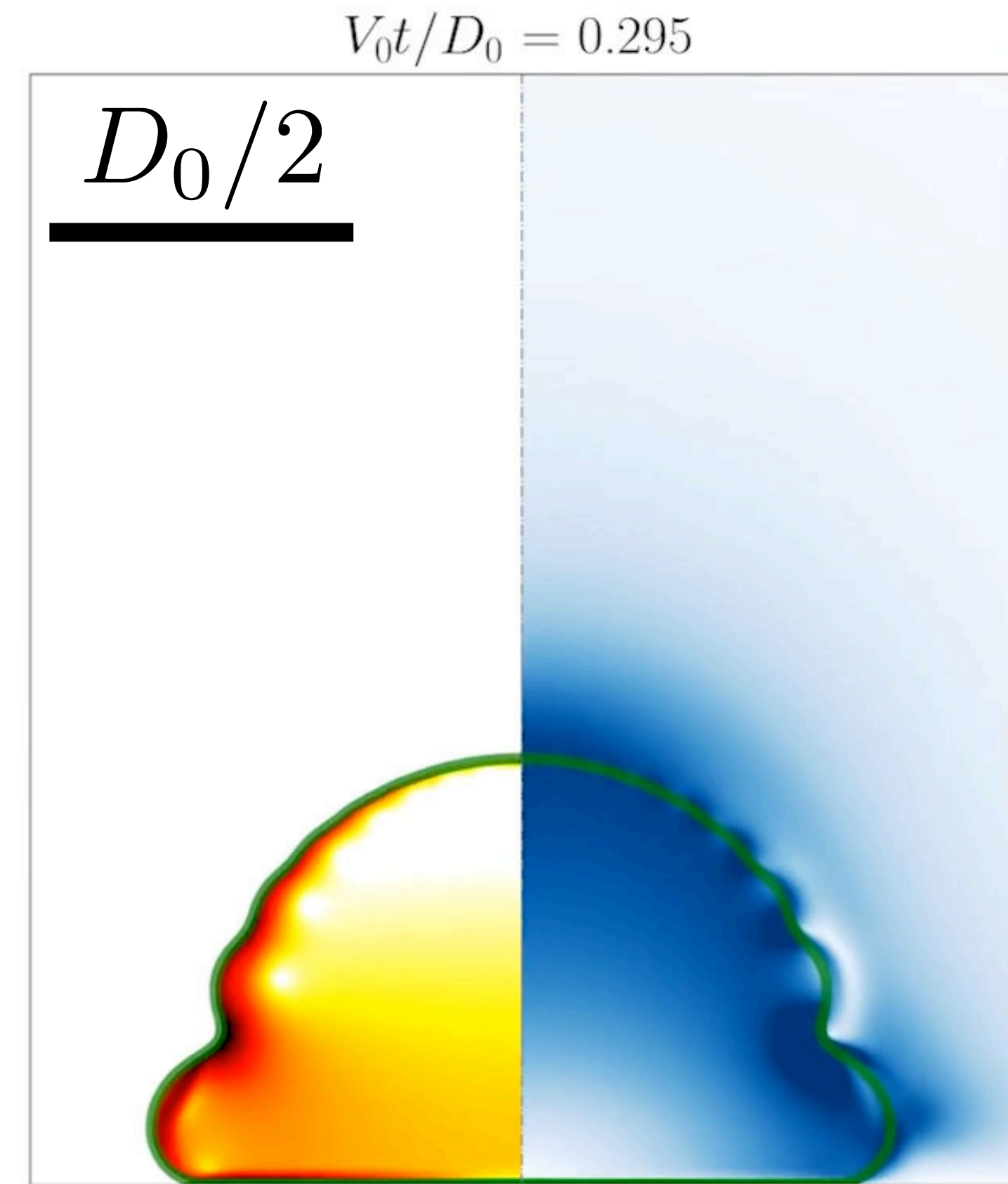
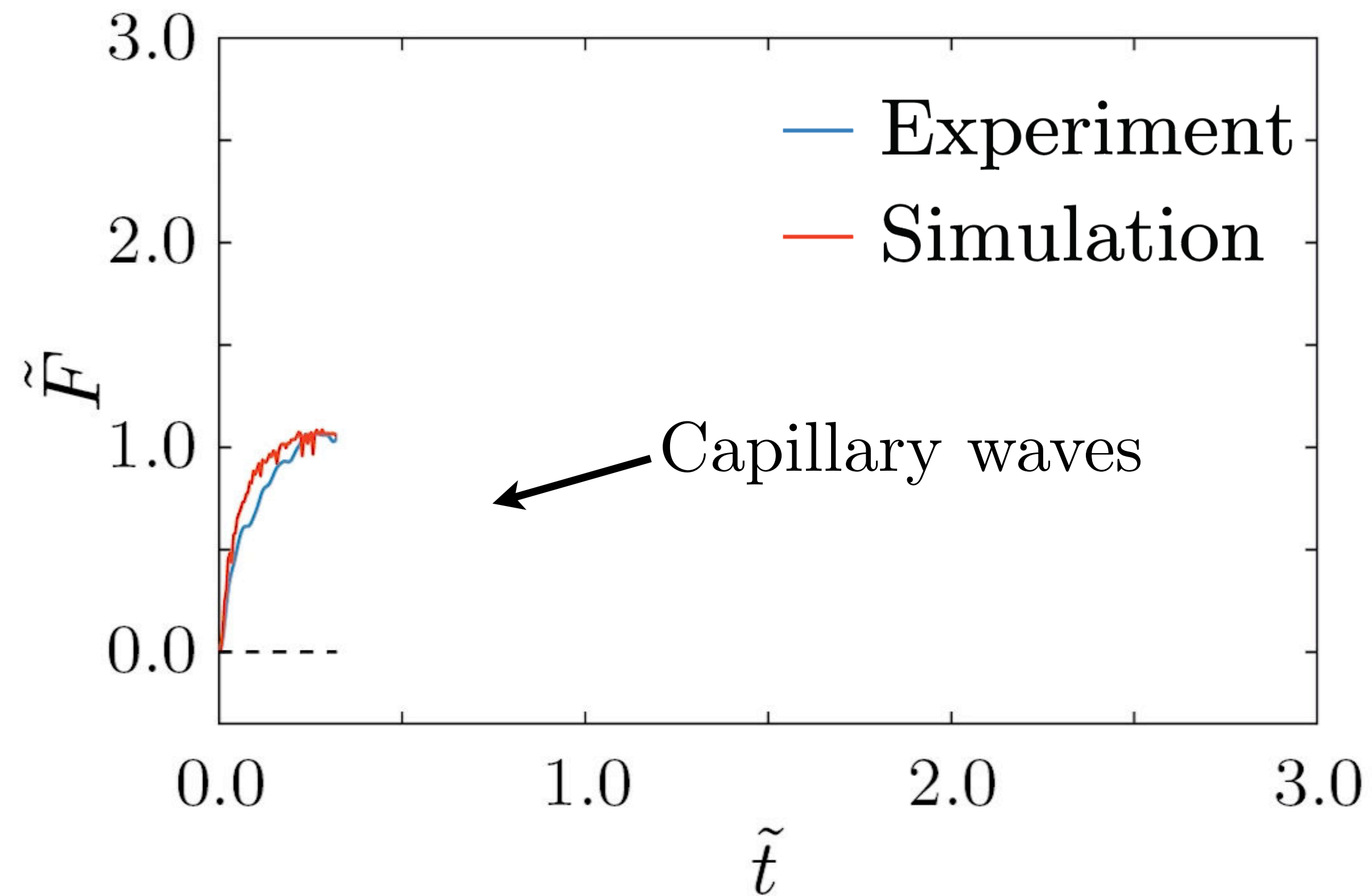
$$\dot{E}_d^{\text{local}} = 2\eta (\mathcal{D} : \mathcal{D})$$

$$\mathcal{D} = (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) / 2$$

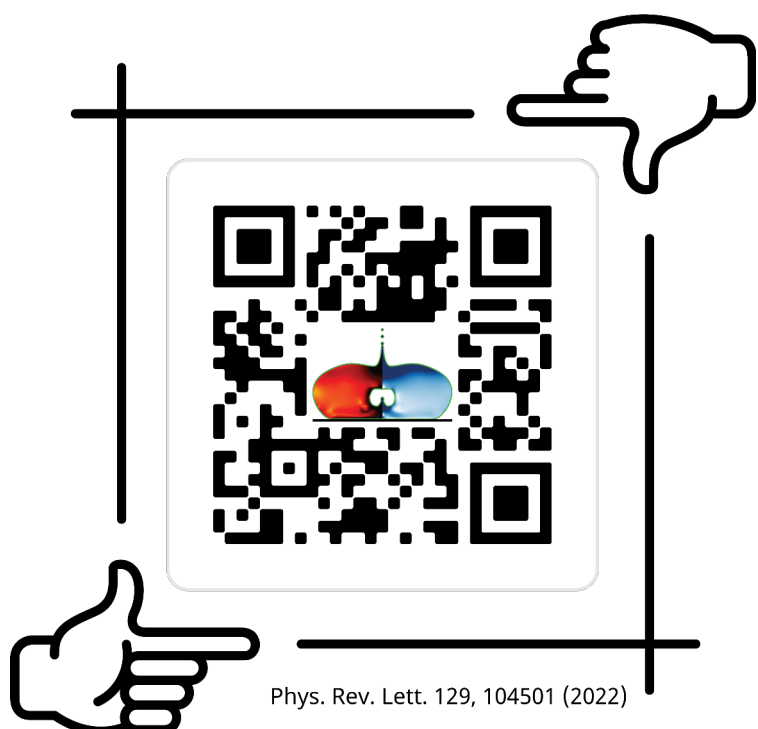


$$We = 9$$

$$Oh = 0.0025$$

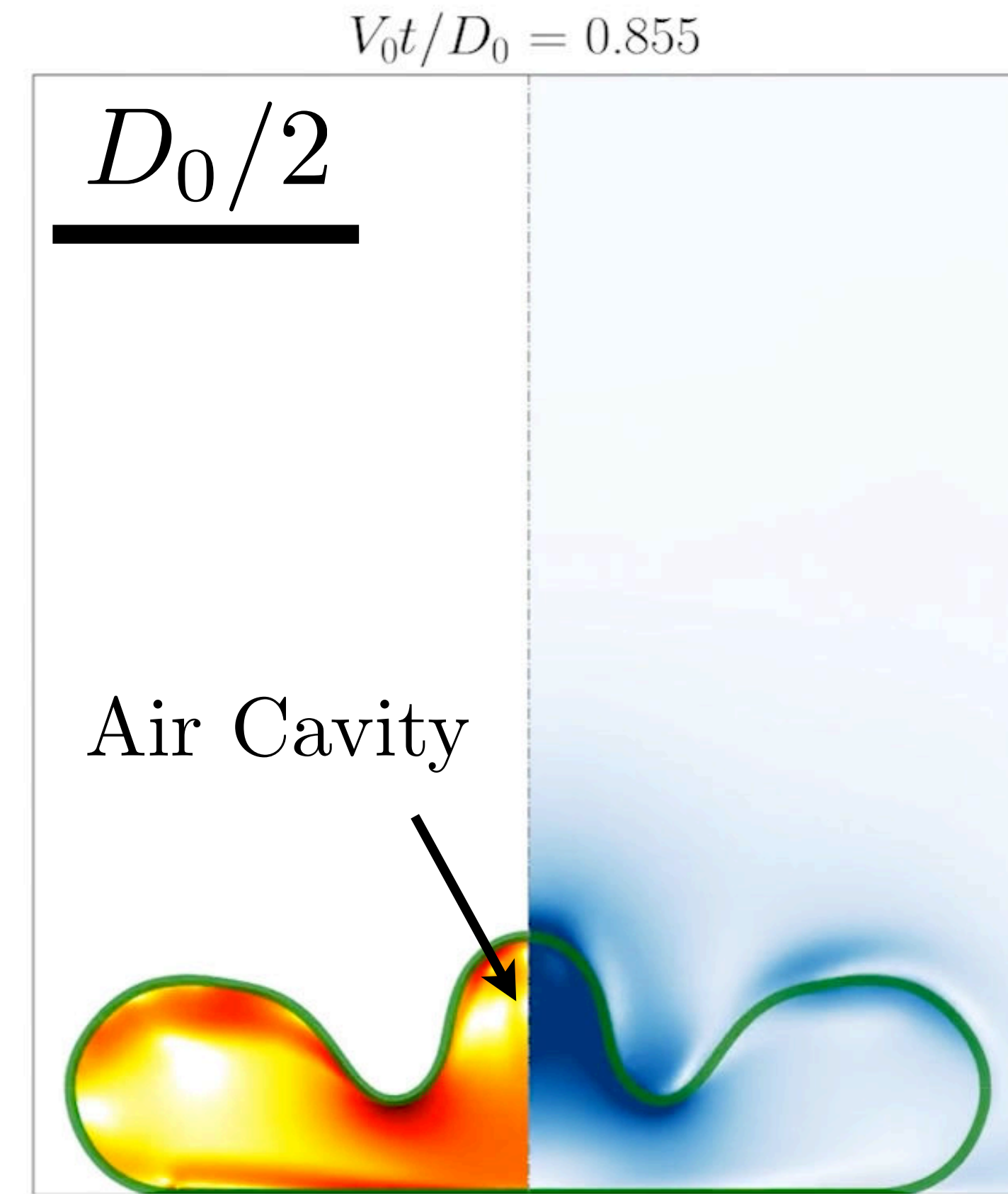
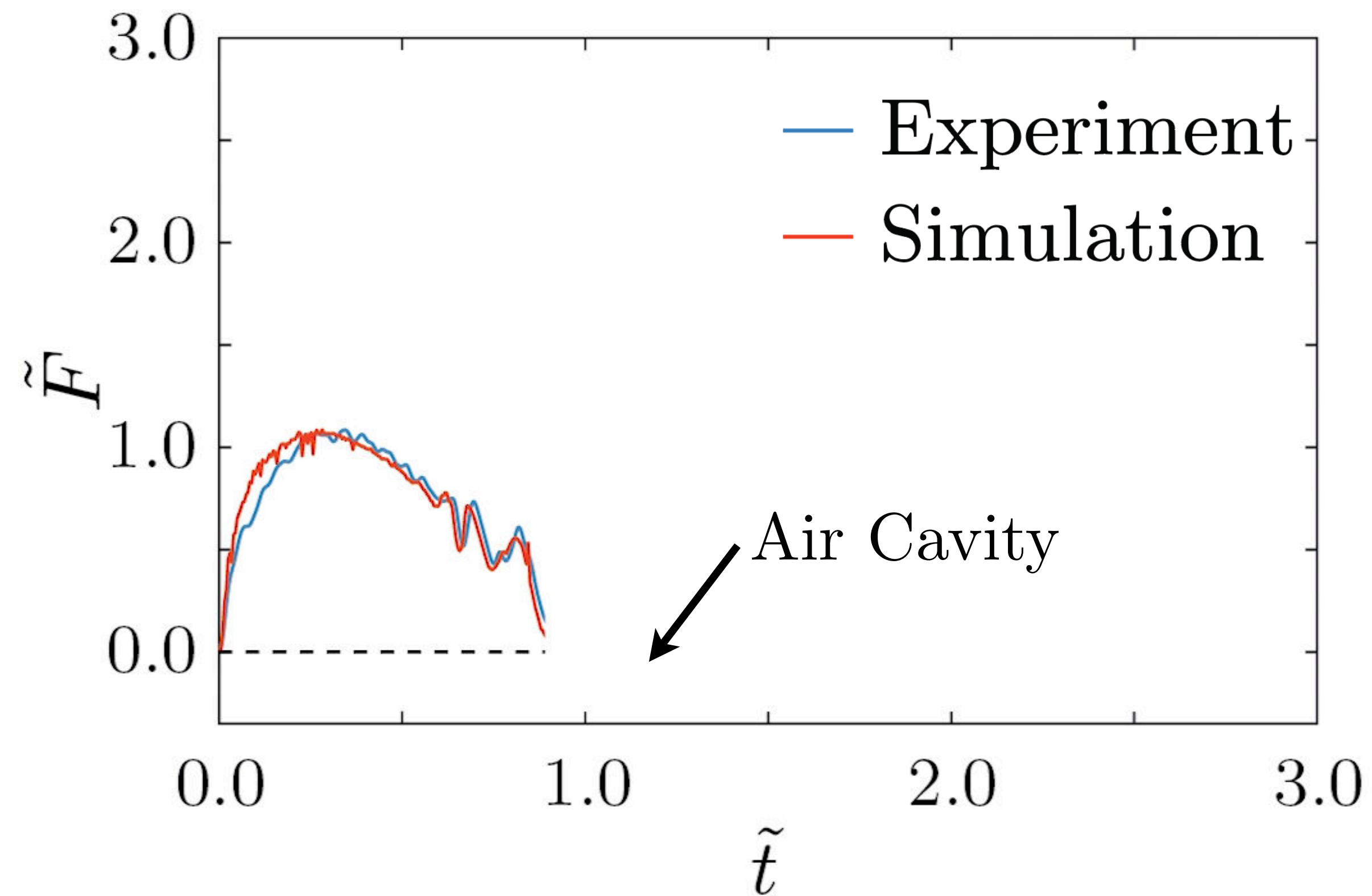
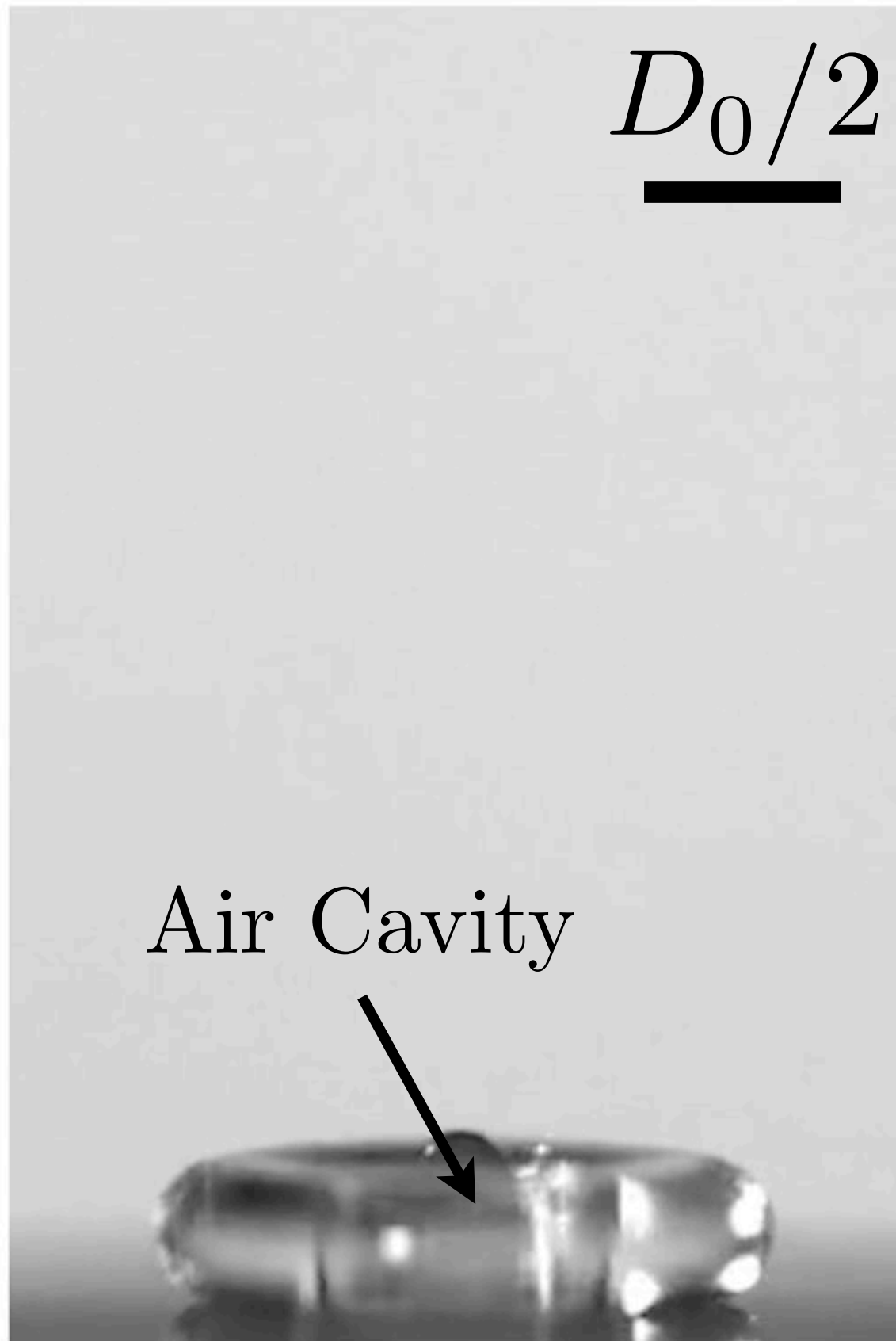


Renardy, **Popinet**, Duchemin, Renardy, **Zaleski**, Josserand,  
Drumright-Clarke, Richard, Clanet, Quéré  
J. Fluid Mech. 484, 69 (2003)



$$We = 9$$

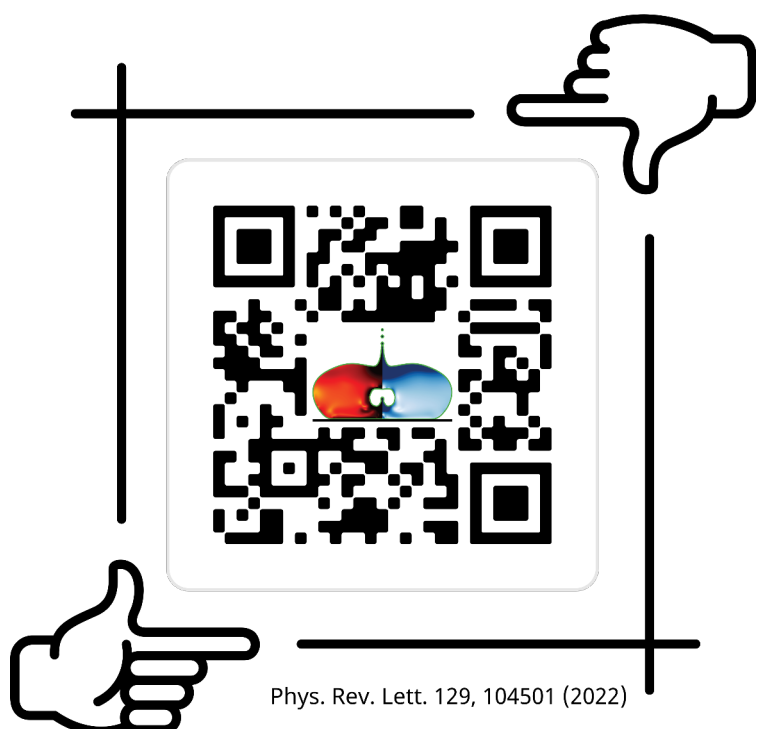
$$Oh = 0.0025$$



## Air bubble entrainment !

Bartolo, Josserand, & Bonn

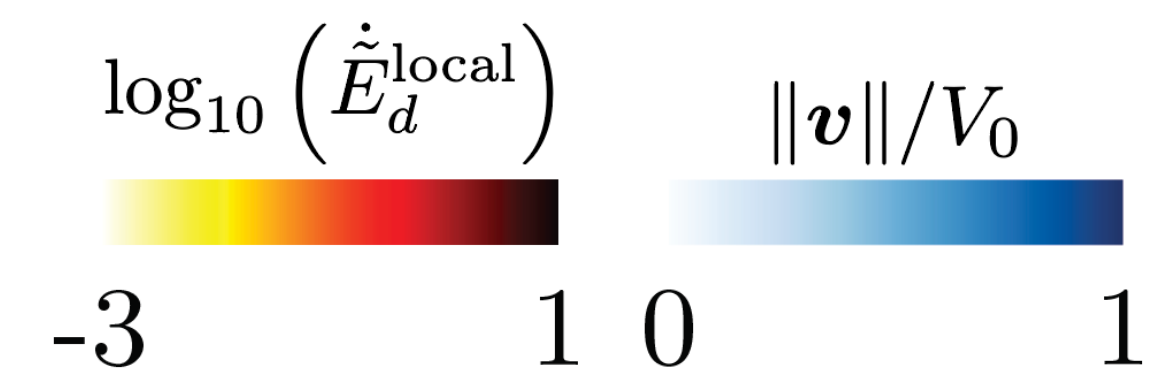
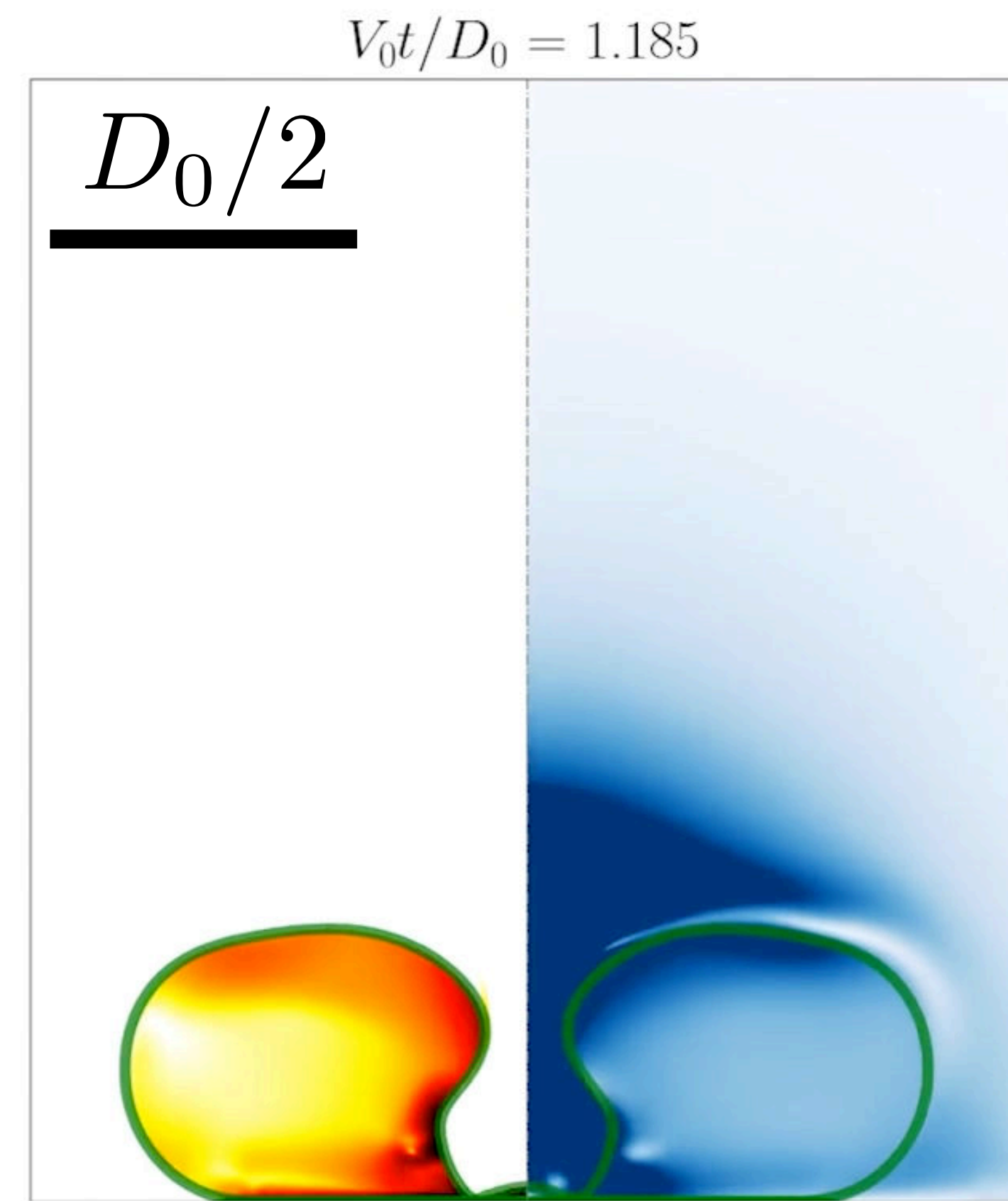
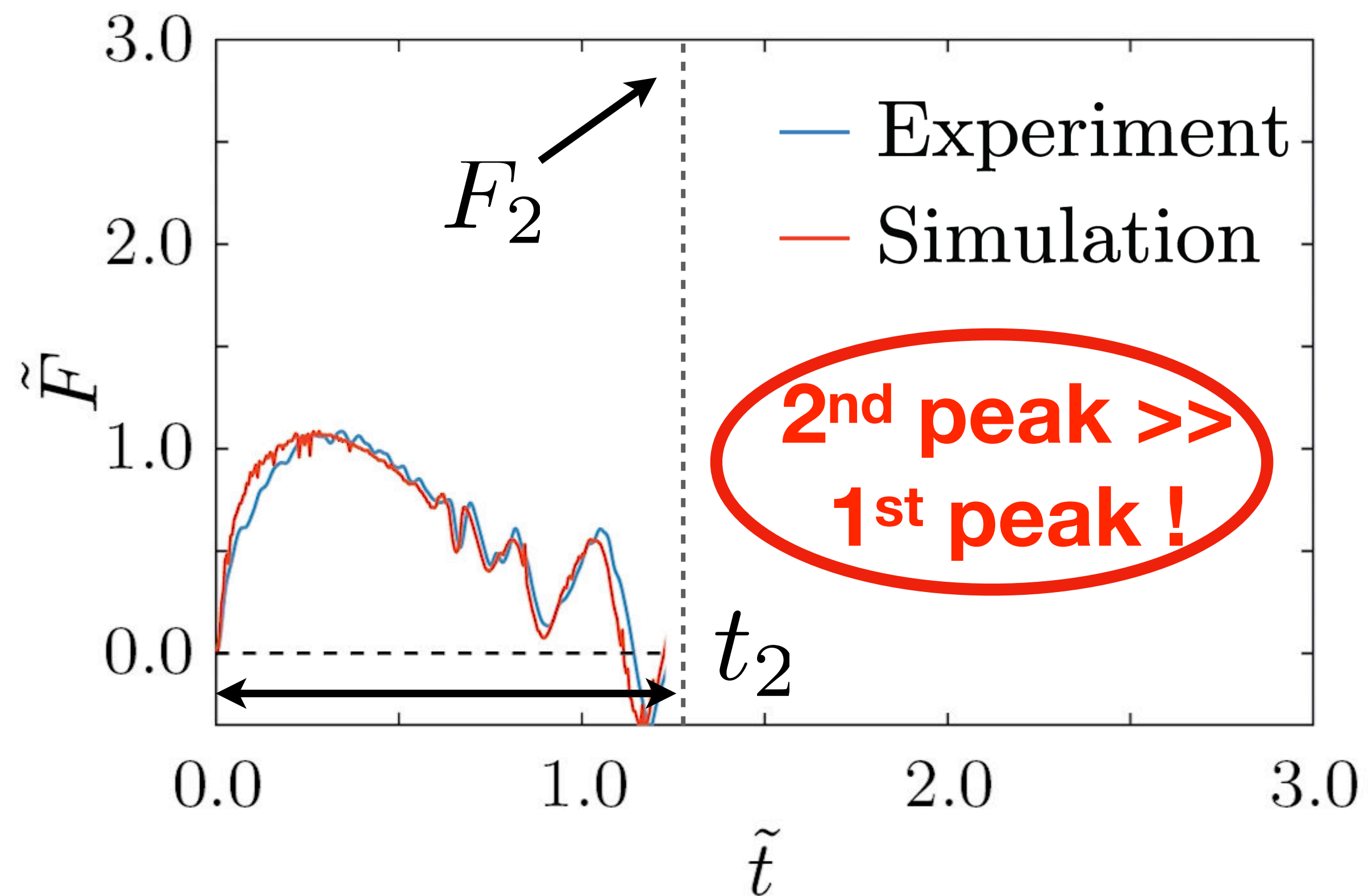
Phys. Rev. Lett. 96, 124501 (2006)





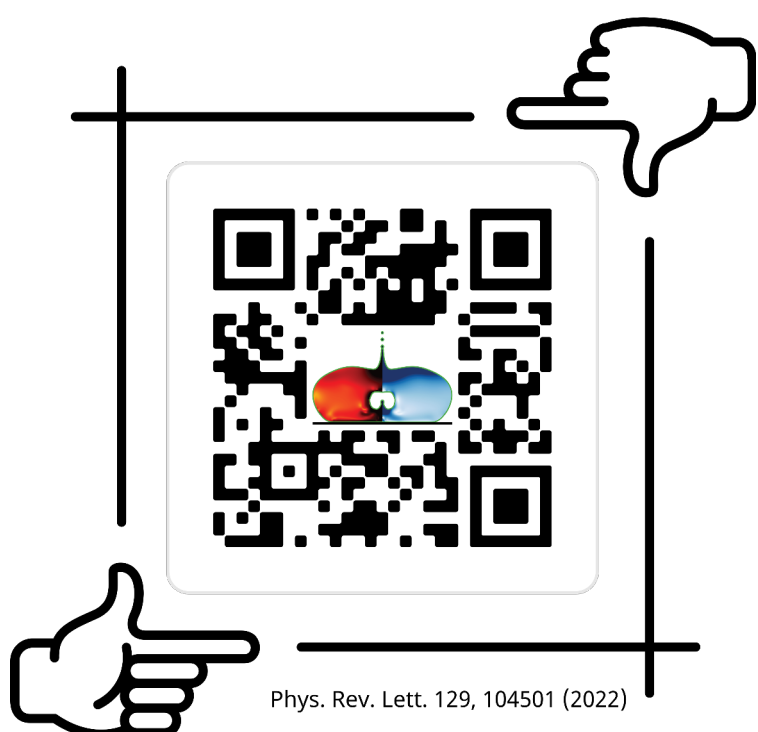
$$We = 9$$

$$Oh = 0.0025$$



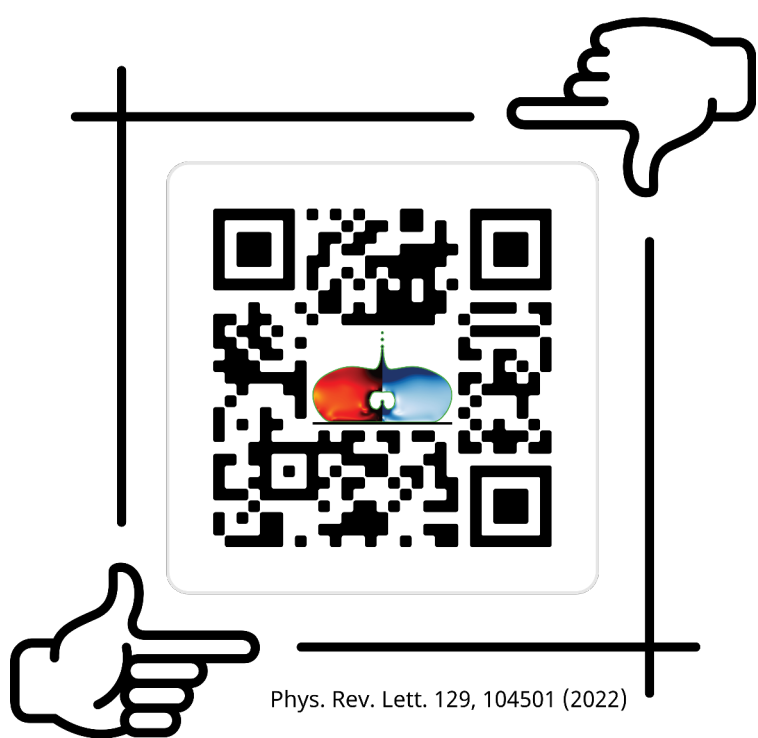
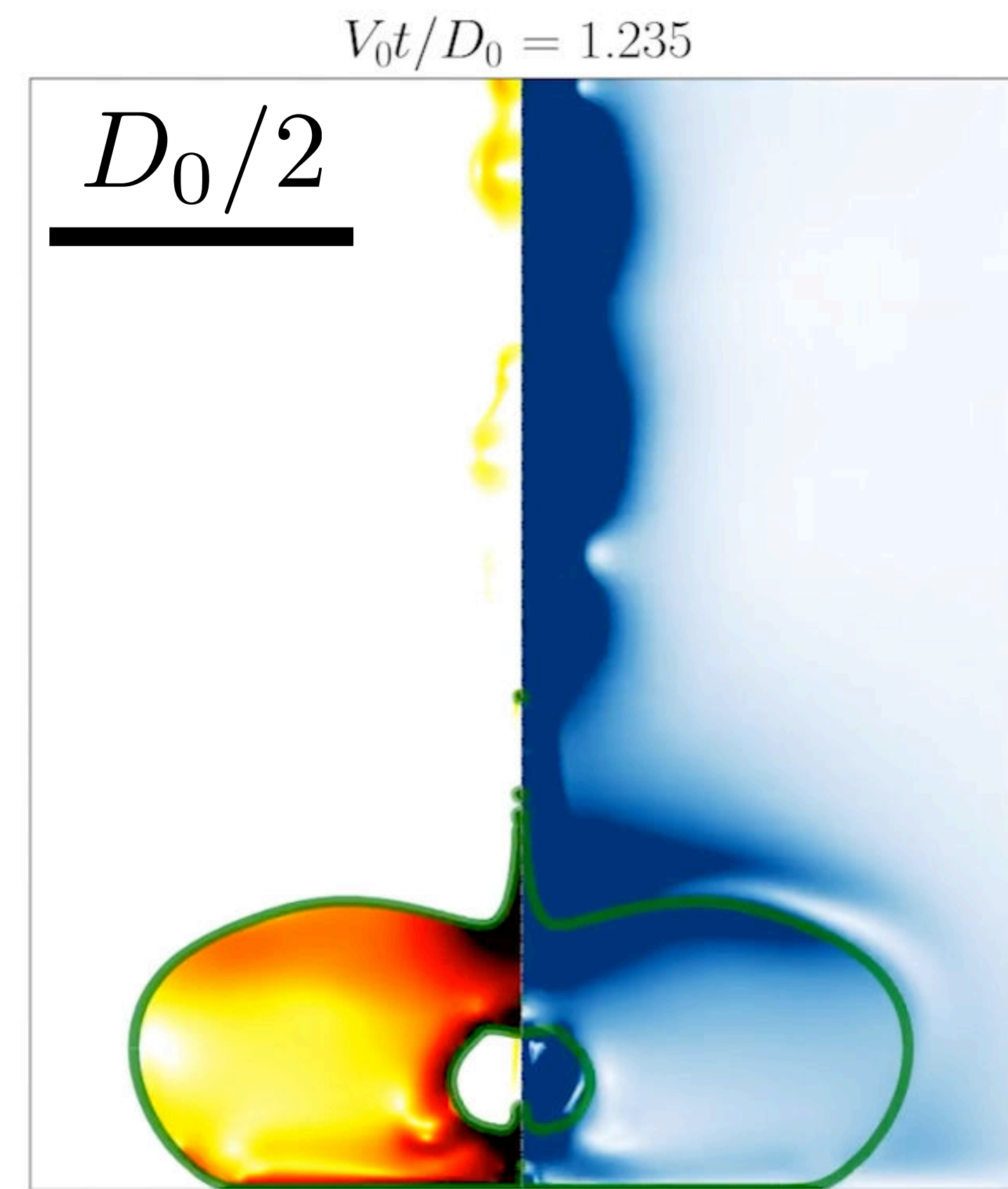
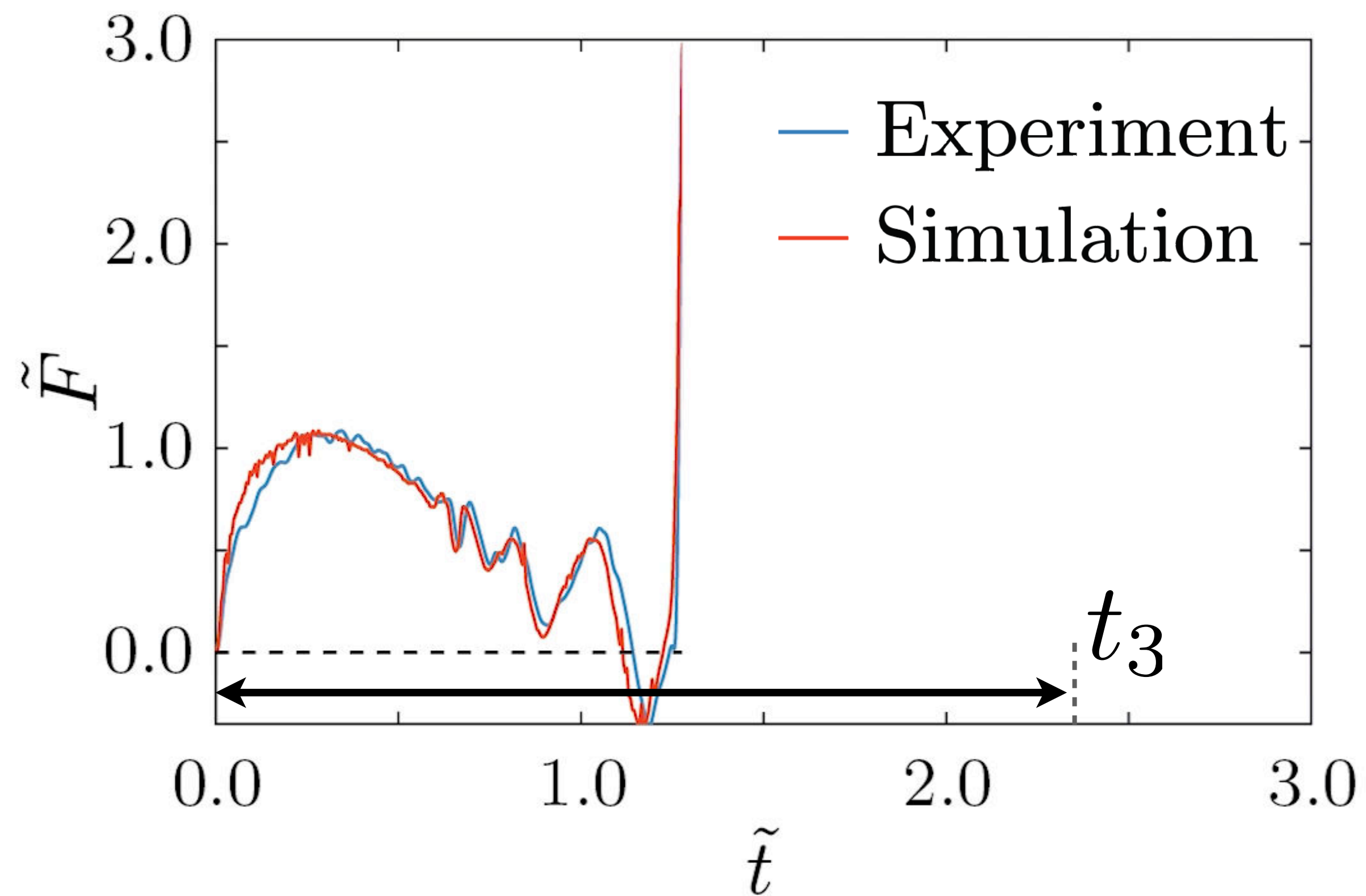
## Closure of air bubble leads to singular jet!

Bartolo, Josserand, & Bonn  
Phys. Rev. Lett. 96, 124501 (2006)



$$We = 9$$

$$Oh = 0.0025$$



Ways to kill this 'singularity' !

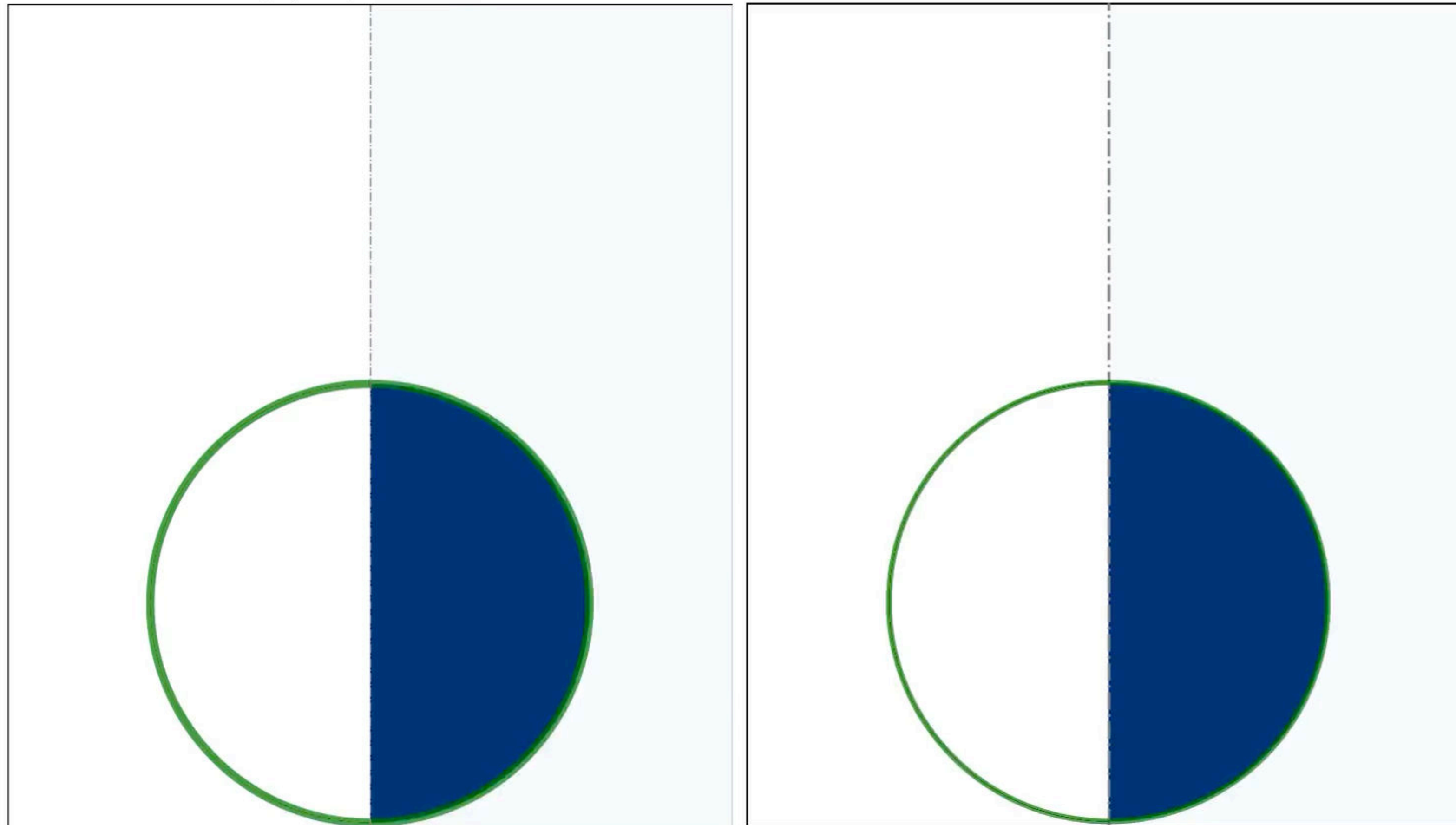


# I. Viscous dissipation

# Jet & bubble

$$We = 9$$

$$V_0 t / D_0 = 0.000$$

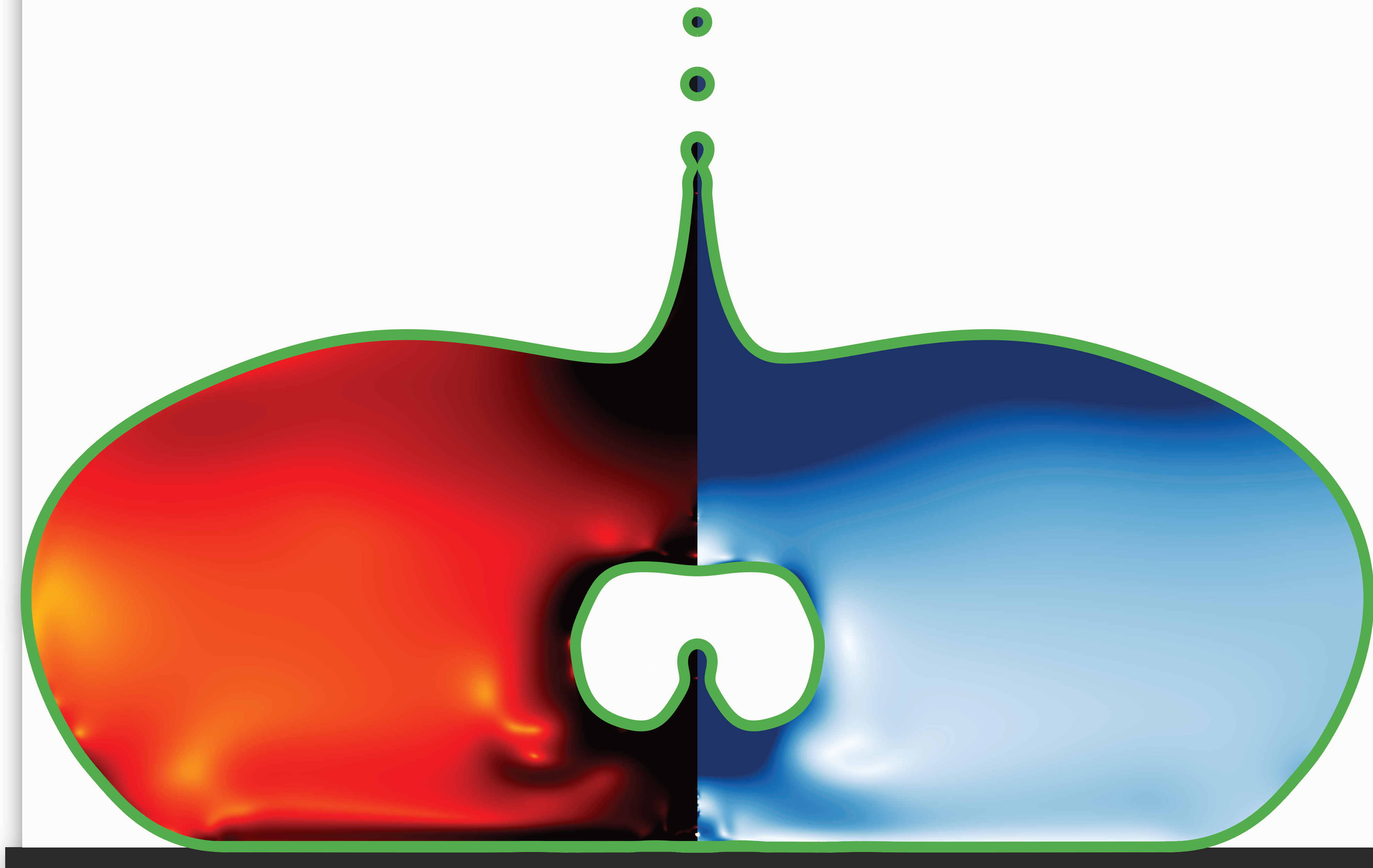


$$Oh = 0.0025$$

$$Oh = 0.005$$

$We = 9$

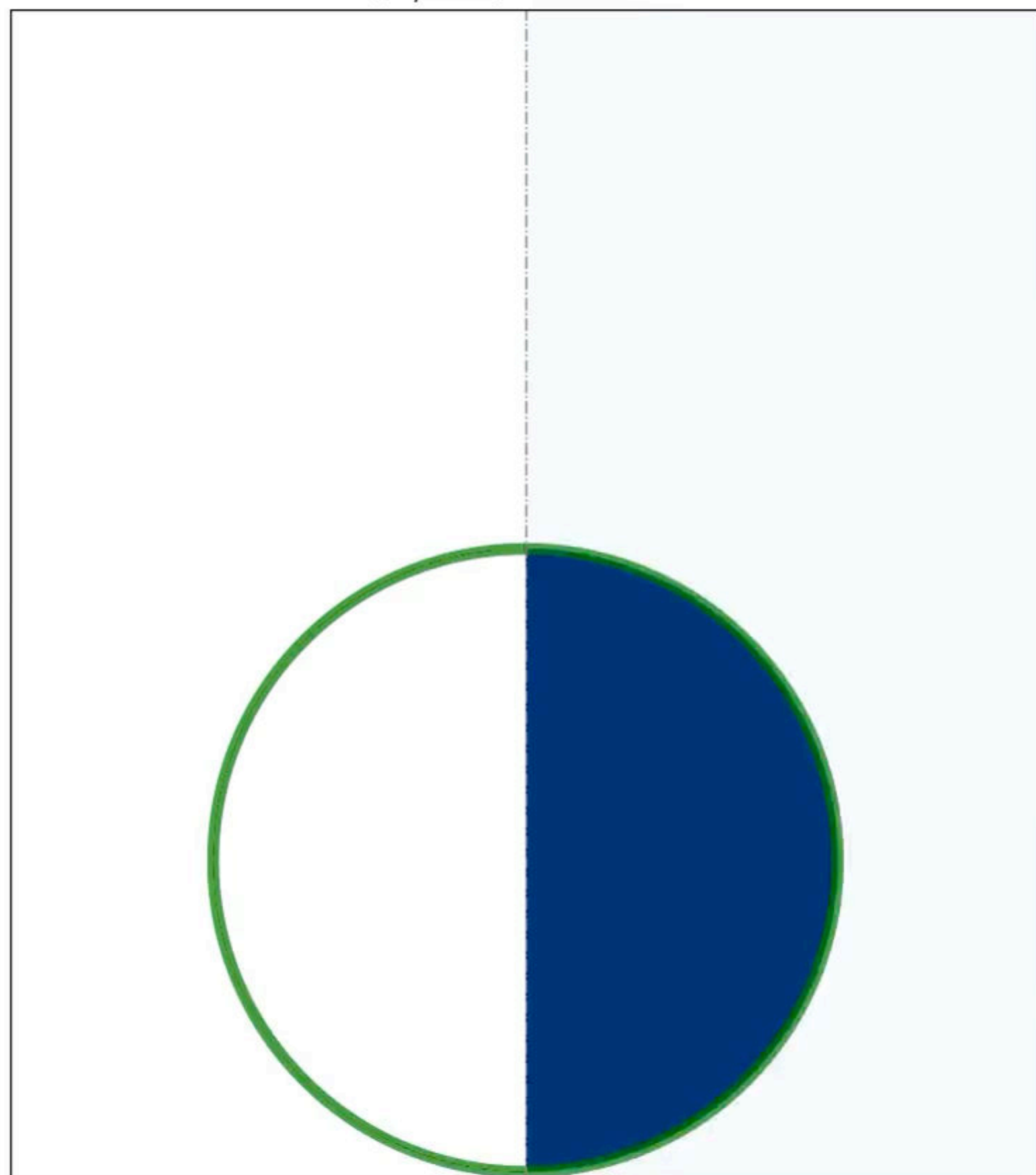
$Oh = 0.0025$



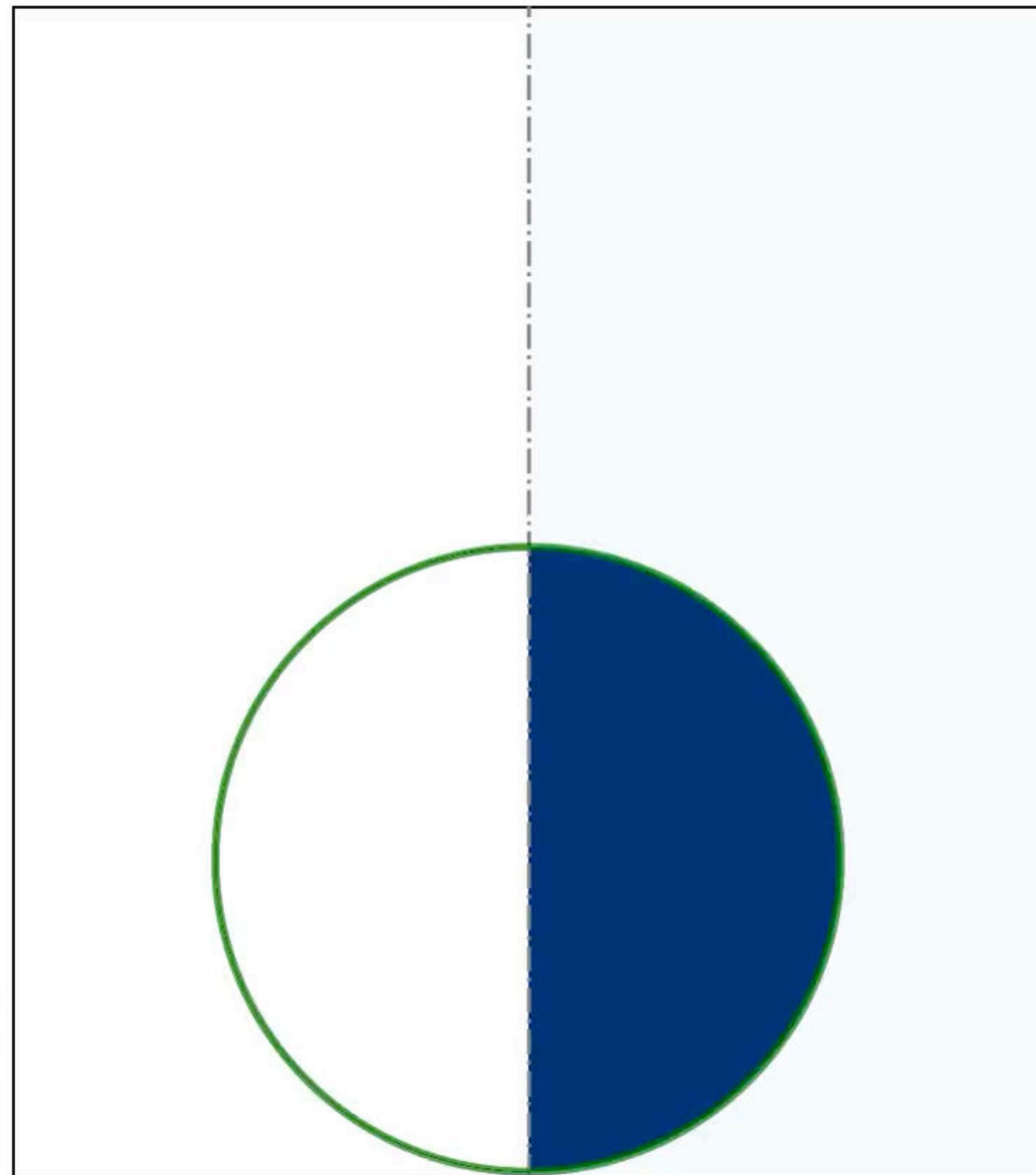
# Singular jet & bubble

$$We = 9$$

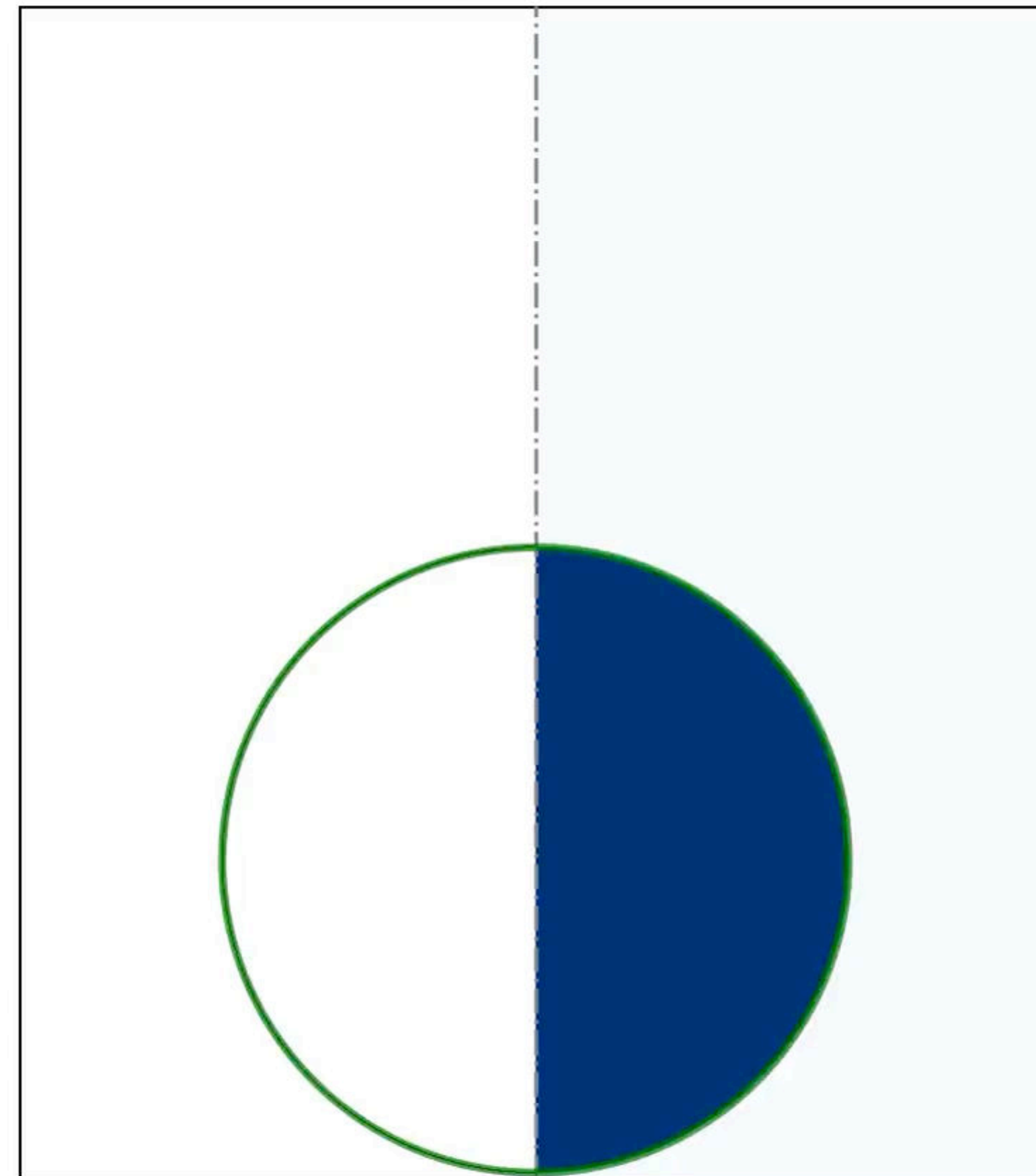
$$V_0 t / D_0 = 0.000$$



$$Oh = 0.0025$$



$$Oh = 0.005$$



$$Oh = 0.05$$

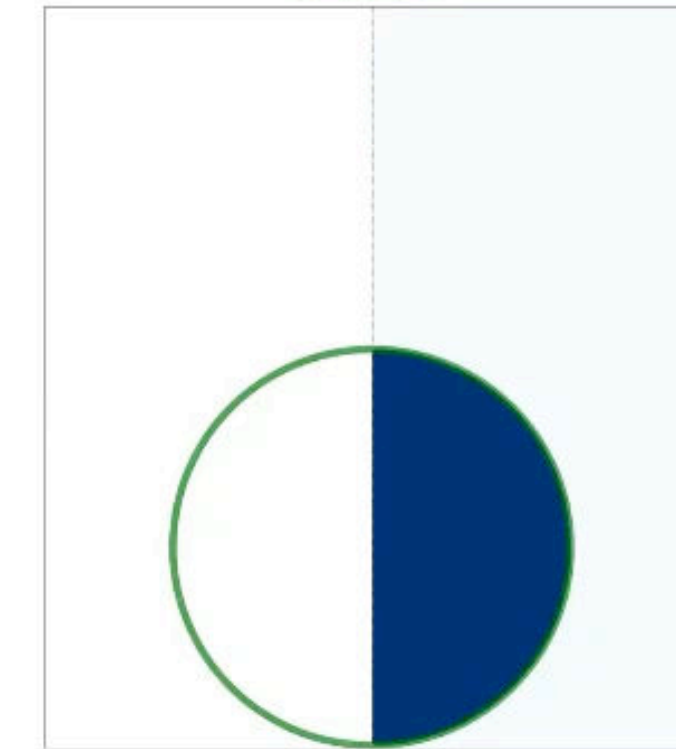
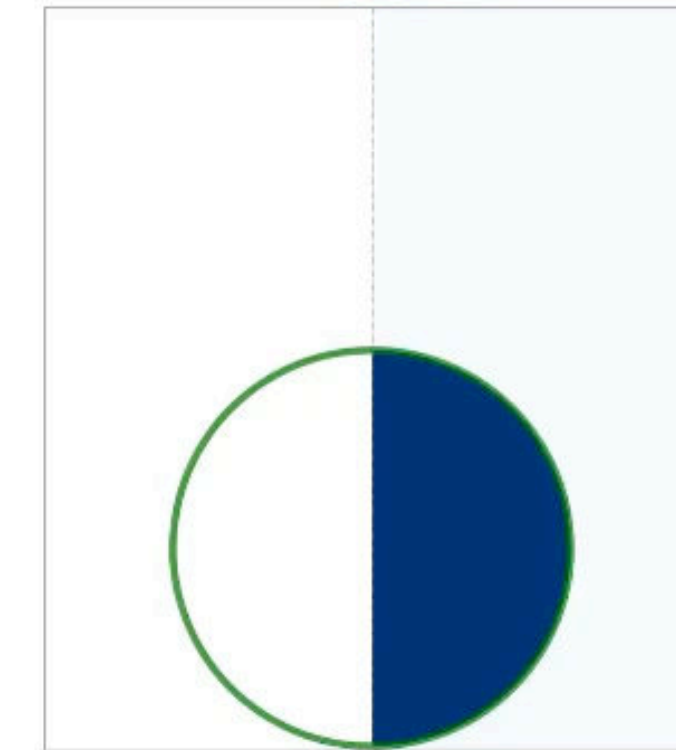
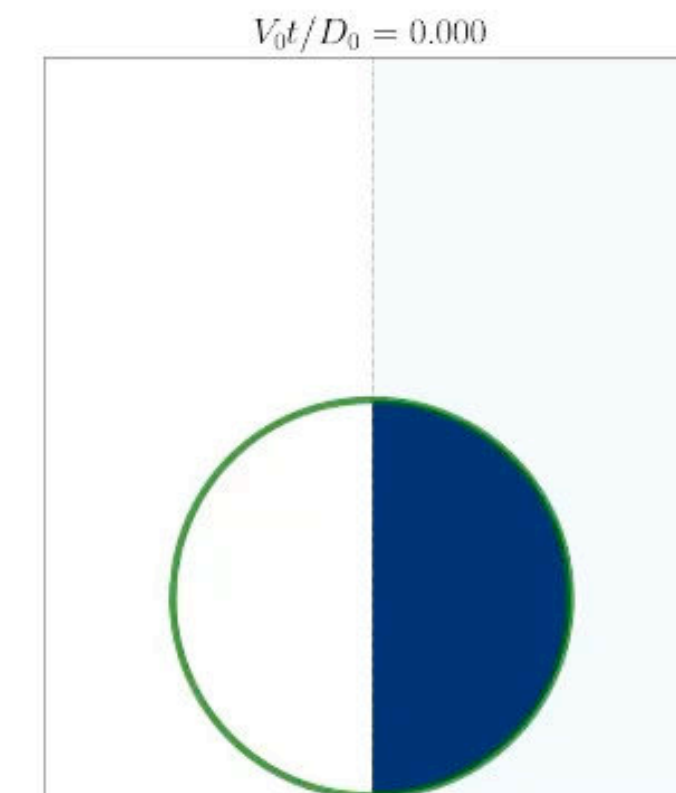
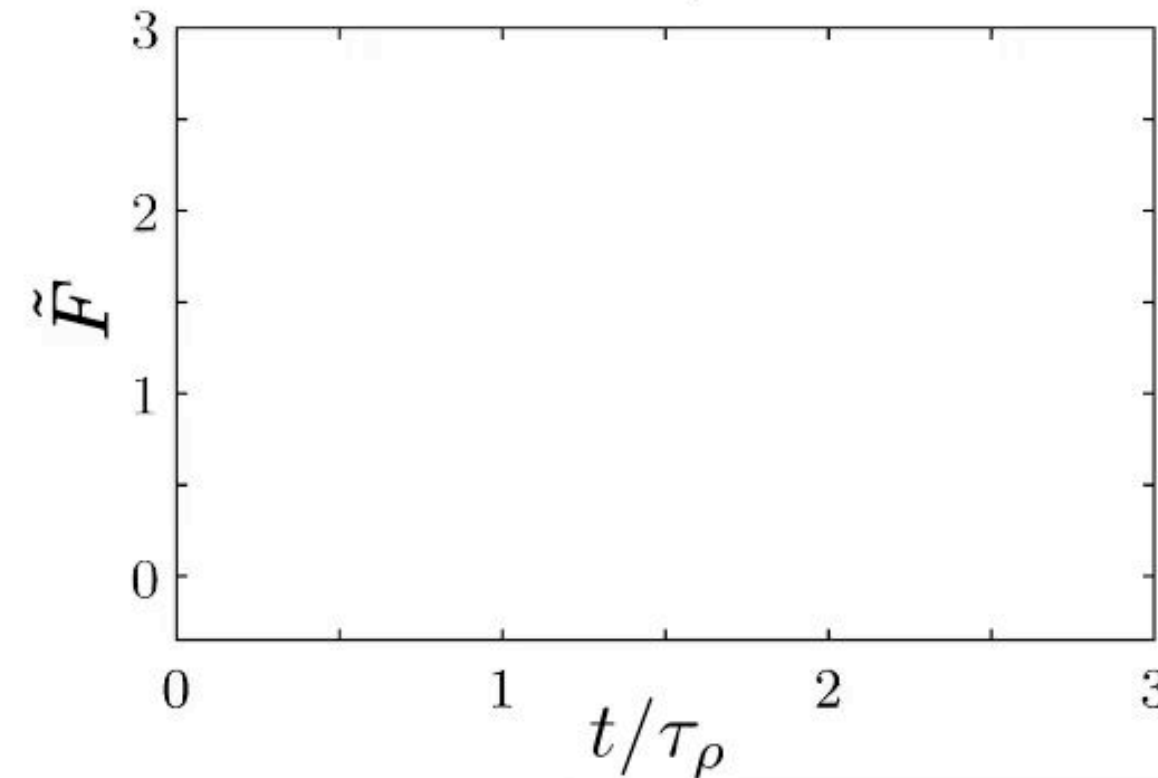
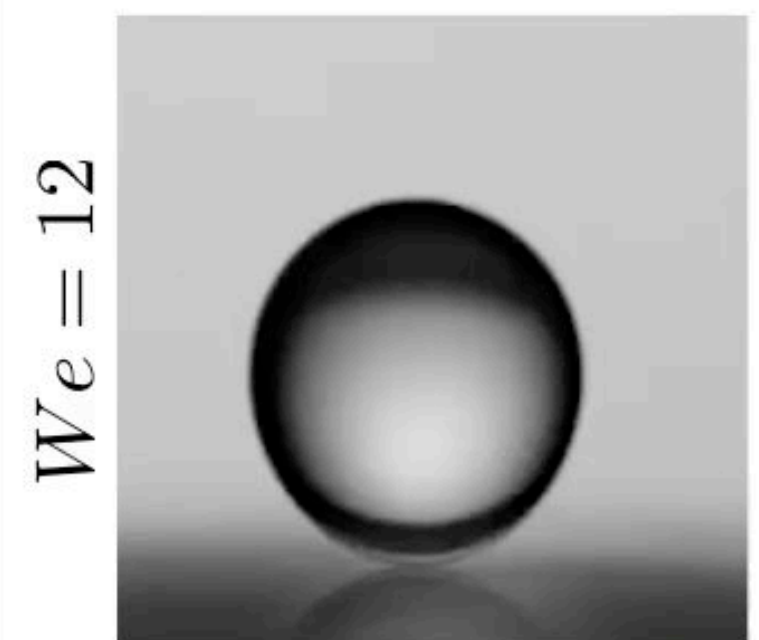
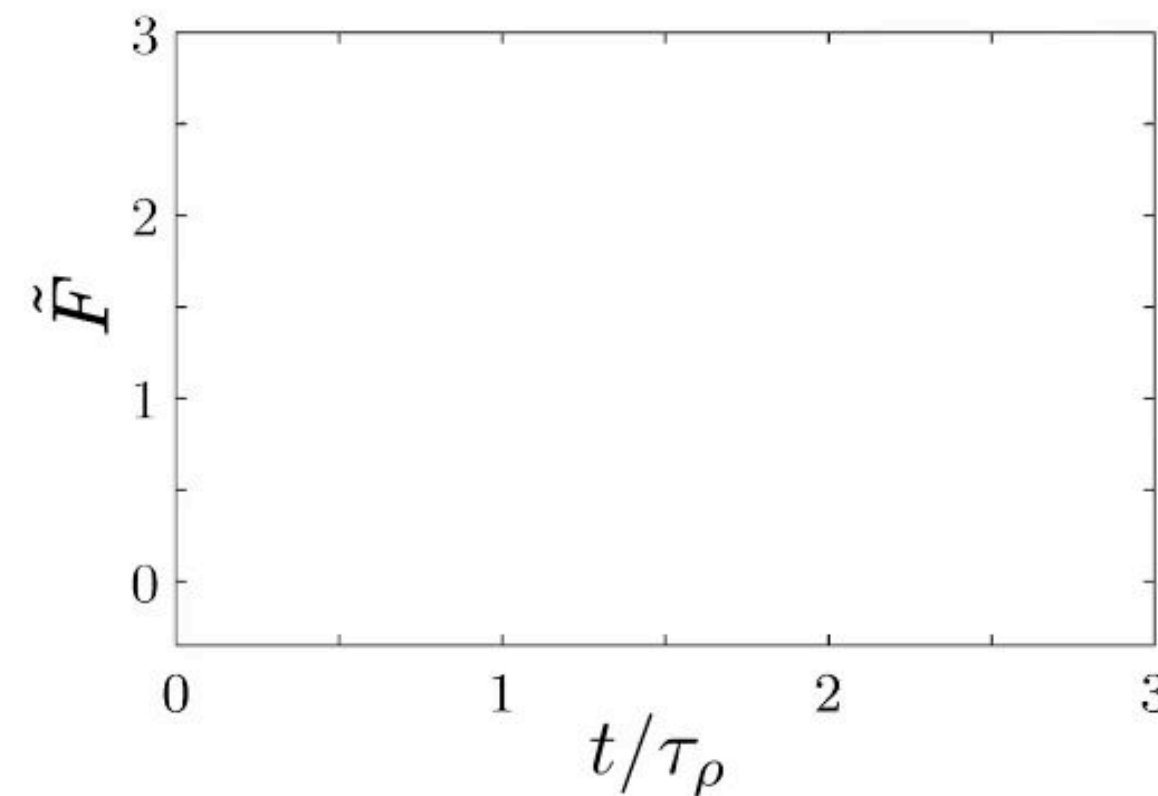
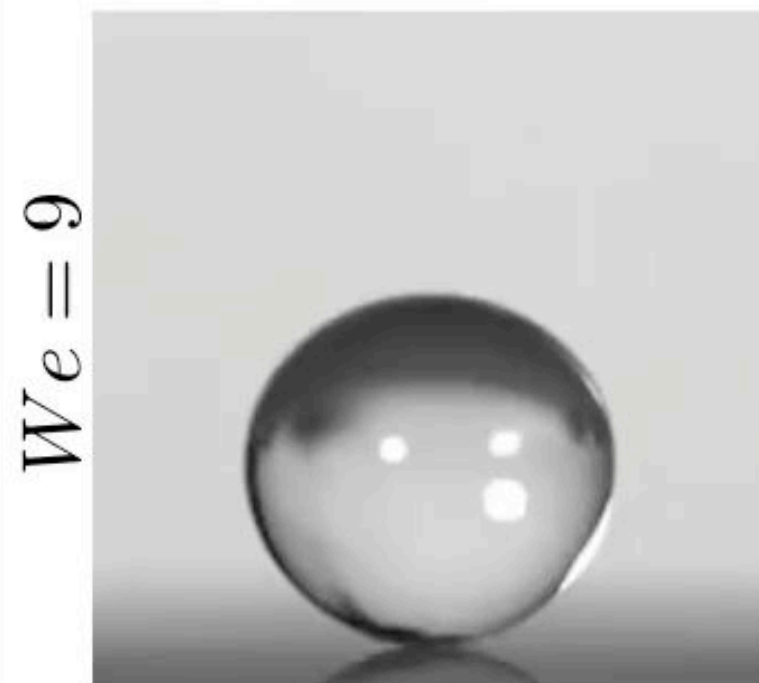
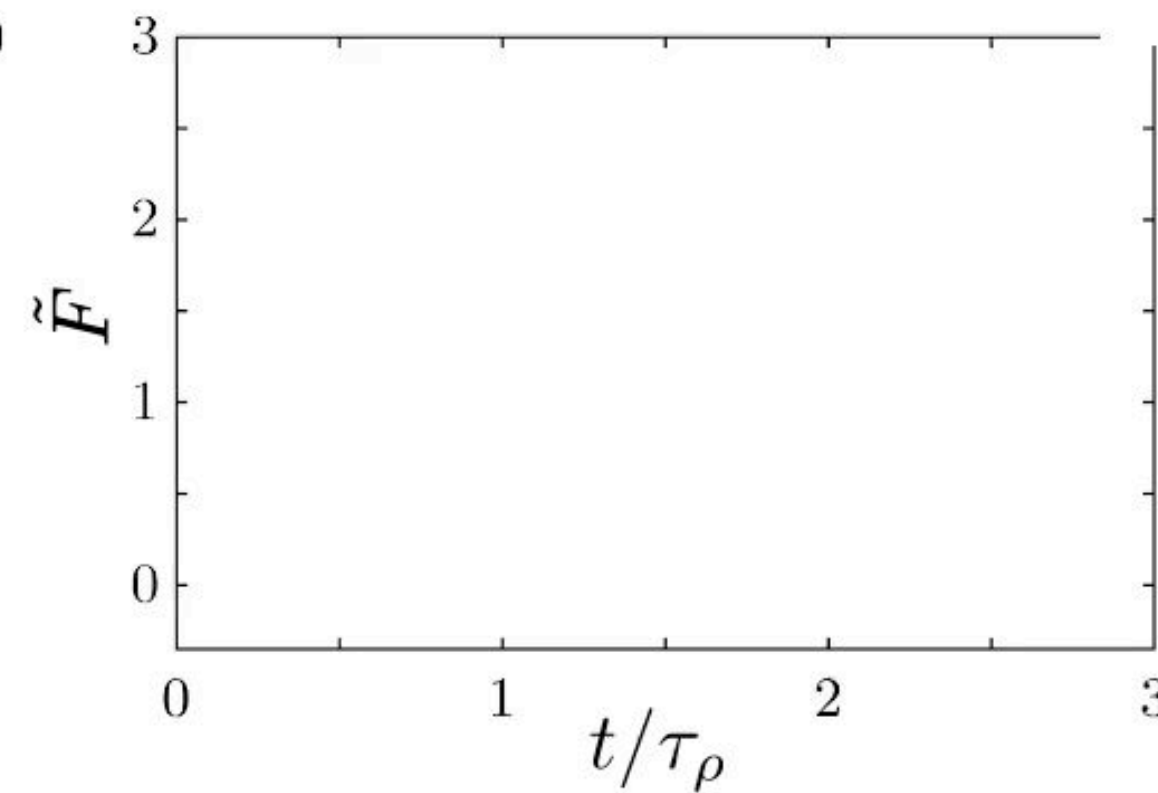
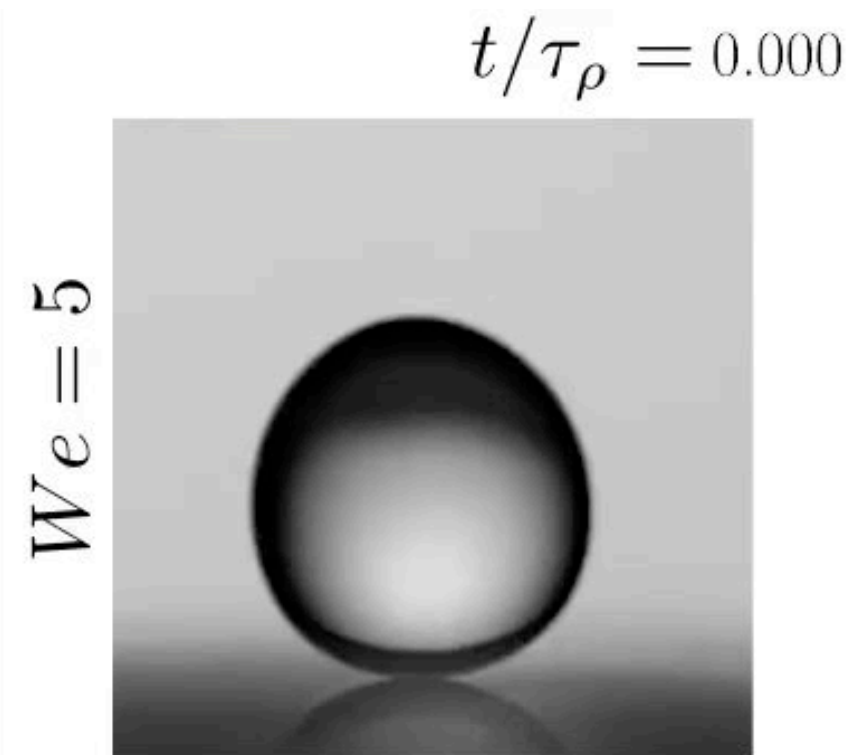
## II. What about the impact conditions?



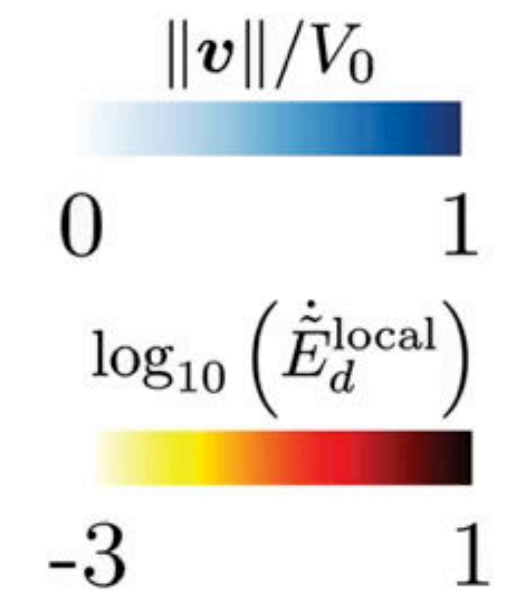
# Bubble entrainment regime is very narrow $We \approx 9$

— Experiment  
— Simulation

$D_0$



$D_0$



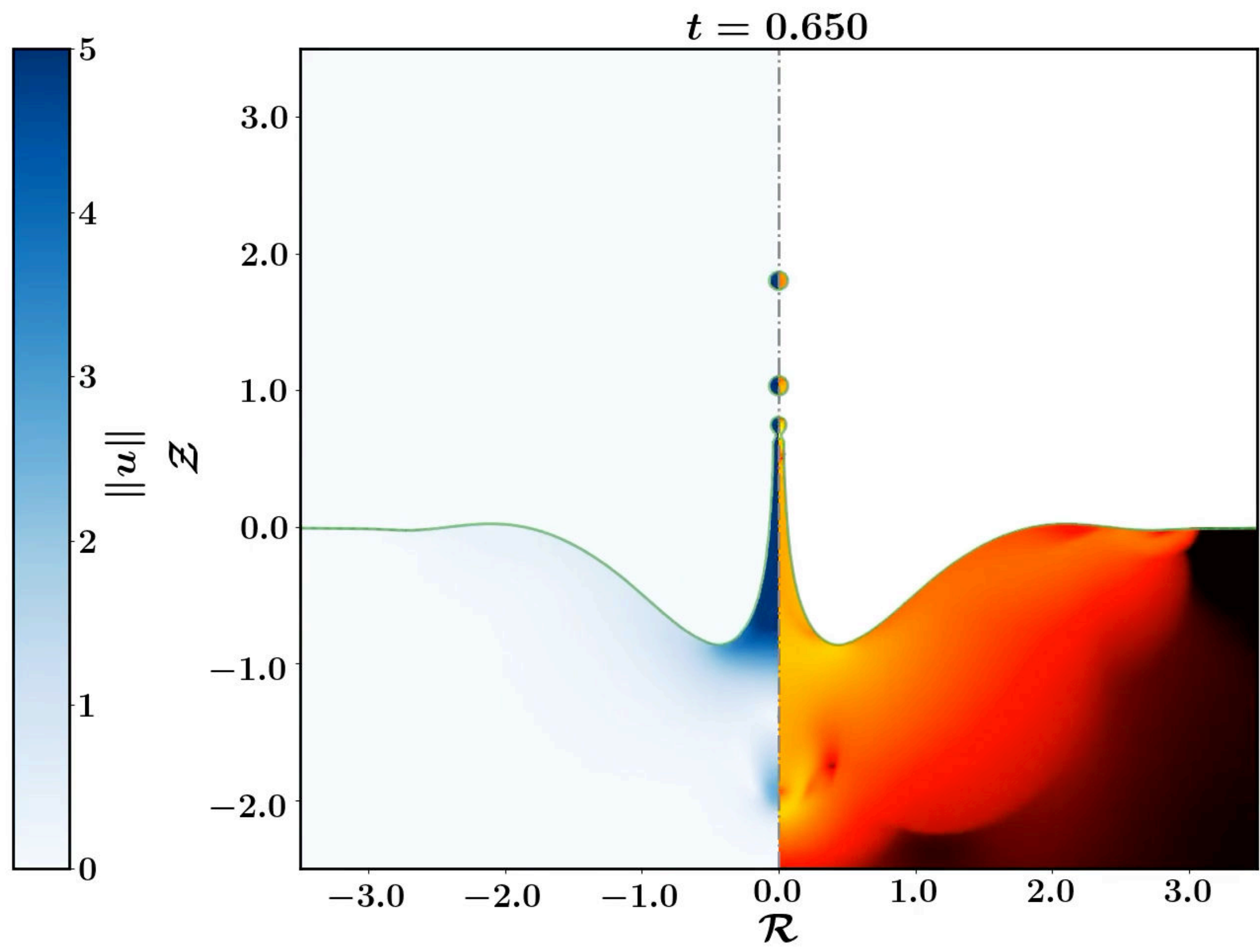
$$We = \frac{\rho_d V_0^2 D_0}{\gamma}$$



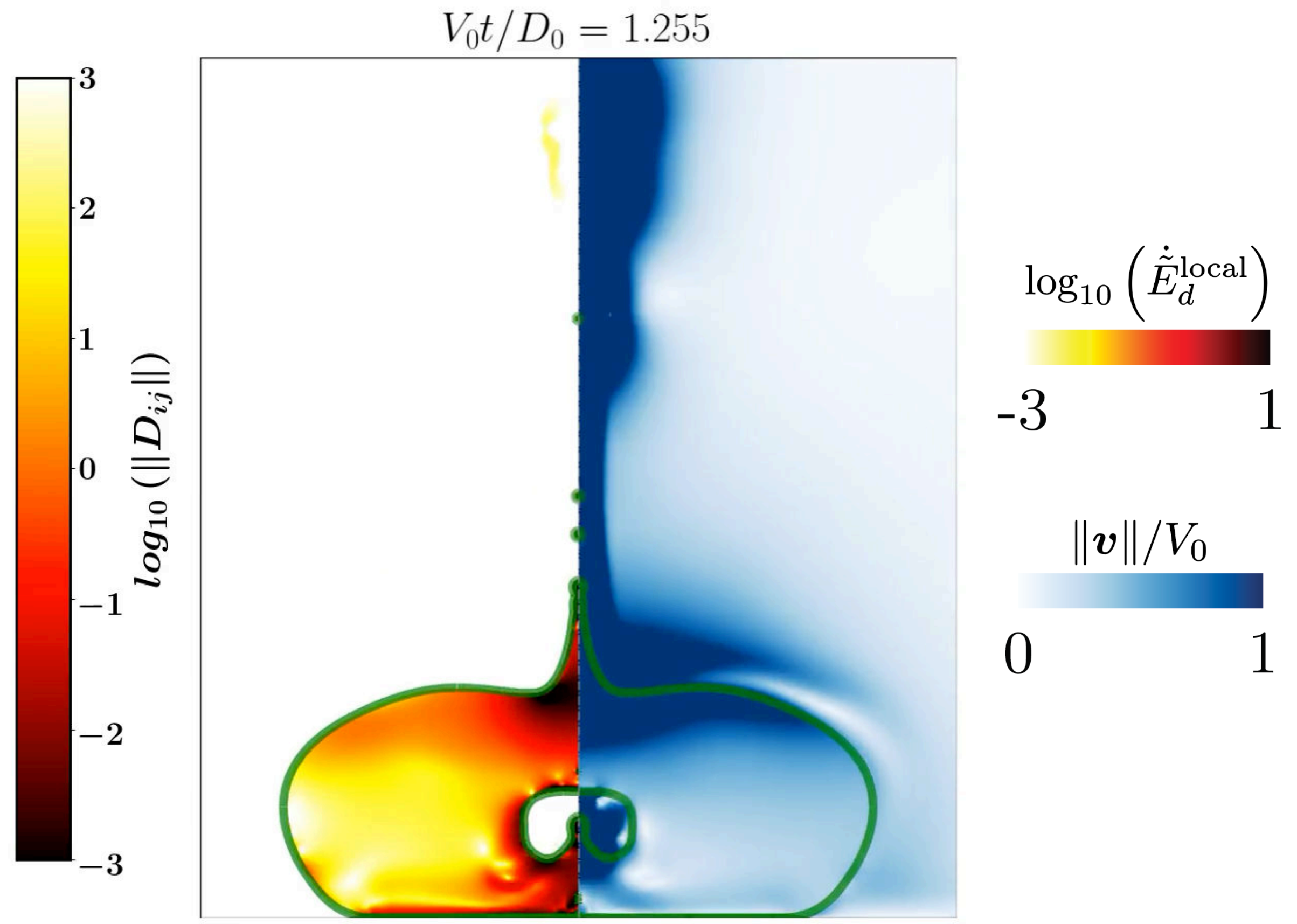




# Throwback Friday: Worthington jets



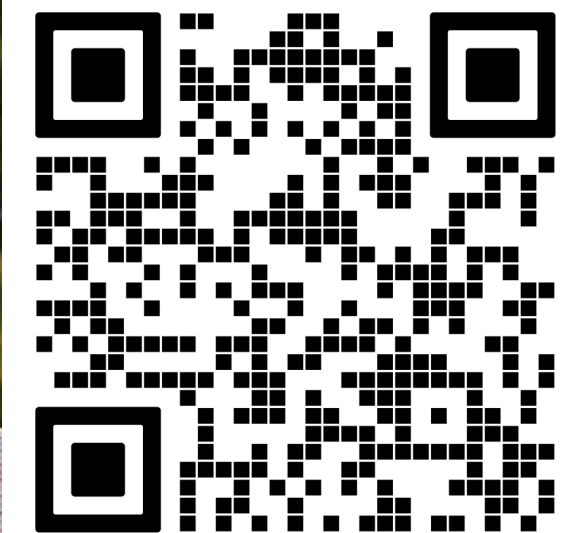
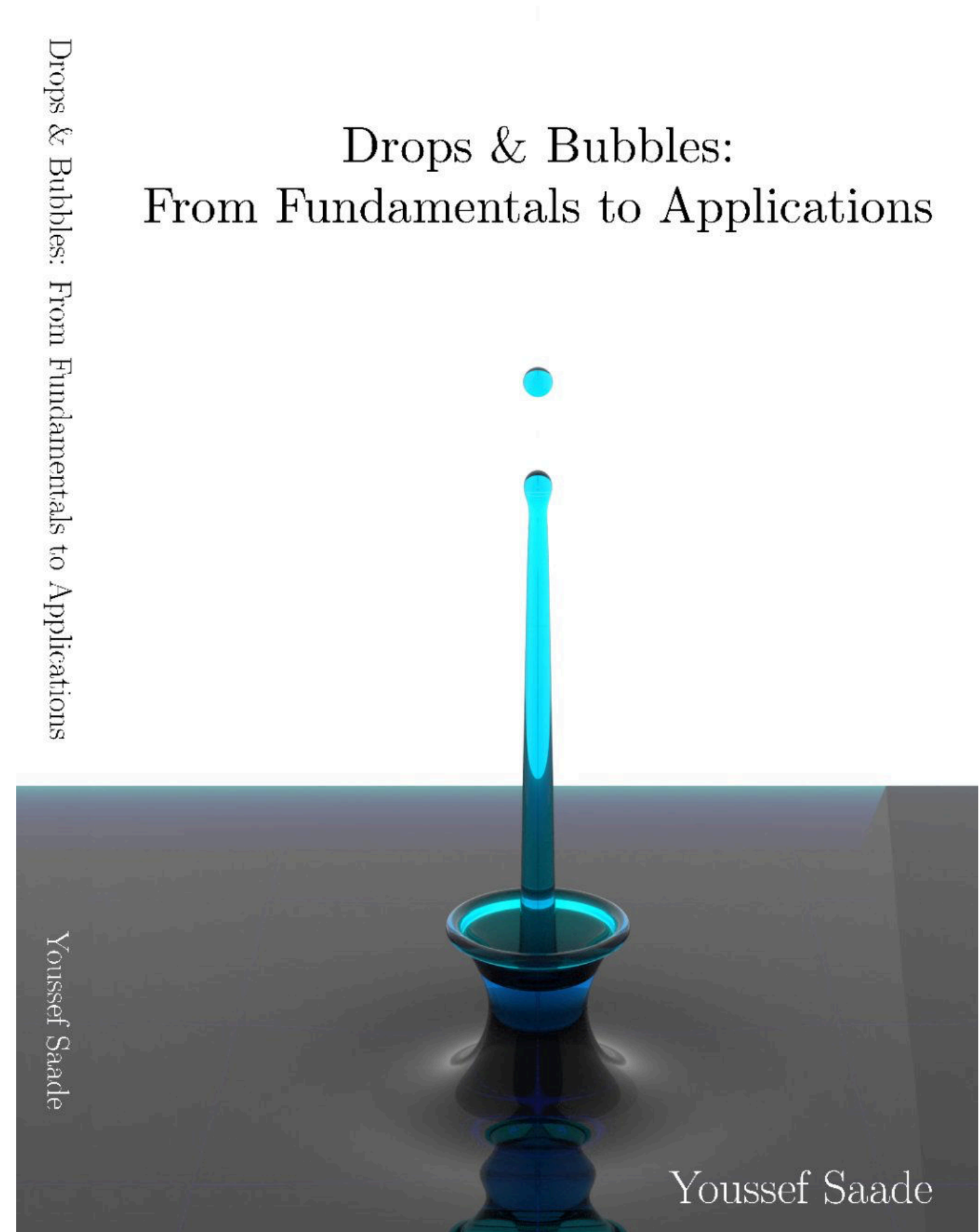
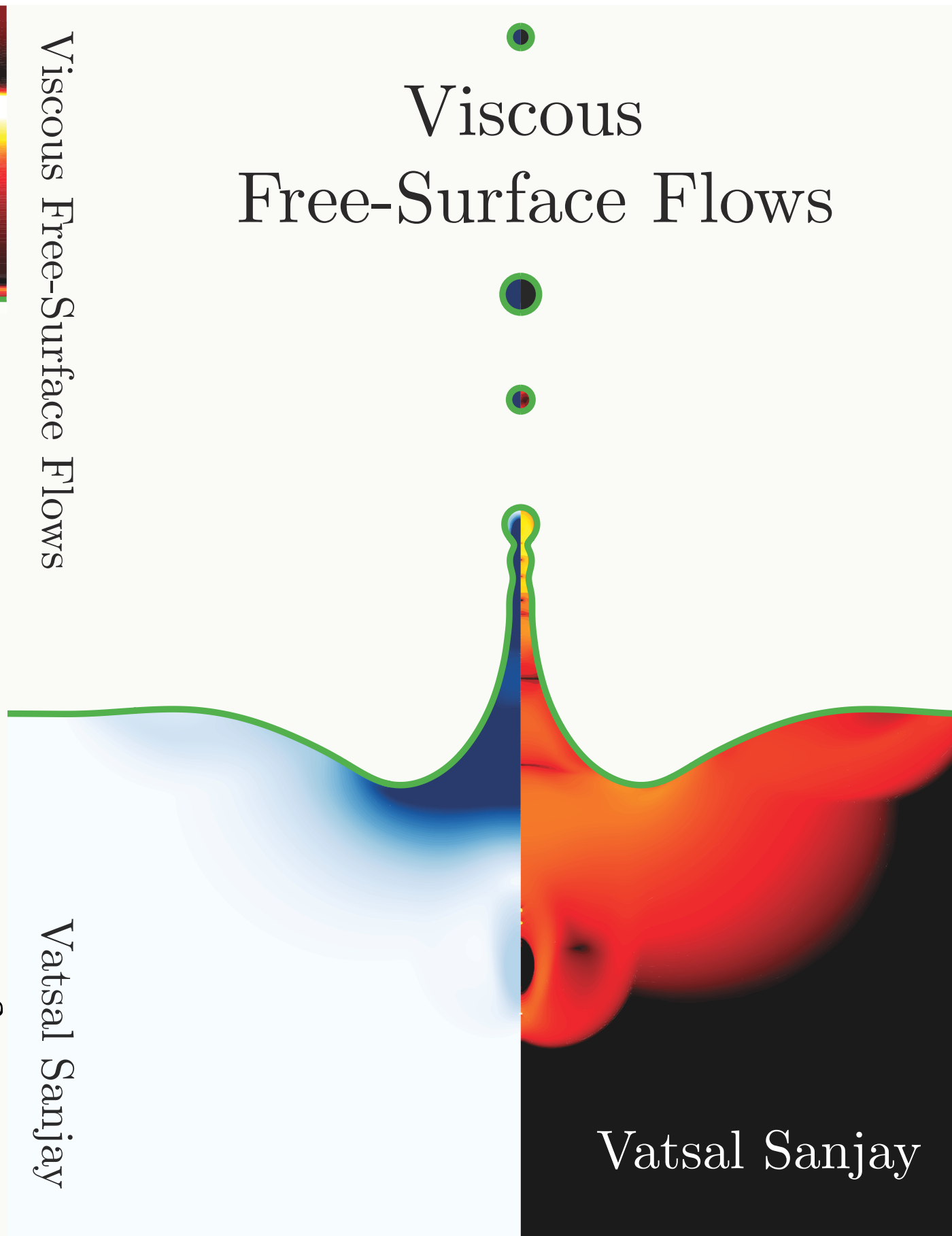
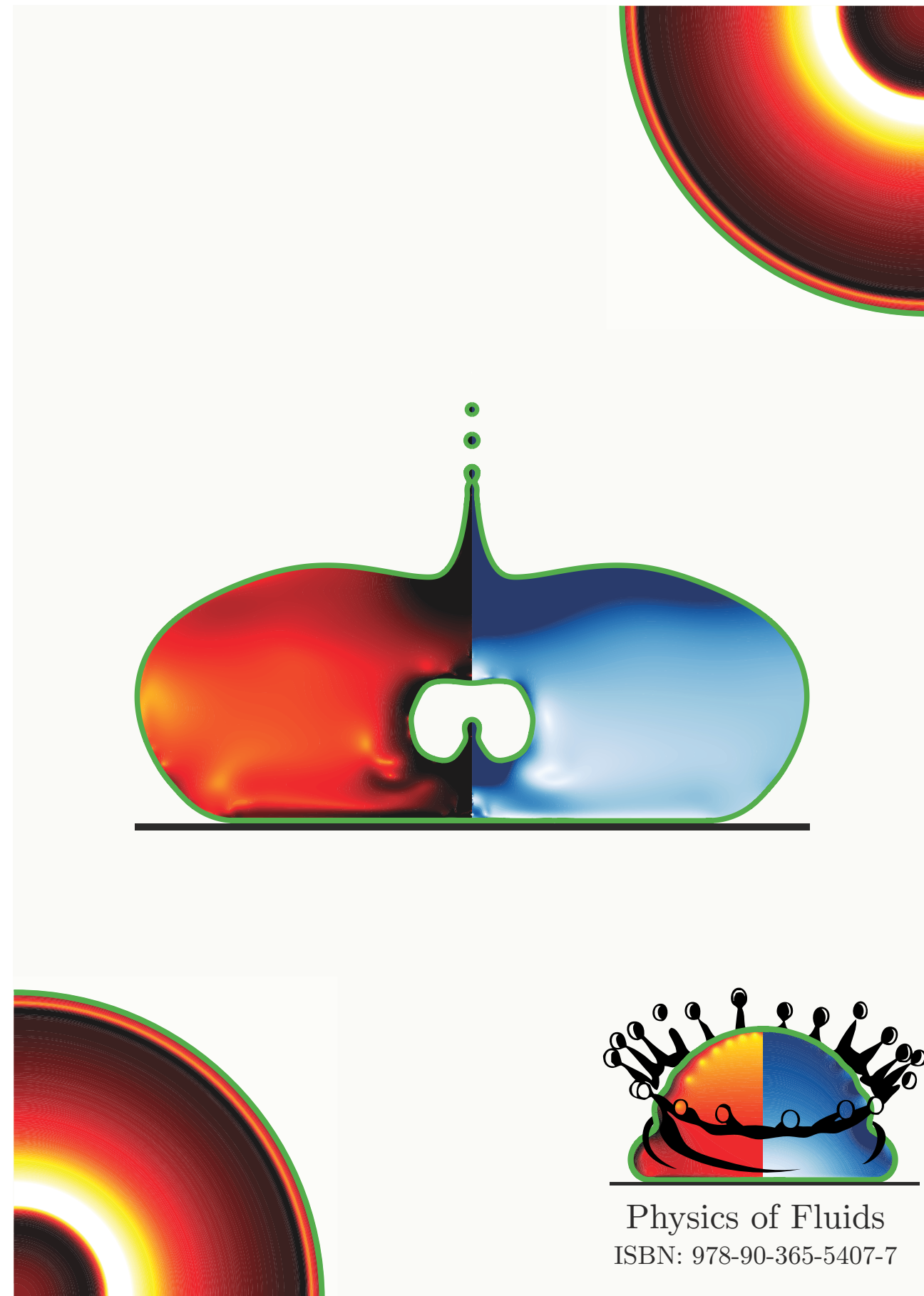
BGUM 2019



BGUM 2023



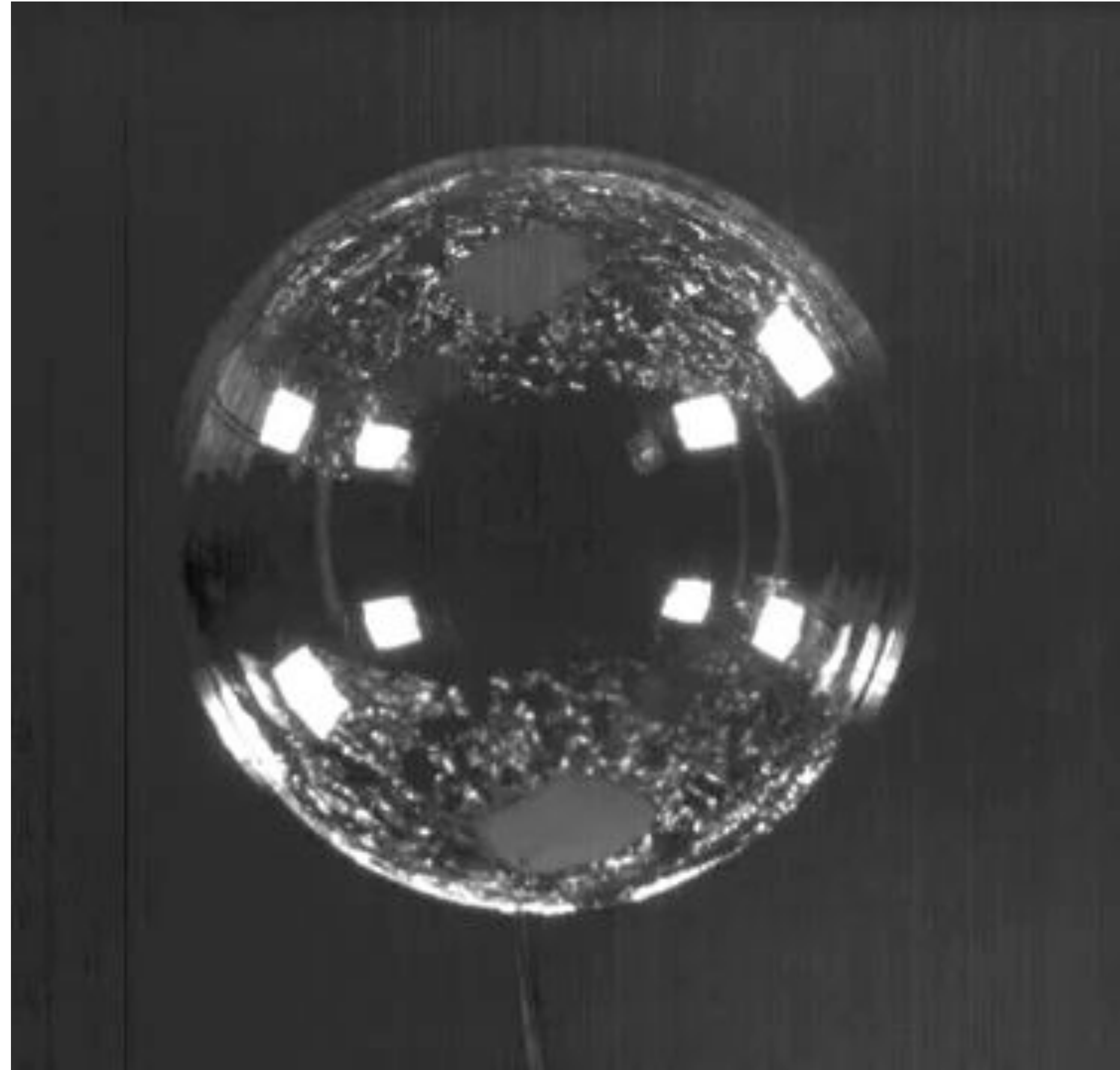
# Worthington-type jets in the last 4 years

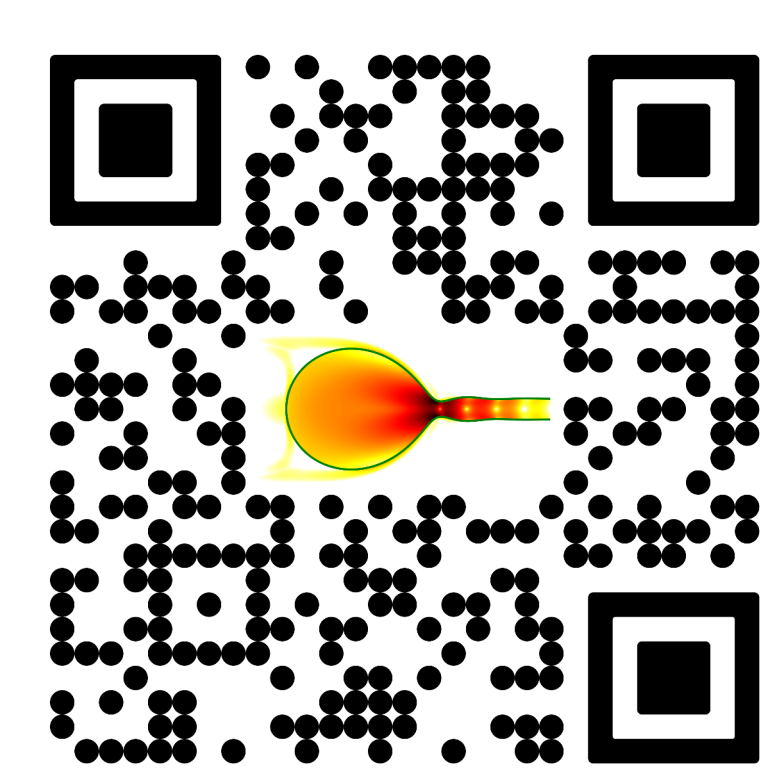


More free-surface flows



# (Another) Real life examples





# Taylor-Culick Retractions

Detlef Lohse



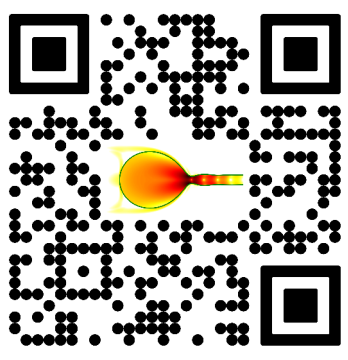
Udo Sen



Pallav Kant





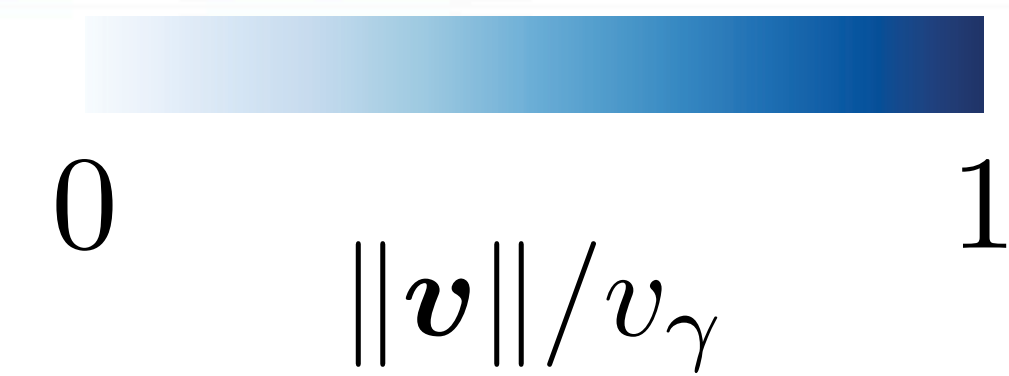
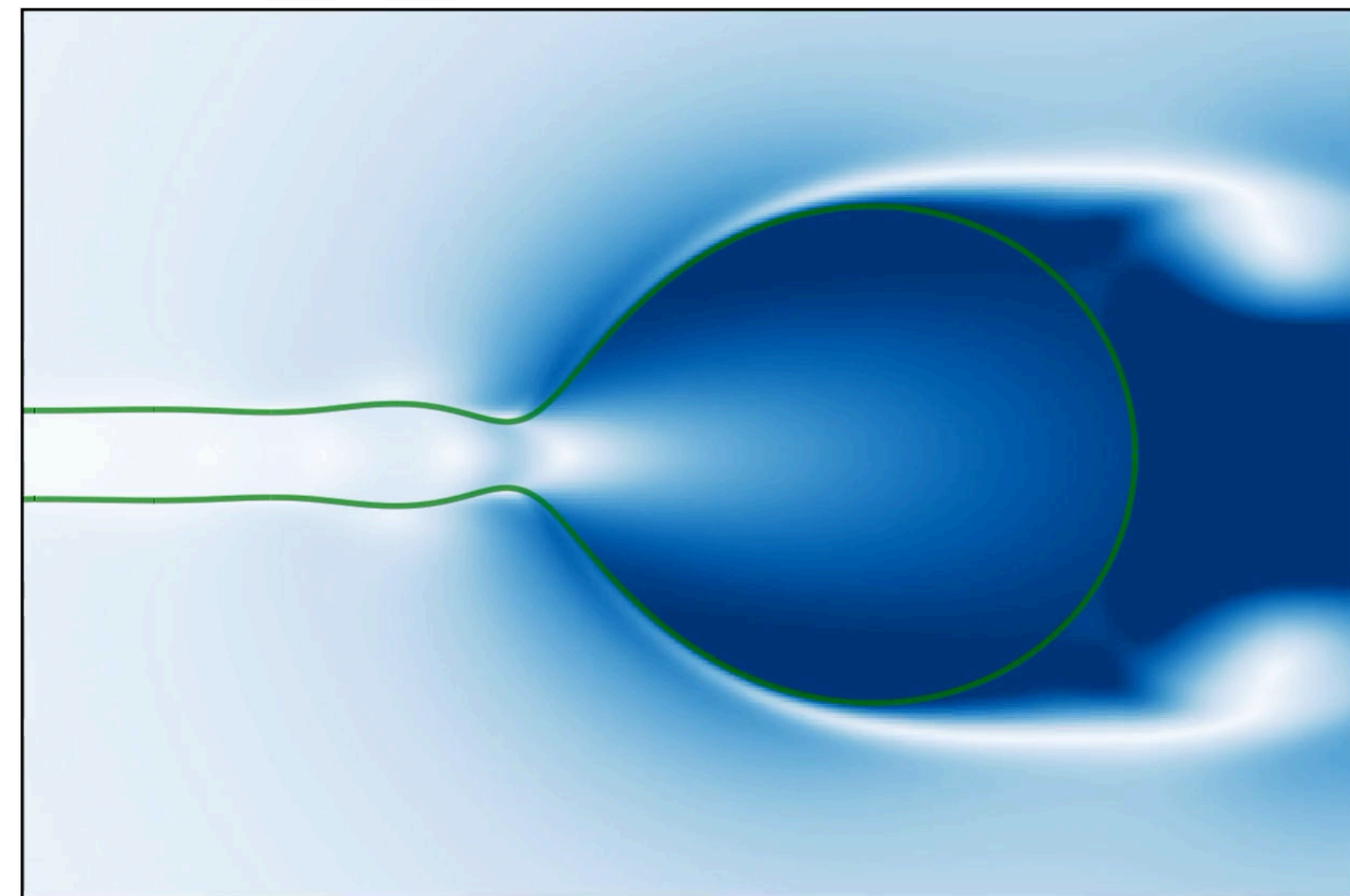
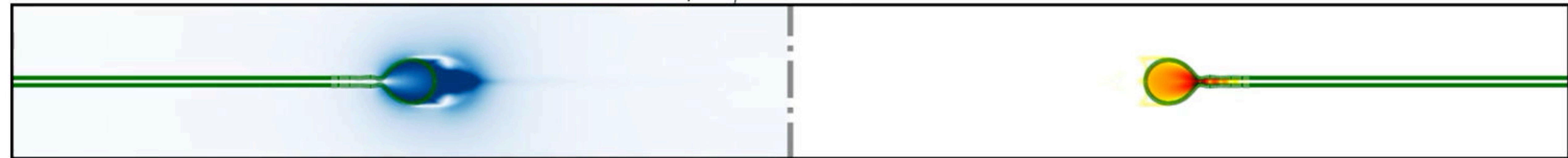


# Classical Taylor-Culick retraction

$$Oh_f = 0.05$$

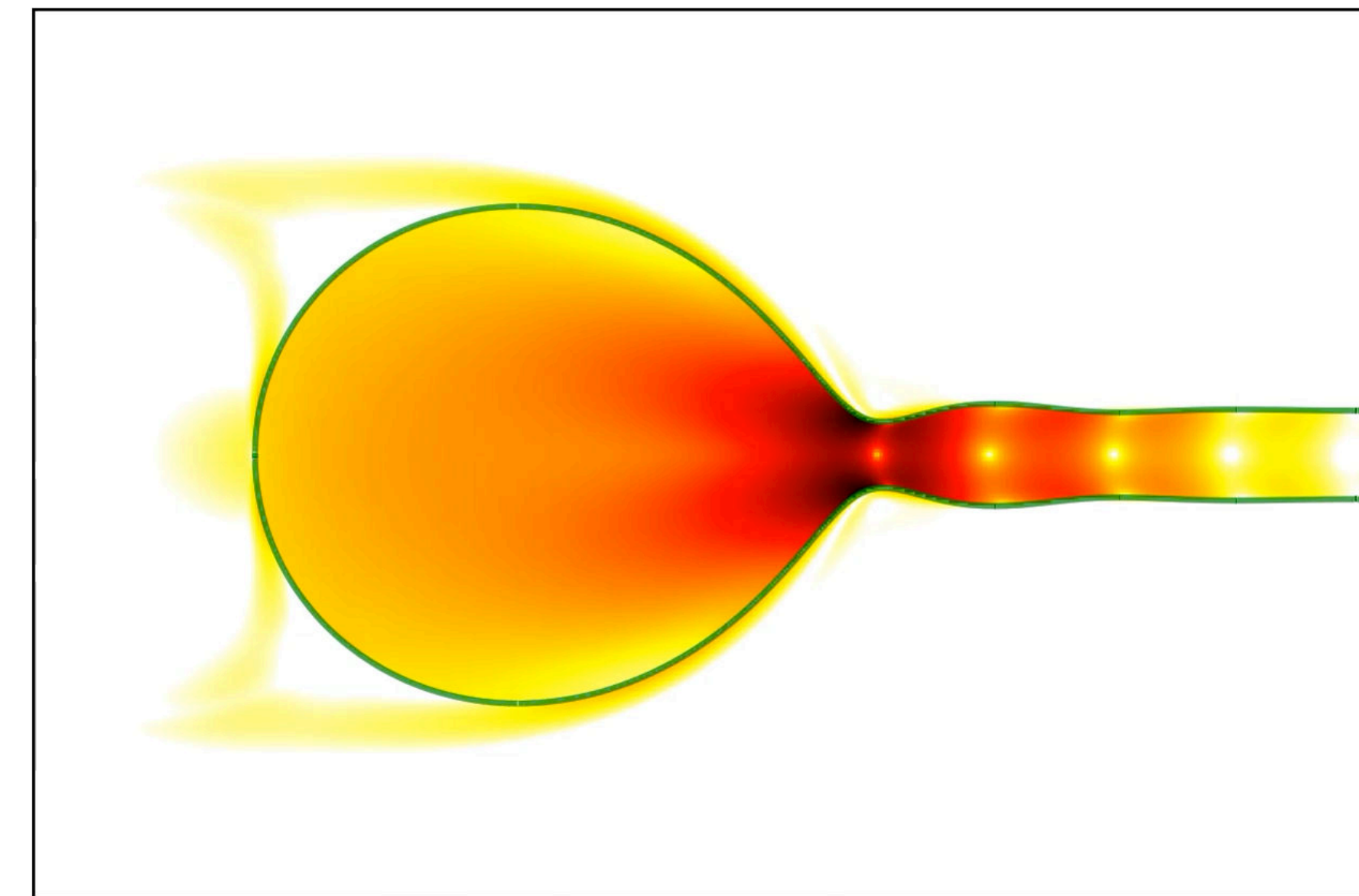
$$t/\tau_\gamma = 50.500$$

$$Oh = \frac{\eta}{\sqrt{\rho_f (2\gamma_{af}) h_0}}$$

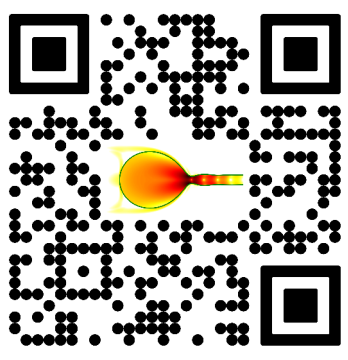


$$\mathcal{D} = \left( \nabla \mathbf{v} + (\nabla \mathbf{v})^T \right) / 2$$

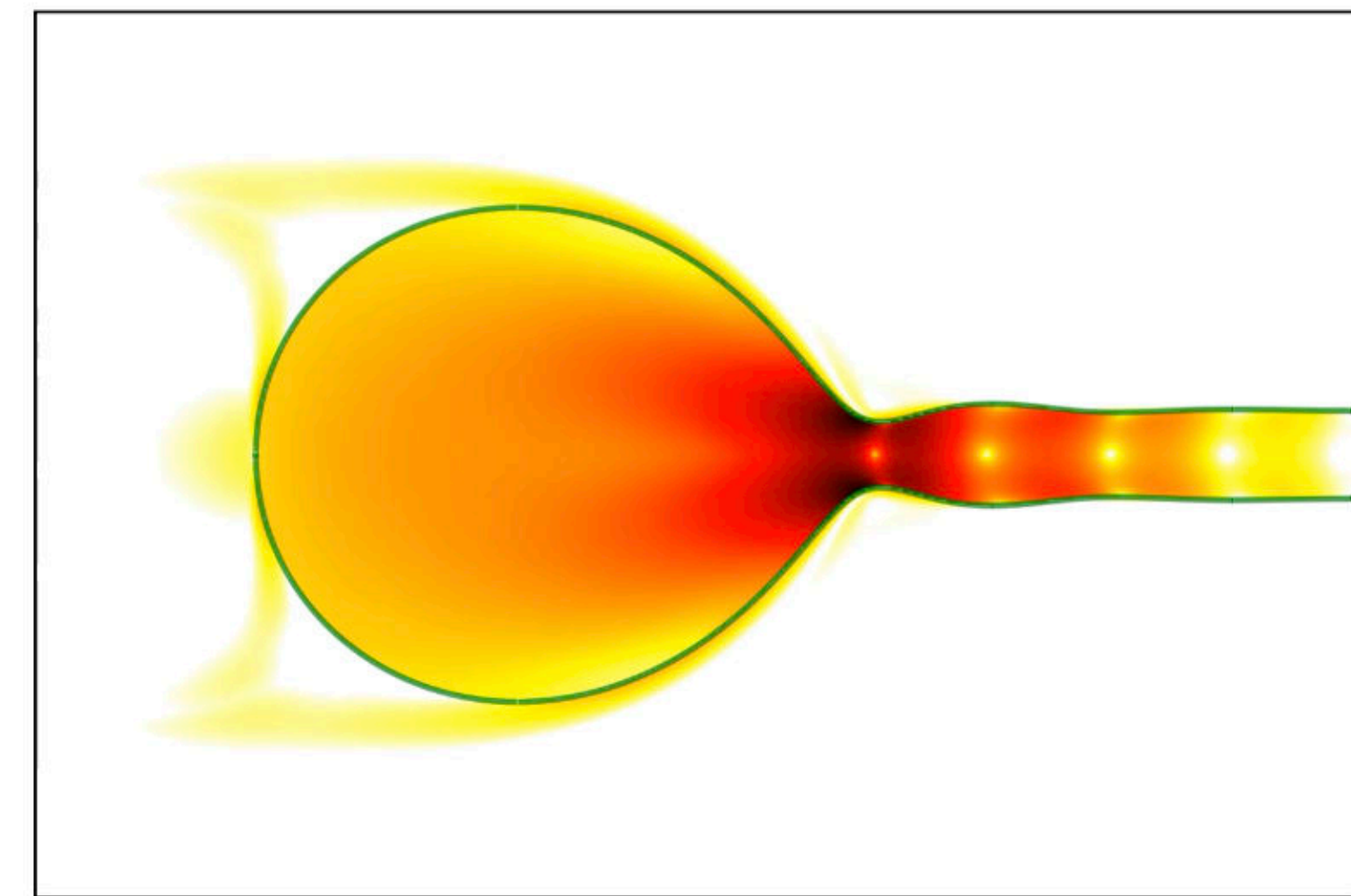
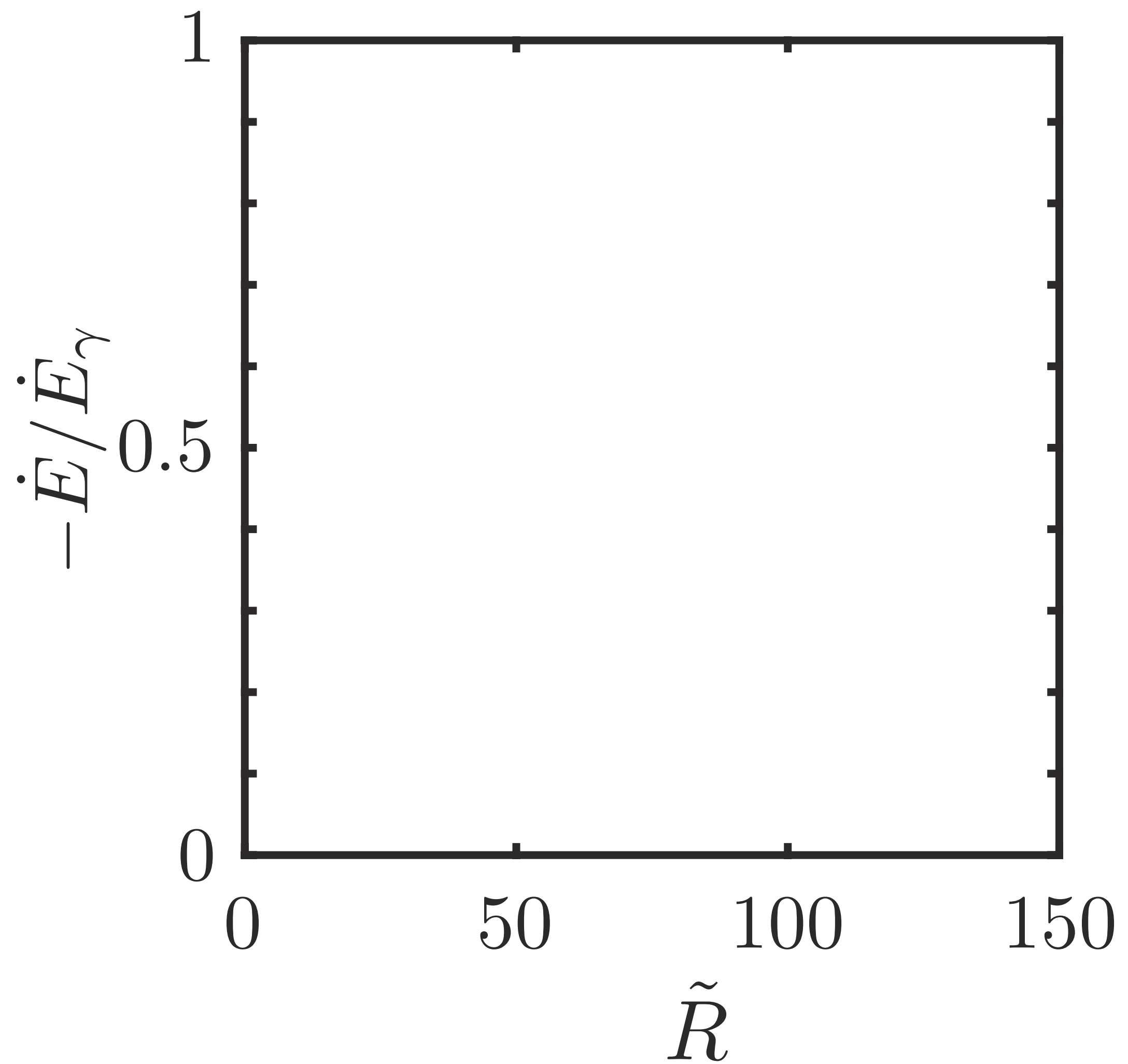
$$\tau_\gamma = \sqrt{\frac{\rho_f h_0^3}{2\gamma_{af}}} v_\gamma = \sqrt{\frac{2\gamma_{af}}{\rho_f h_0}}$$



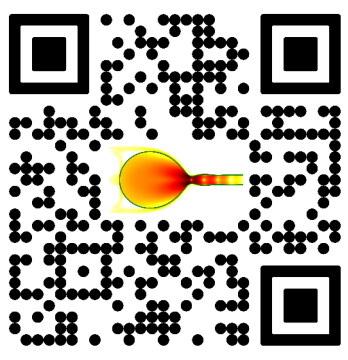




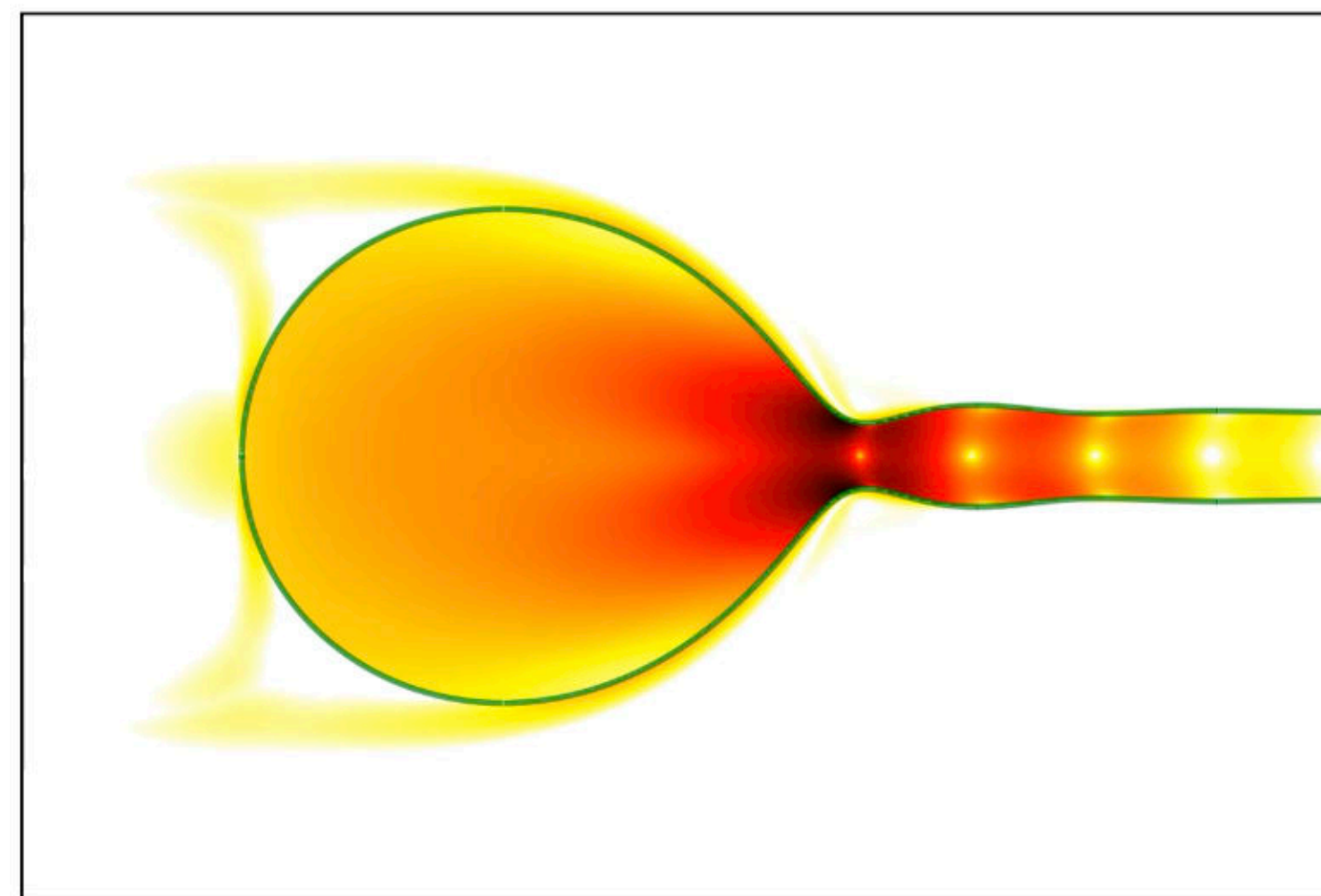
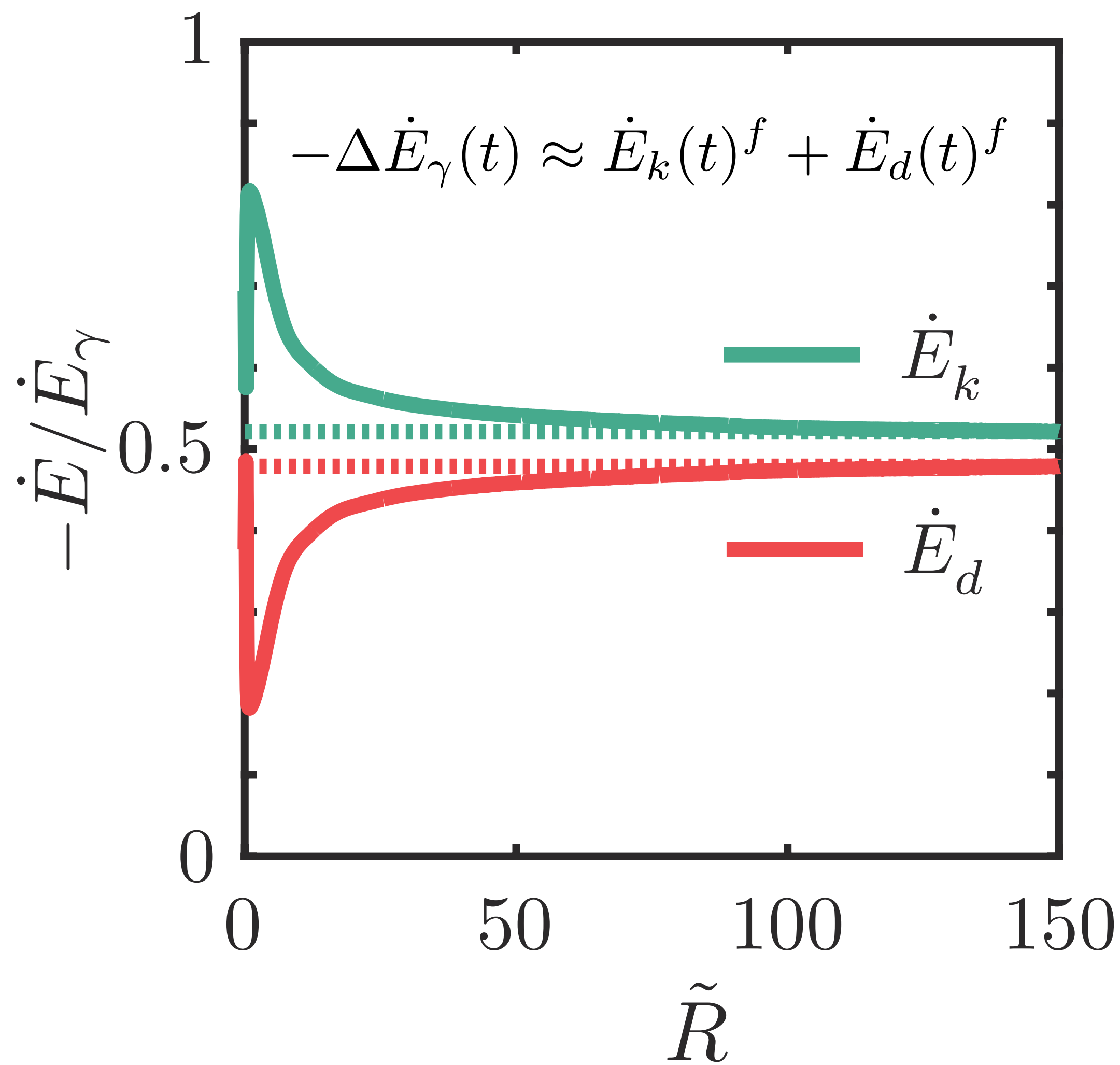
# Energy Budget: Classical Taylor-Culick retraction



$$\log_{10} (2Oh (\mathcal{D} : \mathcal{D}) \tau_\gamma^2)$$



# Energy Budget: Classical Taylor-Culick retraction



$$\dot{E}_d = \frac{1}{2} \frac{dm}{dt} v^2$$



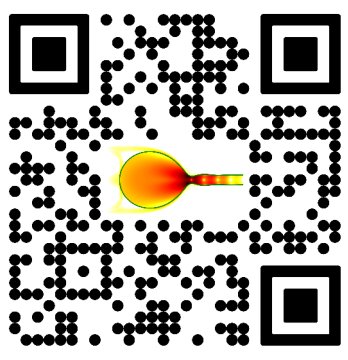
G. I. Taylor



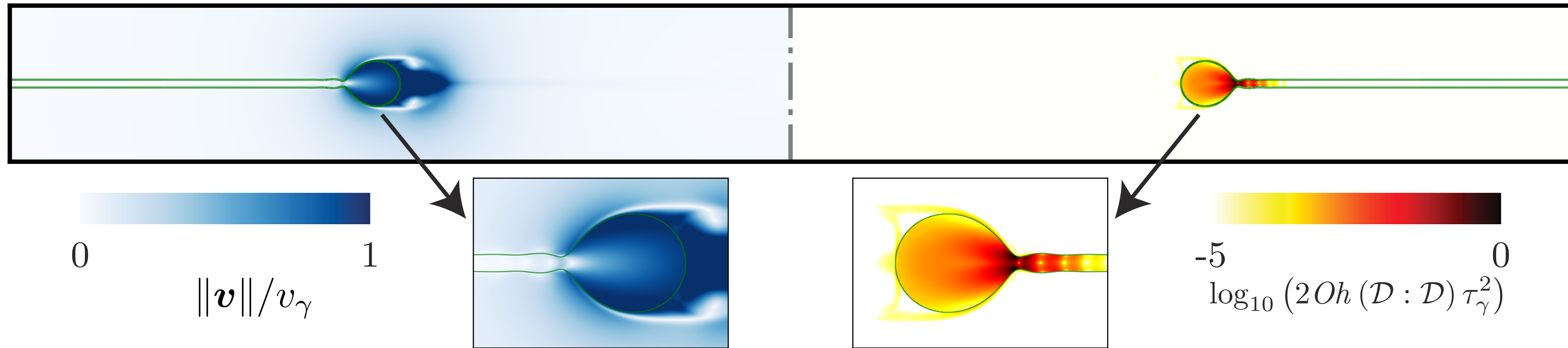
F. E. C. Culick



P.-G. de Gennes



# Synopsis of Classical Taylor-Culick retractions



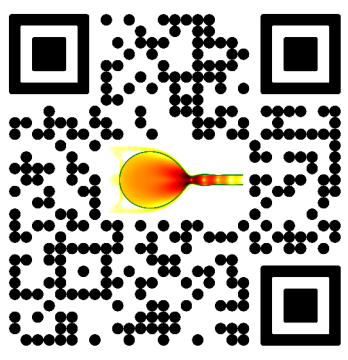
- Even in the inertial limit, **cannot neglect viscous dissipation in the thin film**

$$v_f = v_{TC} = \sqrt{\frac{(2\gamma_f a)}{\rho_f h_0}}$$

$$-\Delta \dot{E}_\gamma(t) \approx \dot{E}_k(t)^f + \dot{E}_d(t)^f$$

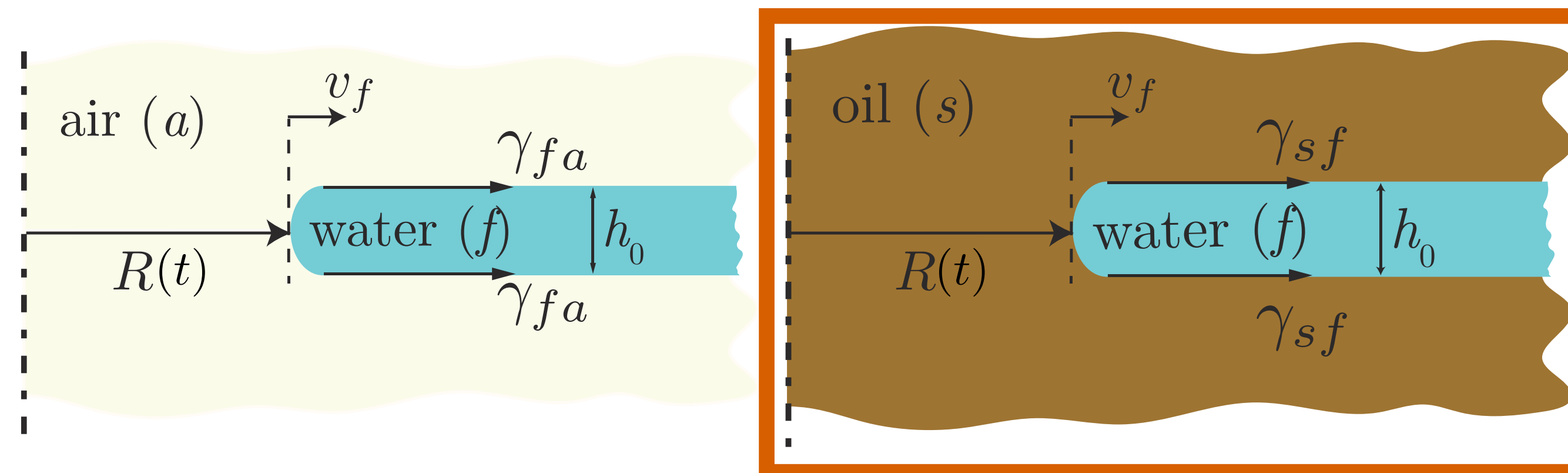
- Dissipation is **independent** of viscosity.

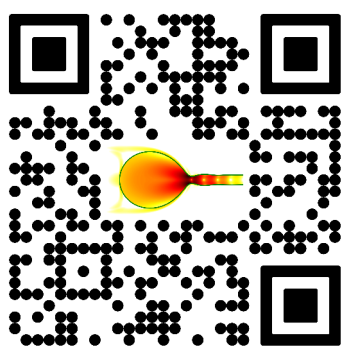
$$\dot{E}_d = \frac{1}{2} \frac{dm}{dt} v^2$$



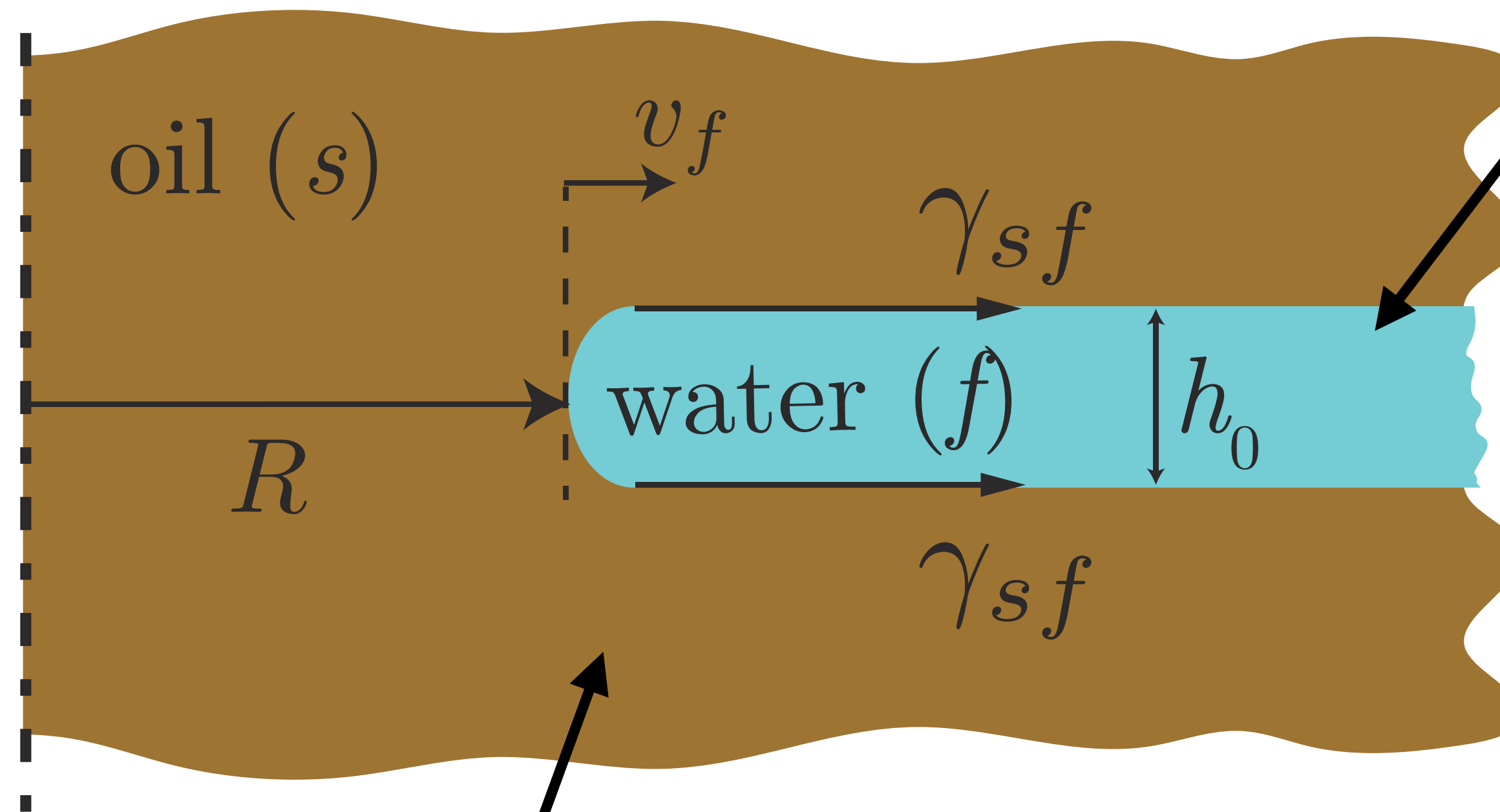
# Beyond the free-surface ...

What happens if the surrounding medium is not air?





# Problem Statement

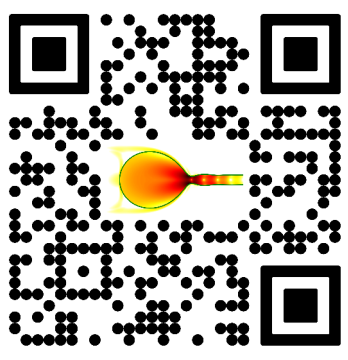


$$Oh_f = \frac{\eta_f}{\sqrt{\rho_f (2\gamma_{fs}) h_0}}$$

$$Oh_s = \frac{\eta_s}{\sqrt{\rho_f (2\gamma_{fs}) h_0}}$$







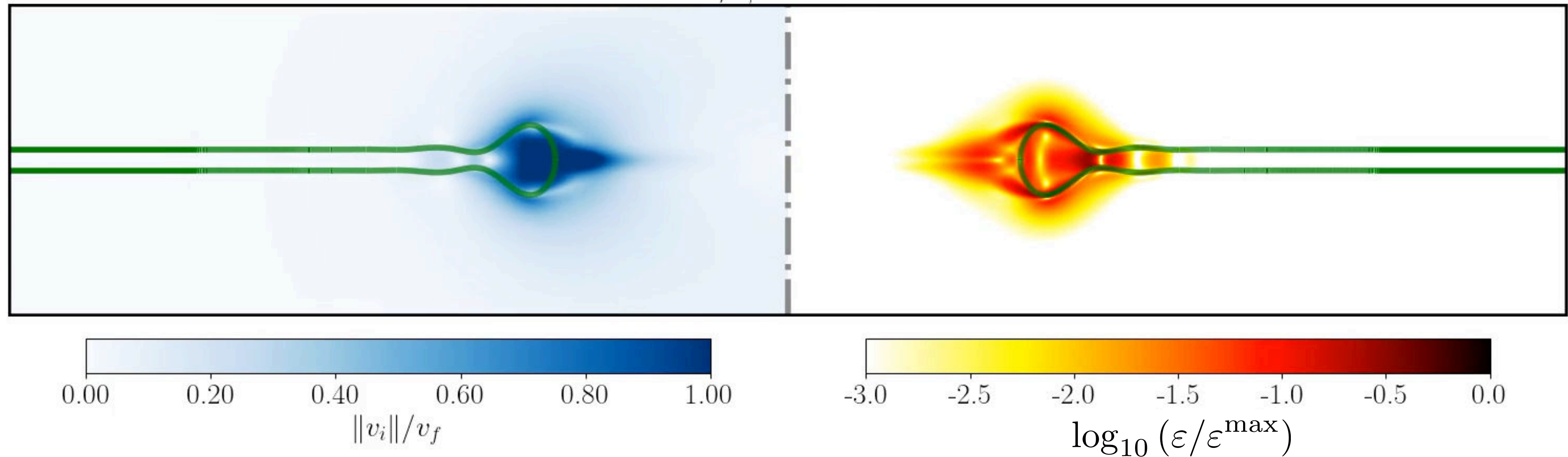
# Retraction in viscous environment: Inertial limit

Control Parameters:  $Oh_s = \frac{\eta_s}{\sqrt{\rho_f (2\gamma_{fs}) h_0}}$

$Oh_f = \frac{\eta_f}{\sqrt{\rho_f (2\gamma_{fs}) h_0}}$

$Oh_f = 0.03 \quad Oh_s = 0.01$

$t/t_\gamma = 20.500$

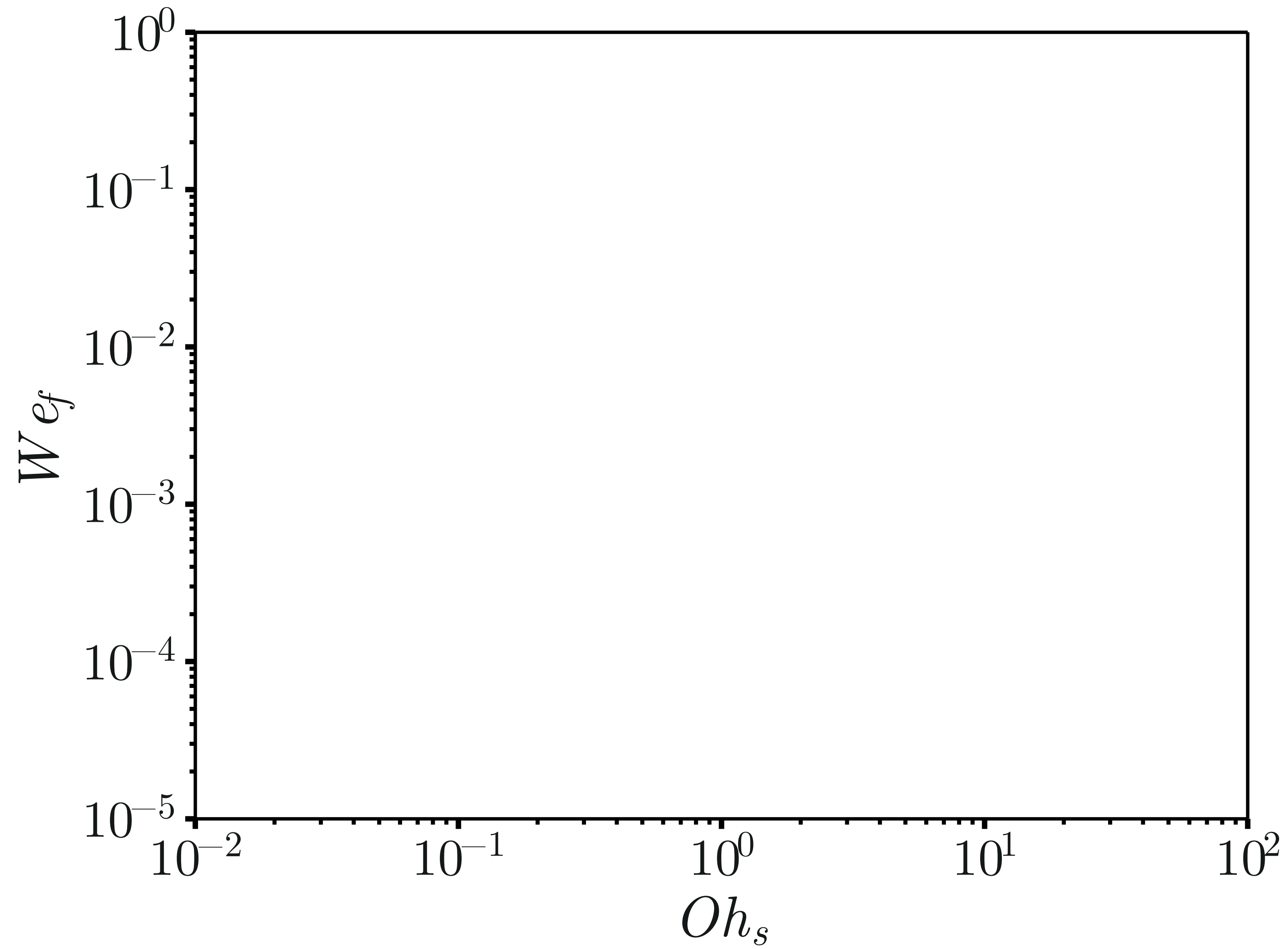
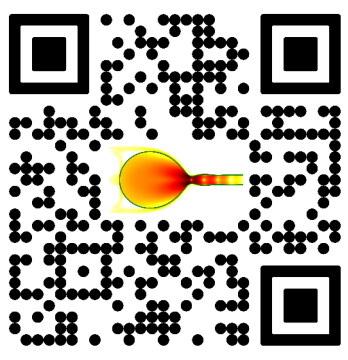


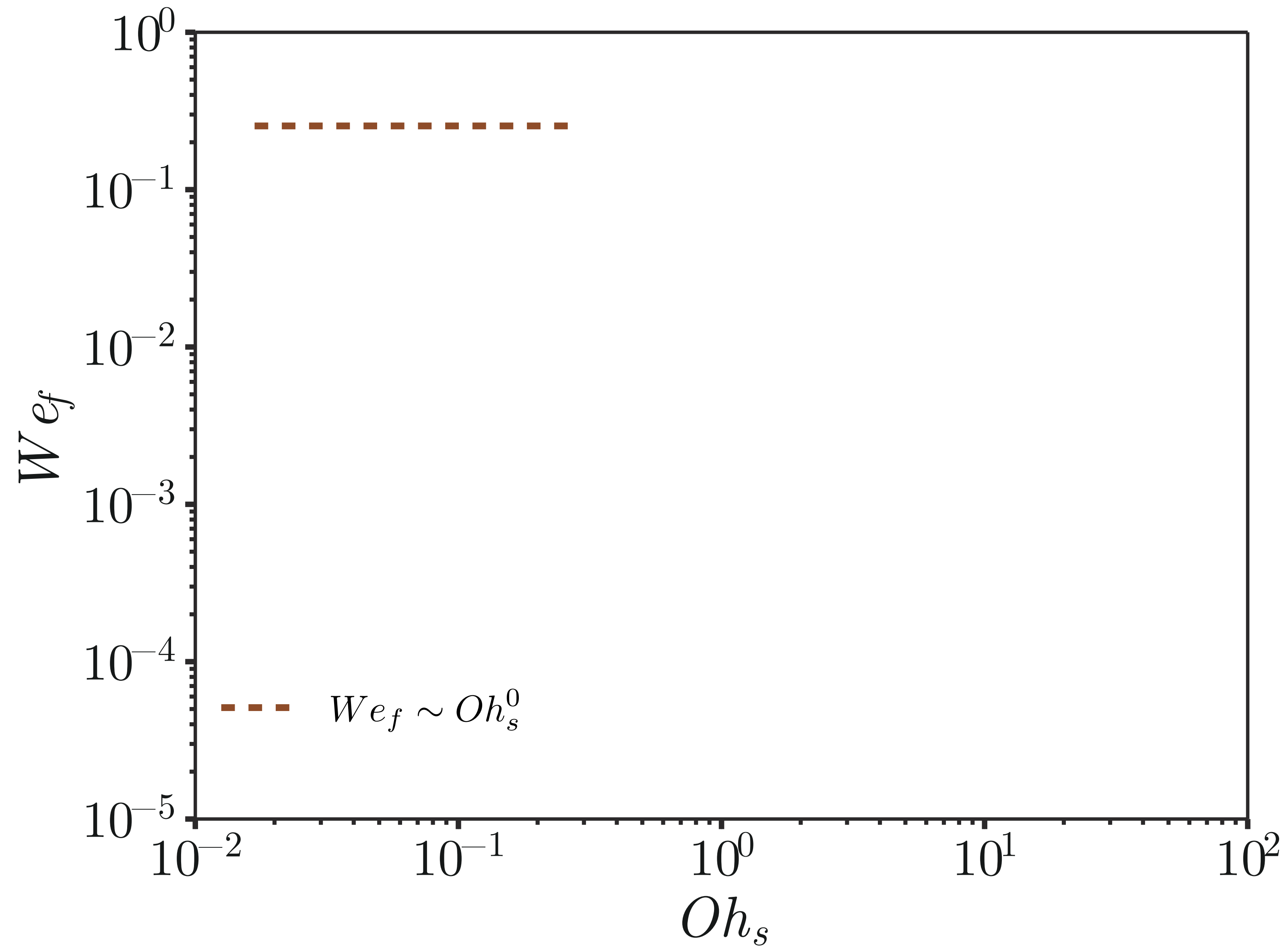
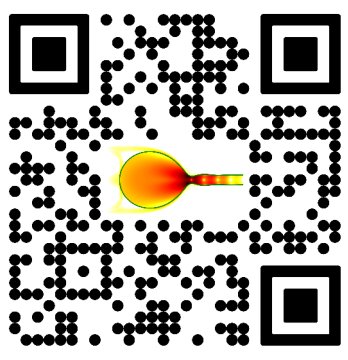
Response:

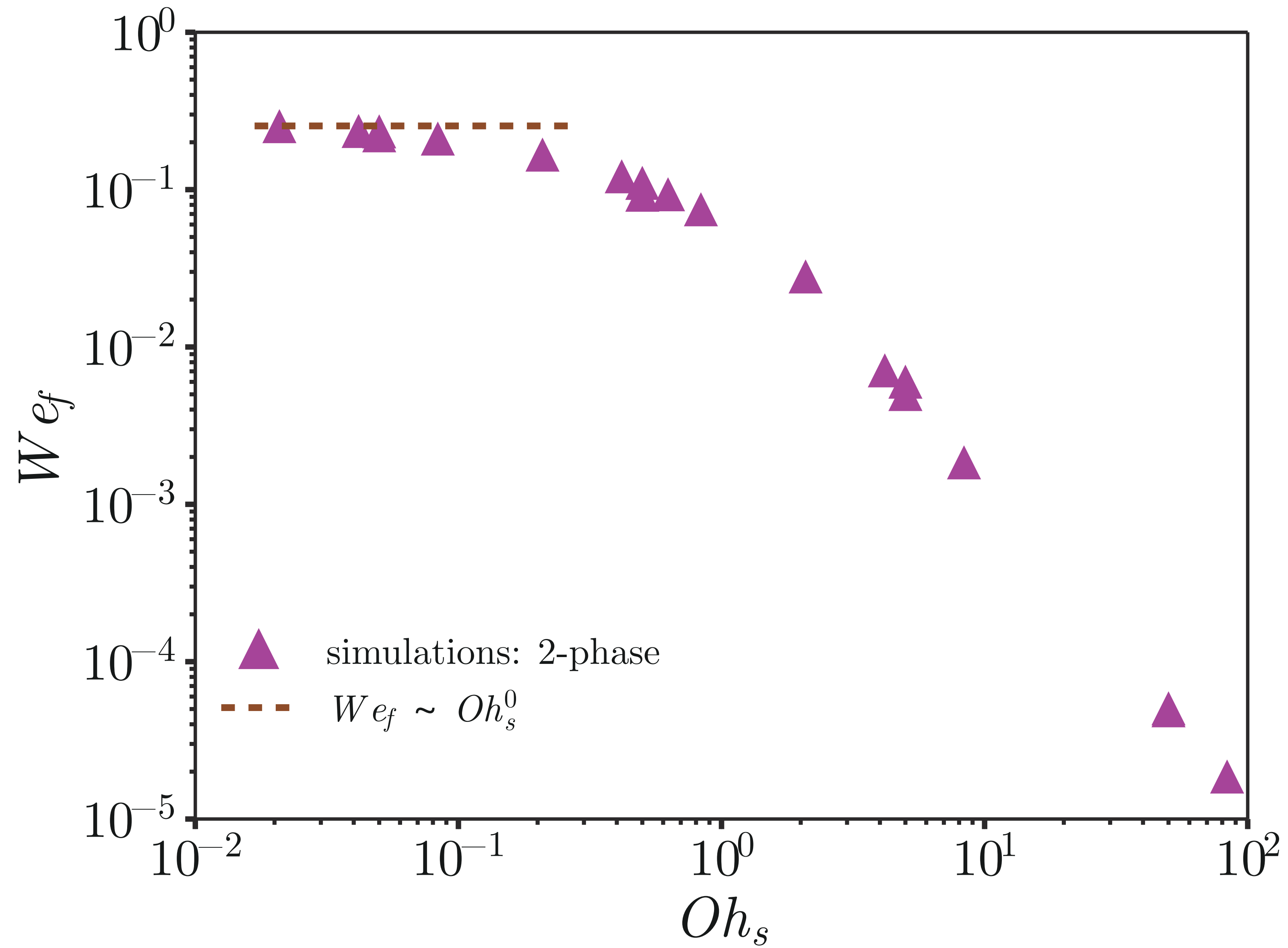
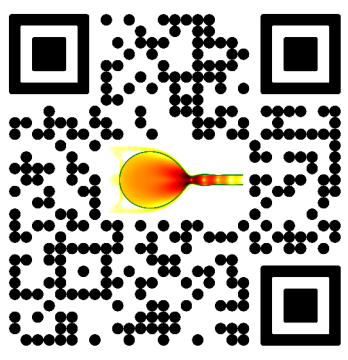
$$We_f = \frac{\rho_f v_f^2 h_0}{2\gamma_{fs}}$$

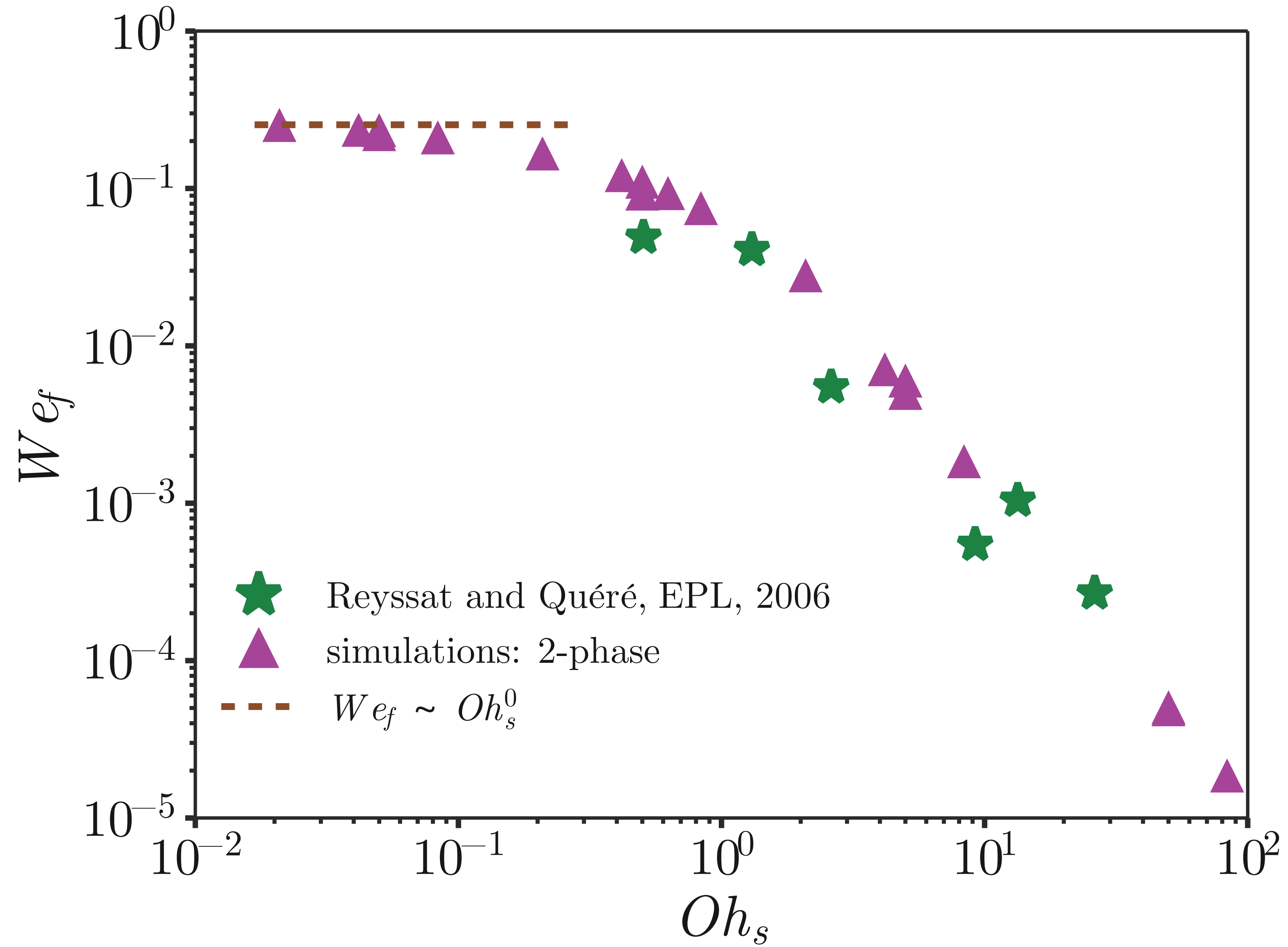
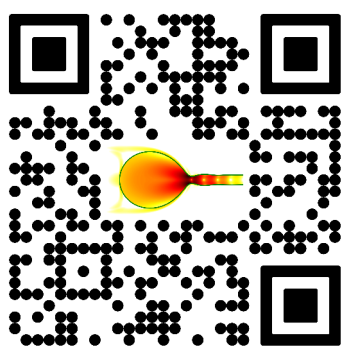
$$\epsilon = 2\eta \mathcal{D}_{ij} \mathcal{D}_{ij}$$

$$\mathcal{D}_{ij} = \frac{1}{2} (\partial_j V_i + \partial_i V_j)$$

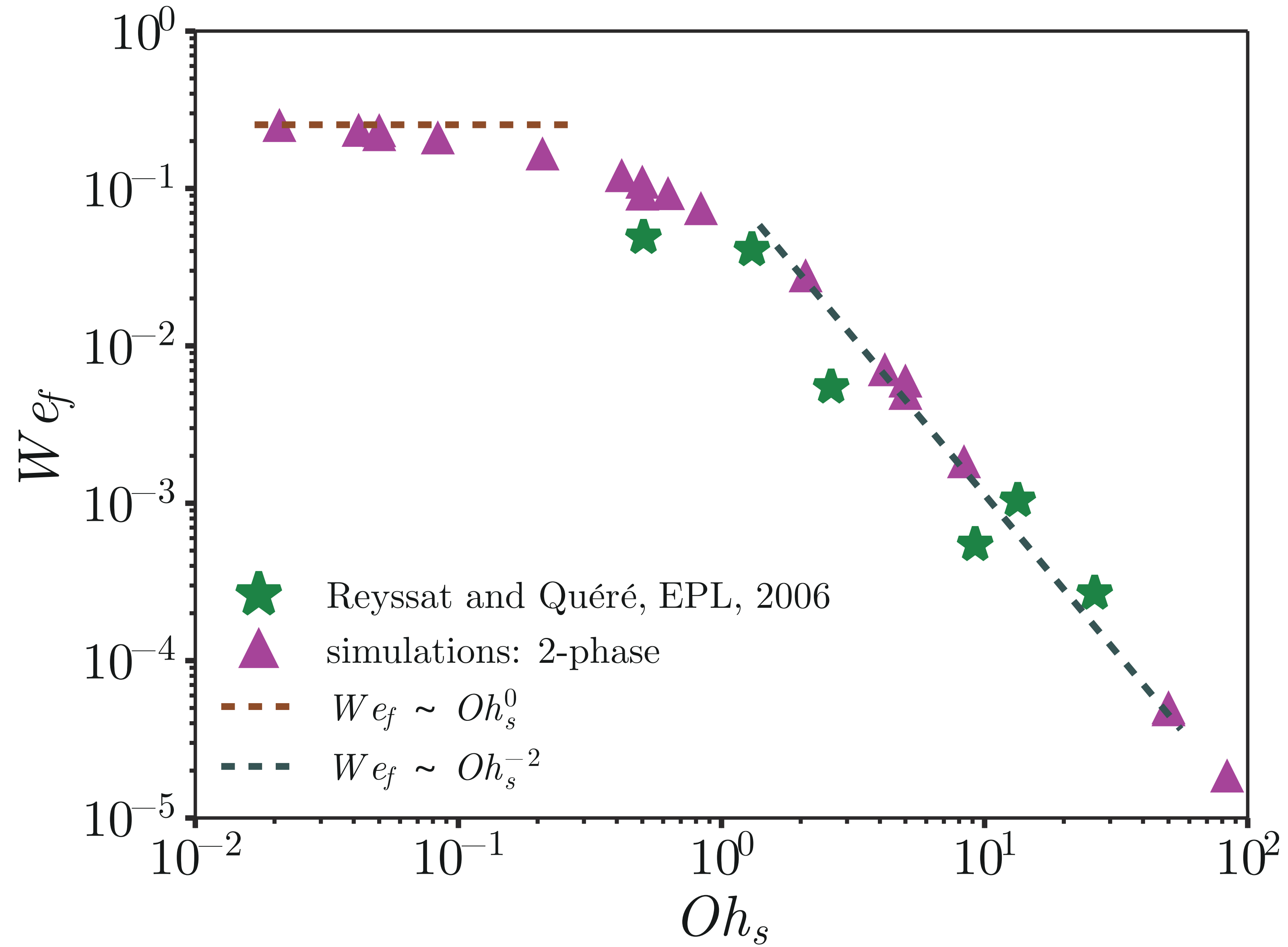
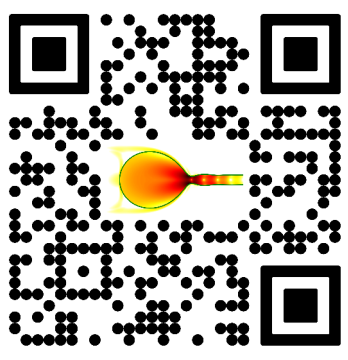


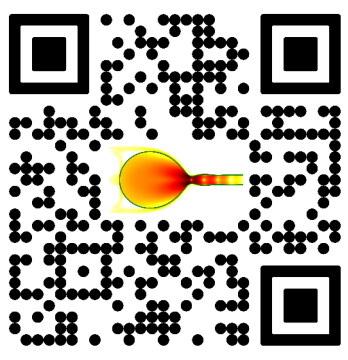












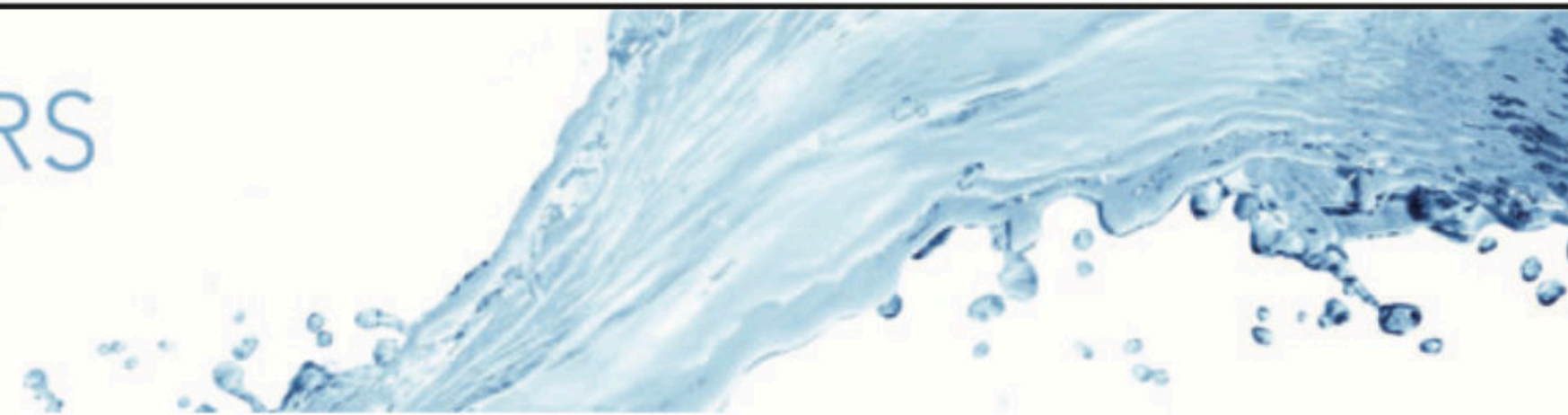
The story continues ...



# The story continues ...

*J. Fluid Mech.* (2022), vol. 948, A14, doi:10.1017/jfm.2022.671

**JM** PAPERS



## Taylor–Culick retractions and the influence of the surroundings

Vatsal Sanjay<sup>1,†</sup>, Uddalok Sen<sup>1,†</sup>, Pallav Kant<sup>1</sup> and Detlef Lohse<sup>1,2,†</sup>

<sup>1</sup>Physics of Fluids Group, Max Planck Center for Complex Fluid Dynamics, Department of Science and Technology, and J. M. Burgers Centre for Fluid Dynamics, University of Twente, P. O. Box 217, 7500 AE Enschede, The Netherlands

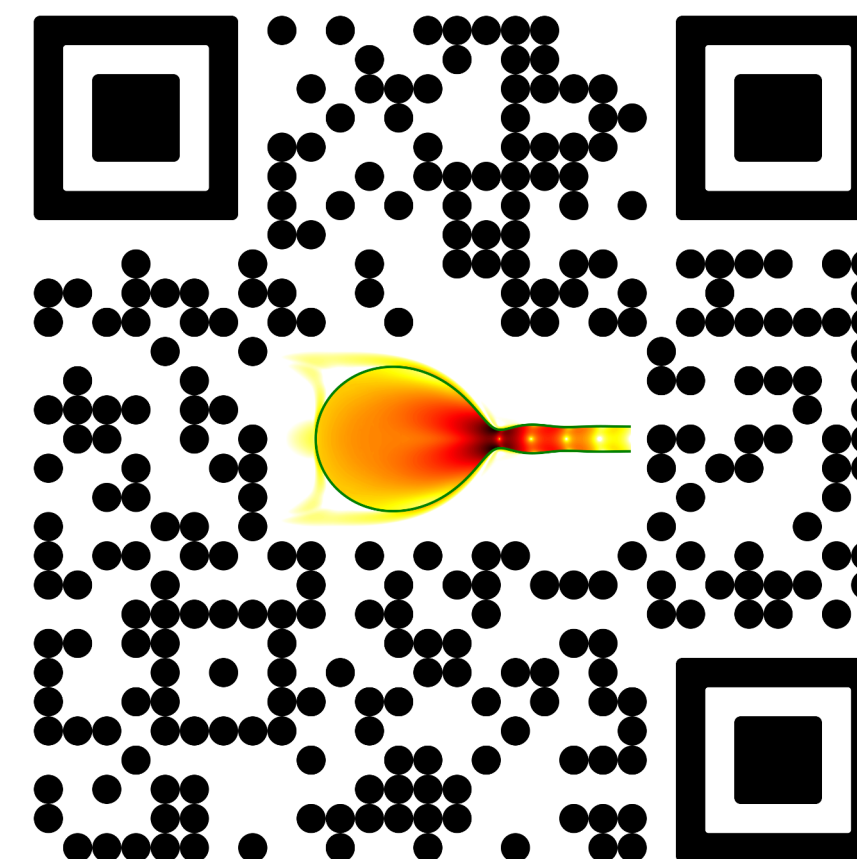
<sup>2</sup>Max Planck Institute for Dynamics and Self-Organization, Am Fassberg 17, 37077 Göttingen, Germany

(Received 27 February 2022; revised 29 May 2022; accepted 18 July 2022)

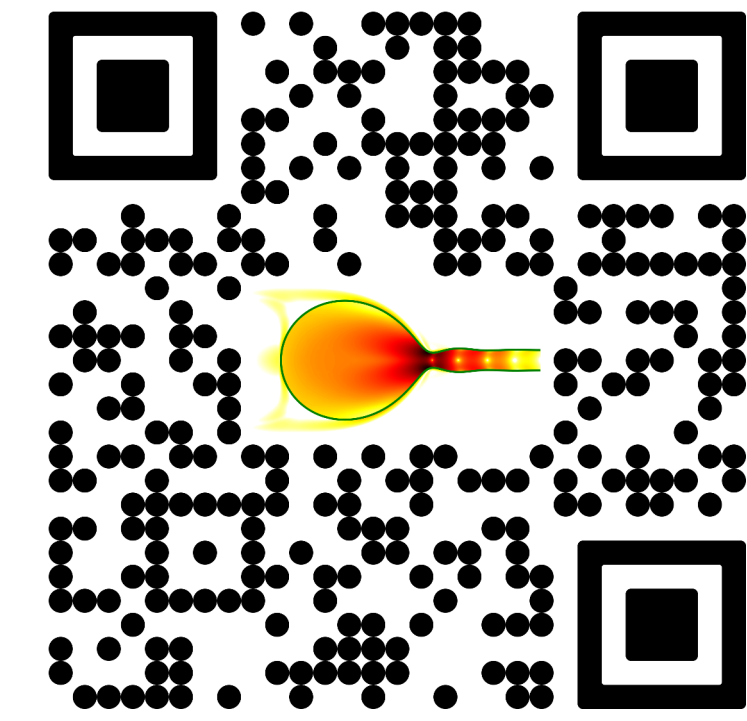


source: [physics.stackexchange.com](https://physics.stackexchange.com)

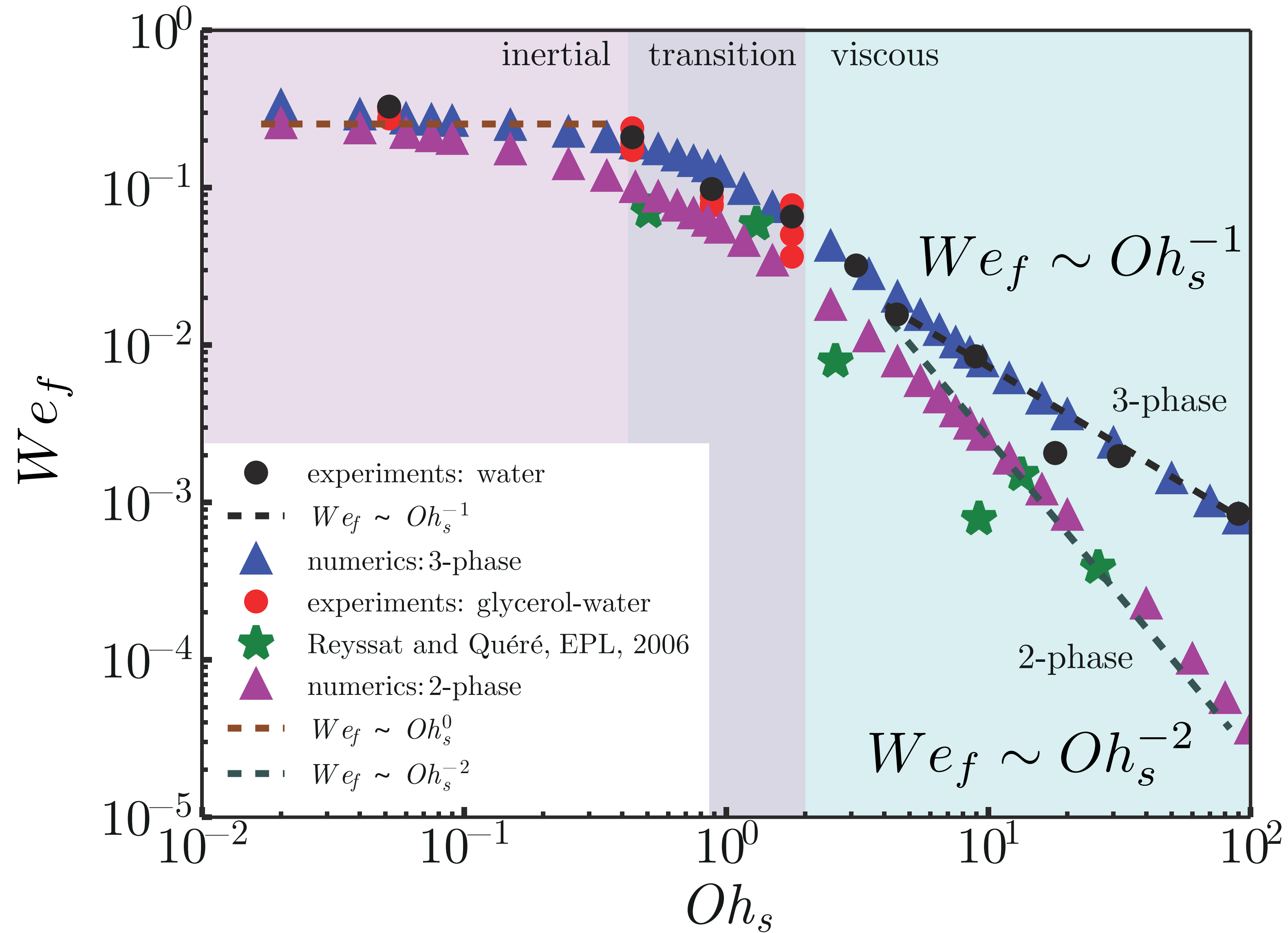
Water films may rupture at the oil-water-air interface, thus dispersing oil droplets further.







# The story continues ...



# More to follow: Elastic Taylor-Culick

$t/t_\gamma = 0.00$

$\|\mathbf{V}\|/V_{TC}$

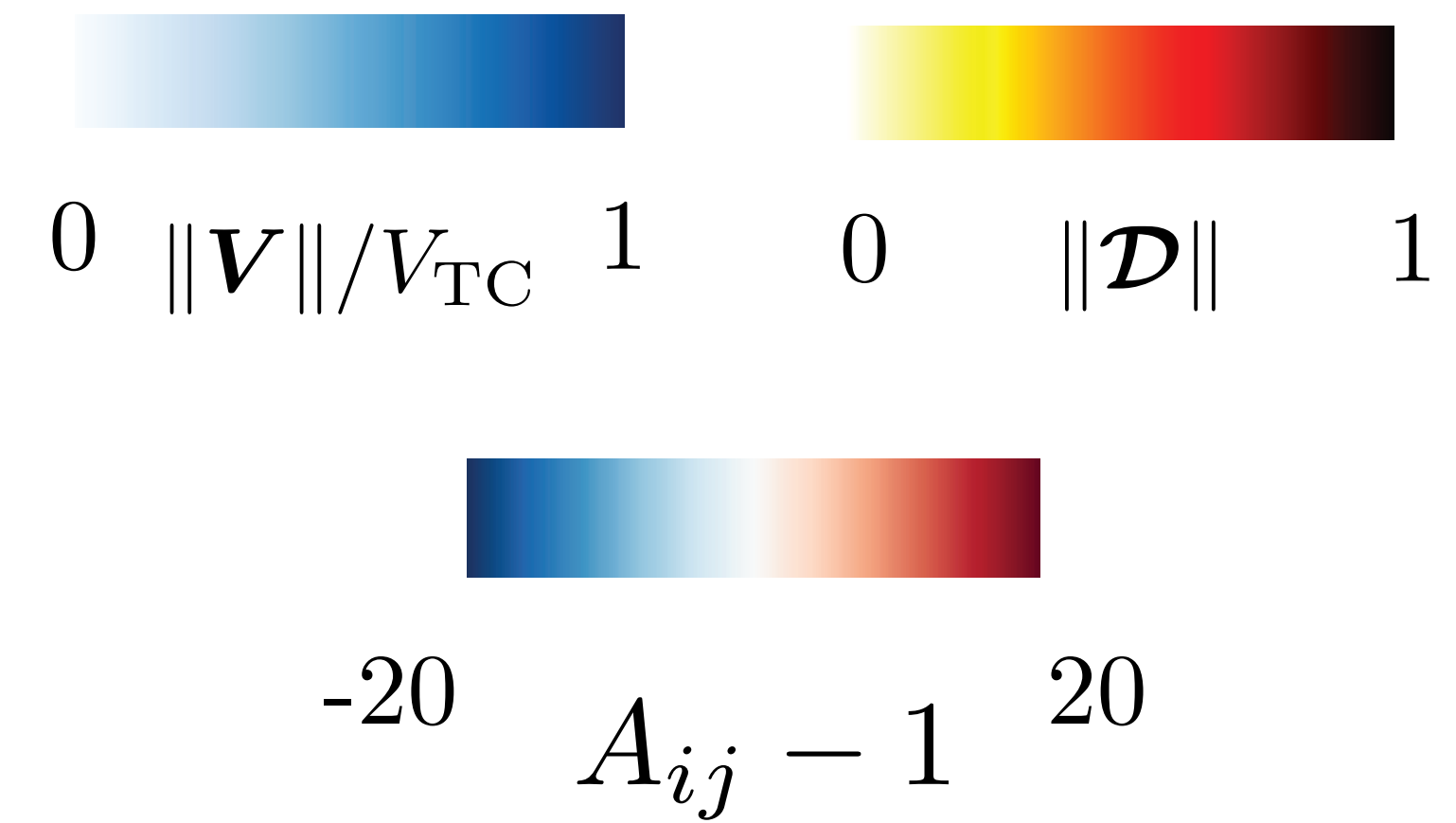
$\|\mathcal{D}\|$

$(A_{rz} - 1)$

$(A_{rr} - 1)$

$(A_{\theta\theta} - 1)$

$(A_{zz} - 1)$



Jacco  
Snoeijer

Vincent  
Bertin

Alex  
Oratis

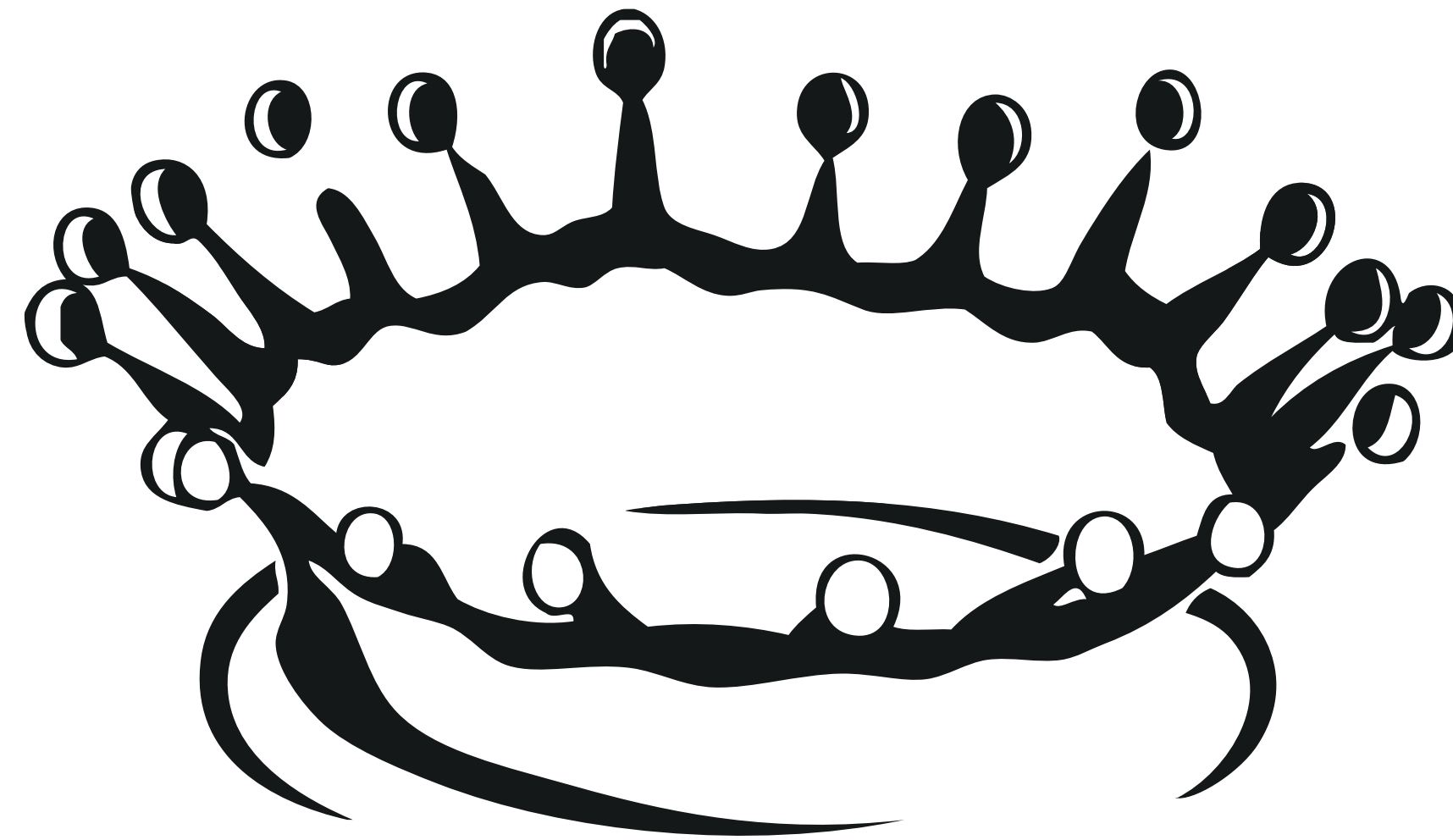
$$Ec = \frac{Gh_0}{\gamma} = 0.01$$

$$De = \lambda / \sqrt{\frac{\rho R_0^3}{\gamma}} \rightarrow \infty$$



One more thing...

# Ph.D. positions across scales



Physics of Fluids



# Ph.D. positions across scales



Physics of Fluids

Flow for future

Flow for Health

Flow for Climate

Flow for HighTech

Flow for Environment

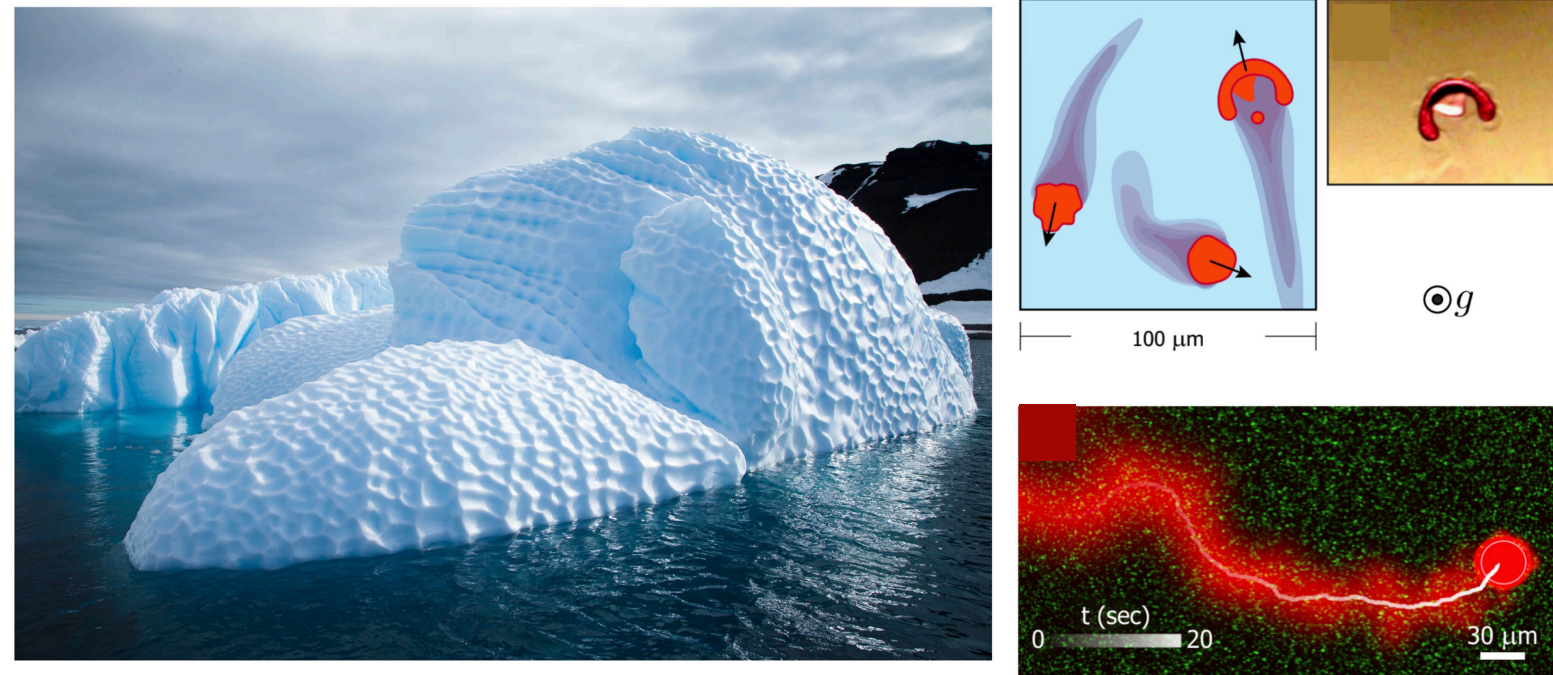
Flow for Energy Transitions

Flow for Agro & Food production

# Ph.D. positions across scales

## MultiMelt

Melting & Dissolution across scales



Physics of Fluids

Flow for future

Flow for Health

**Flow for Climate**

Flow for HighTech

**Flow for Environment**

**Flow for Energy Transitions**

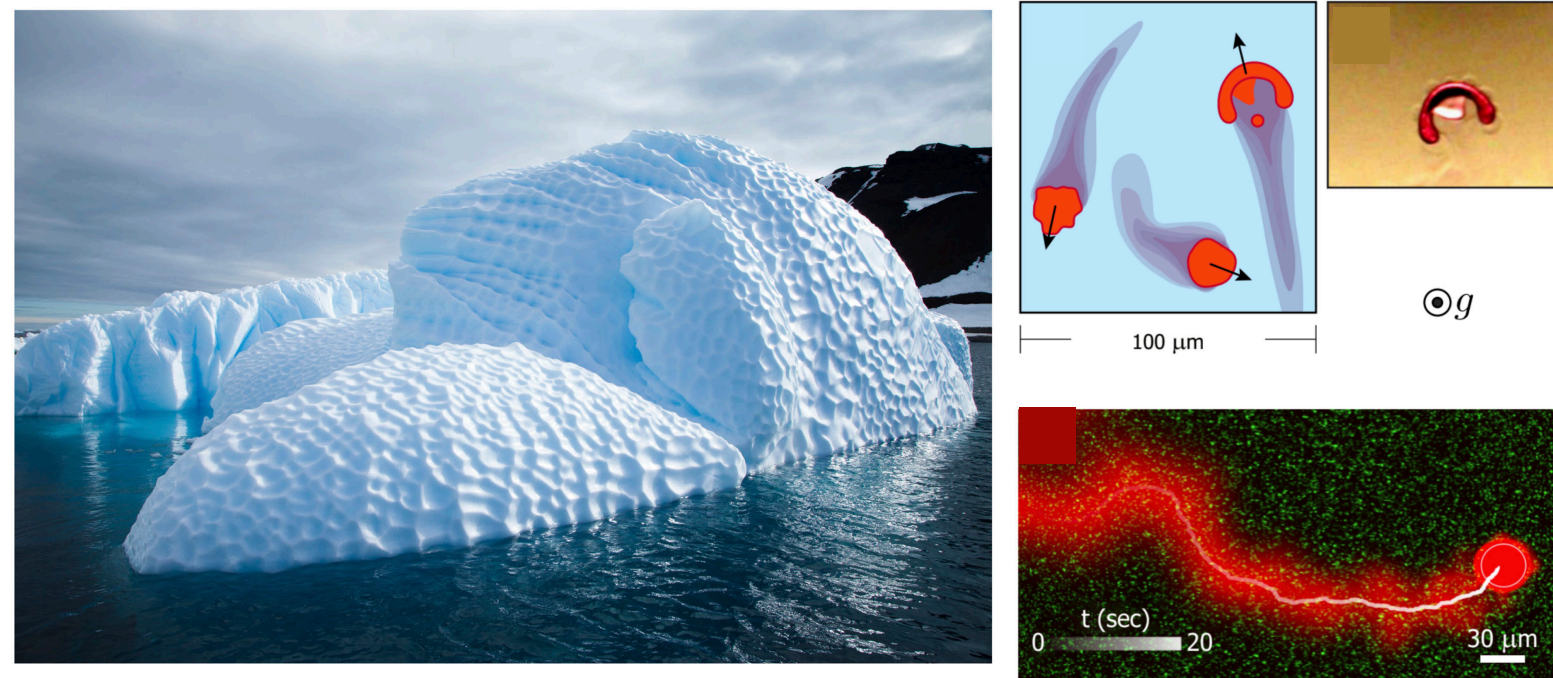
Flow for Agro & Food production



# Ph.D. positions across scales

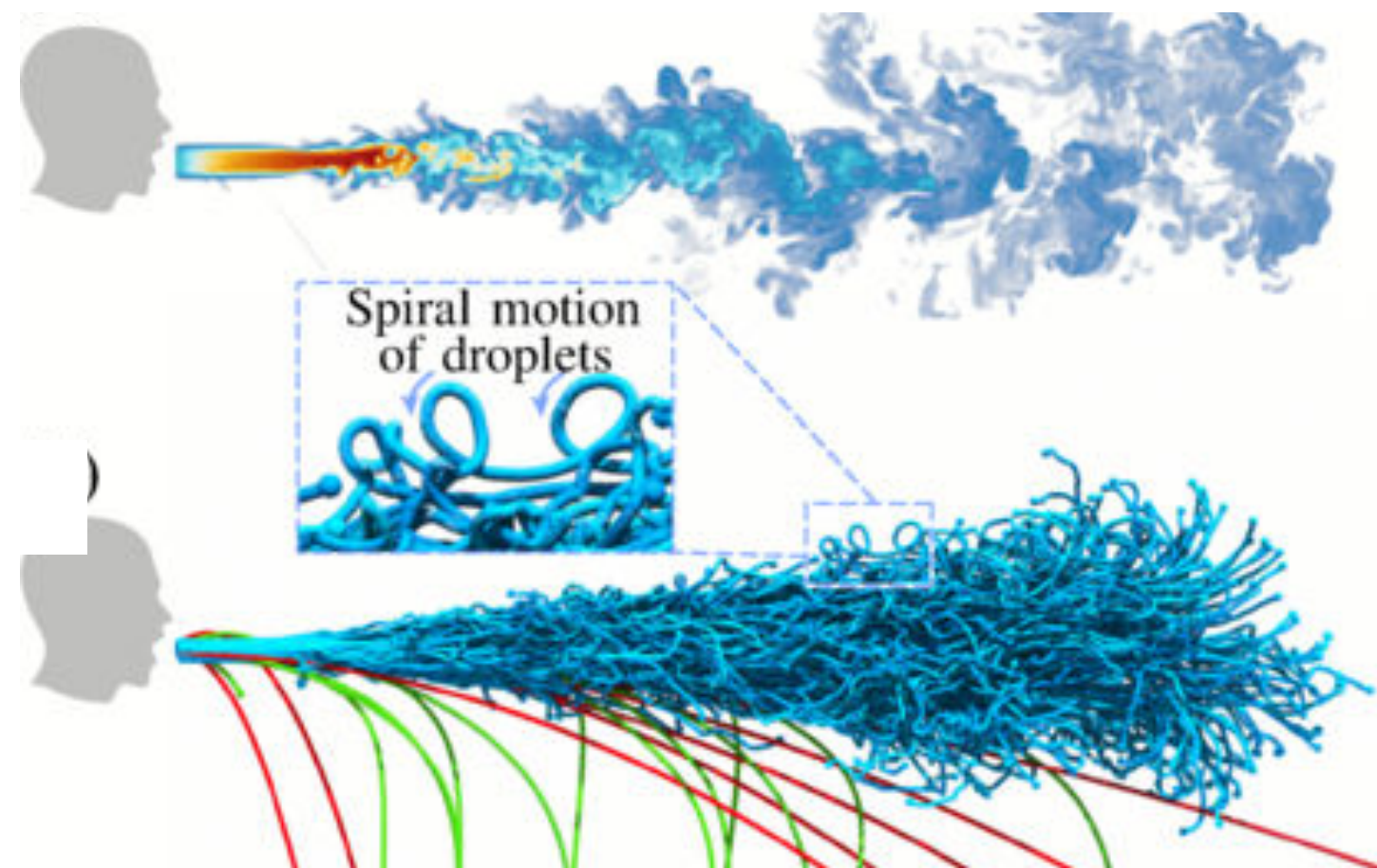
## MultiMelt

Melting & Dissolution across scales



## MIST

Mitigation Strategies for  
Airborne Infection Control



## Physics of Fluids

## Flow for future

**Flow for Health**

**Flow for Climate**

Flow for HighTech

**Flow for Environment**

**Flow for Energy Transitions**

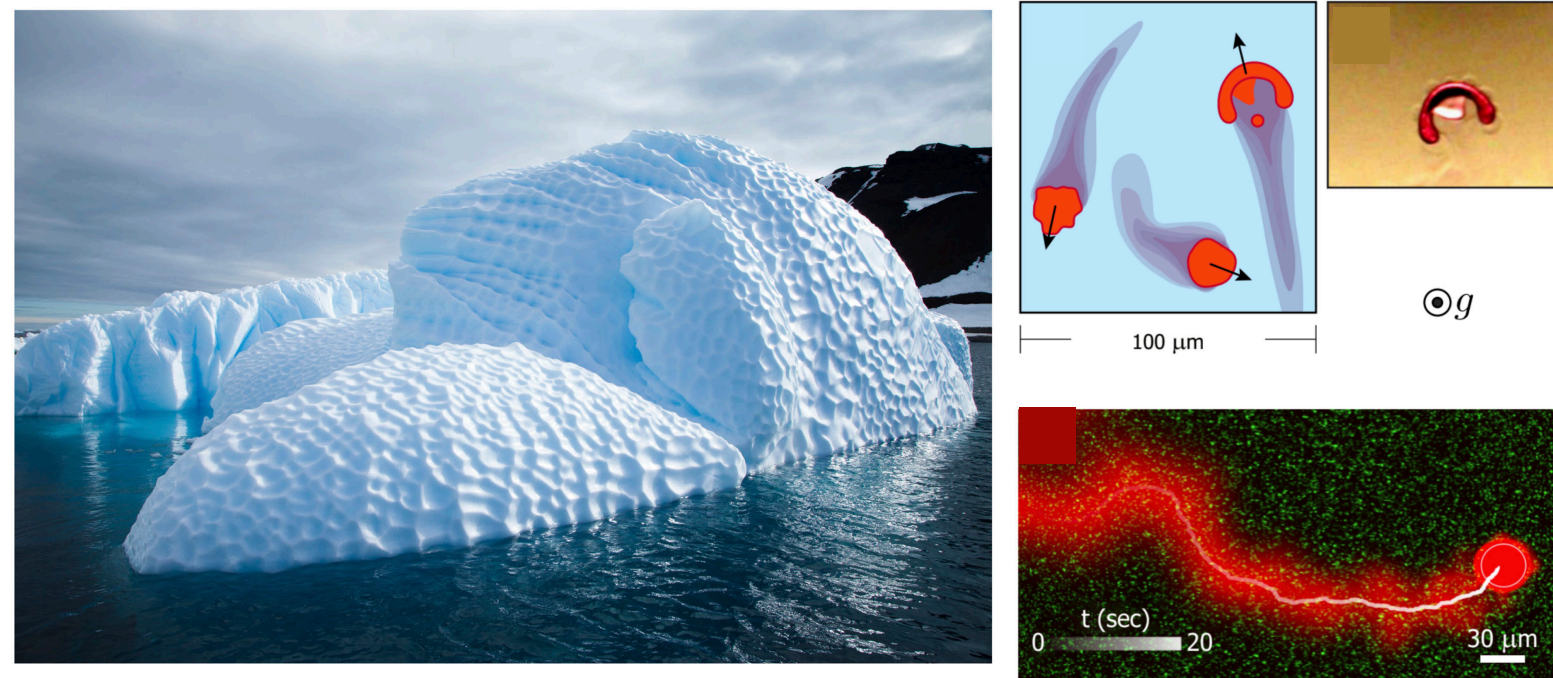
Flow for Agro & Food production



# Ph.D. positions across scales

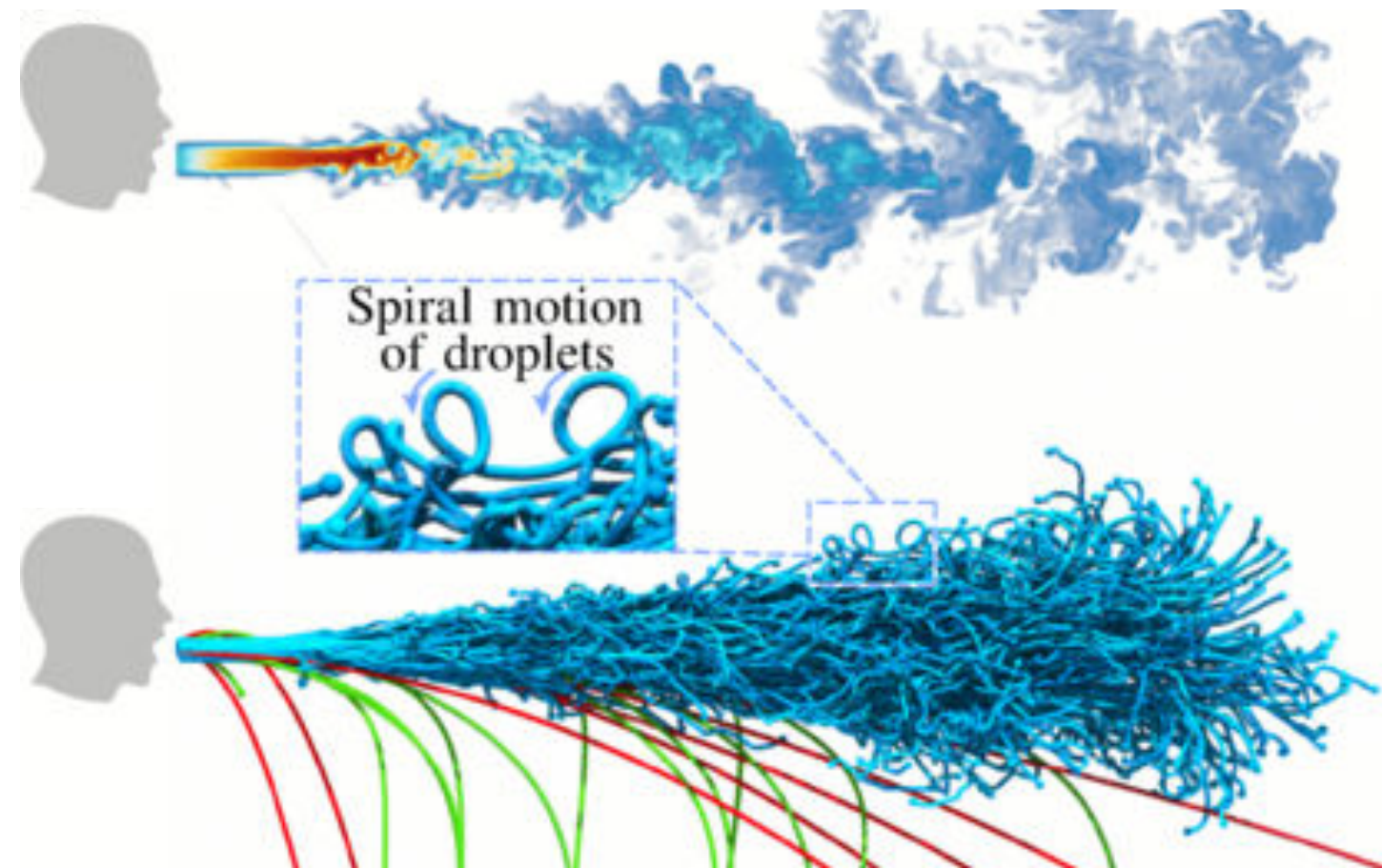
## MultiMelt

Melting & Dissolution across scales



## MIST

Mitigation Strategies for Airborne Infection Control



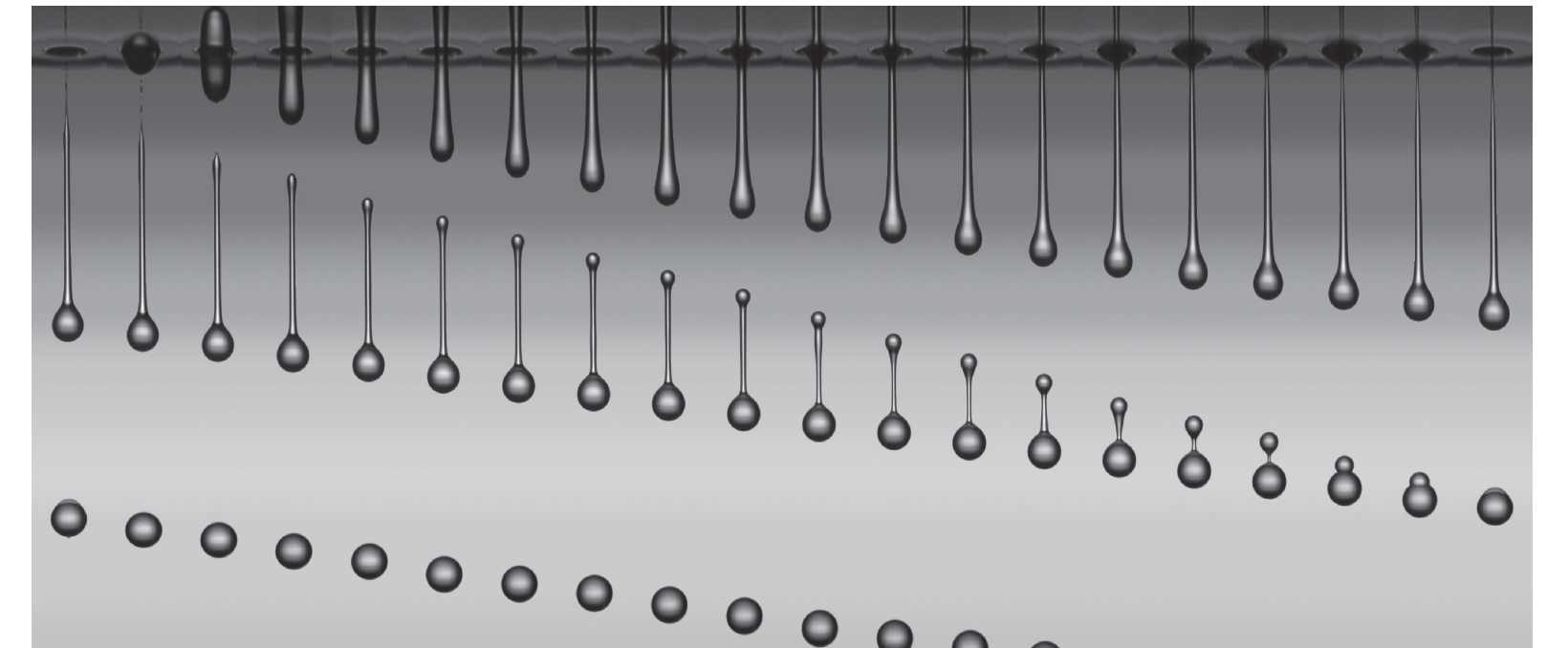
Physics of Fluids

Flow for future

- Flow for Health
- Flow for Climate
- Flow for HighTech
- Flow for Environment
- Flow for Energy Transitions
- Flow for Agro & Food production

## Fundamentals of Inkjet printing-II

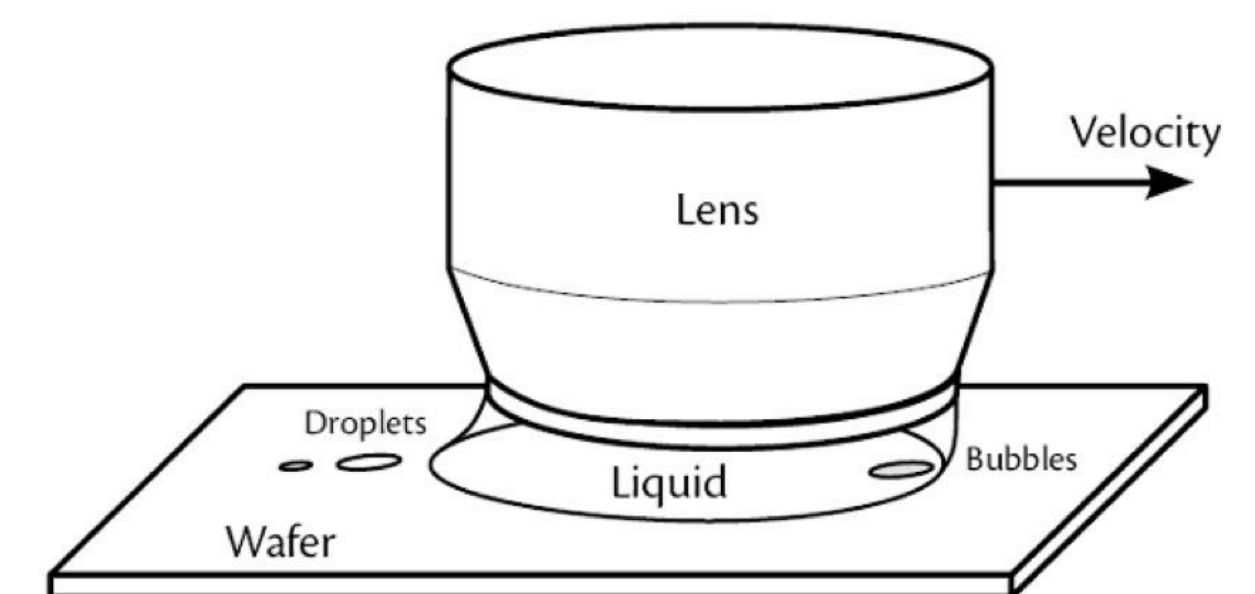
Physicochemical Hydrodynamics of Droplets



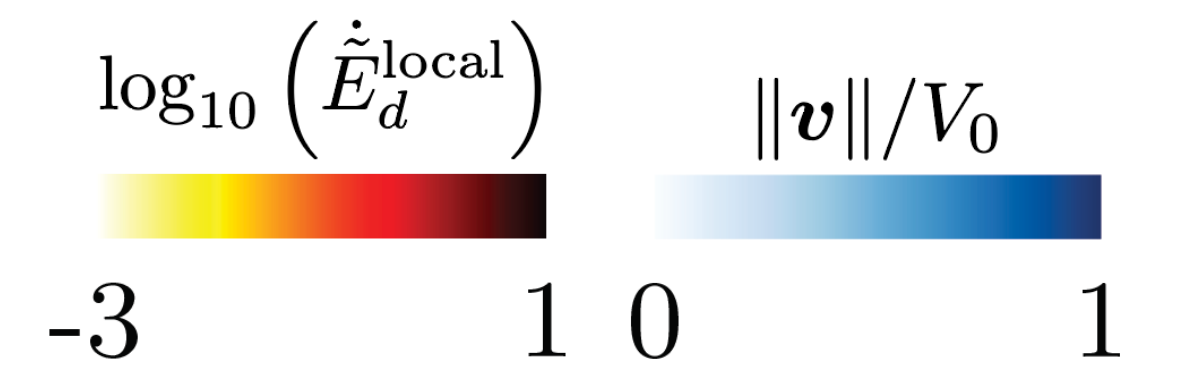
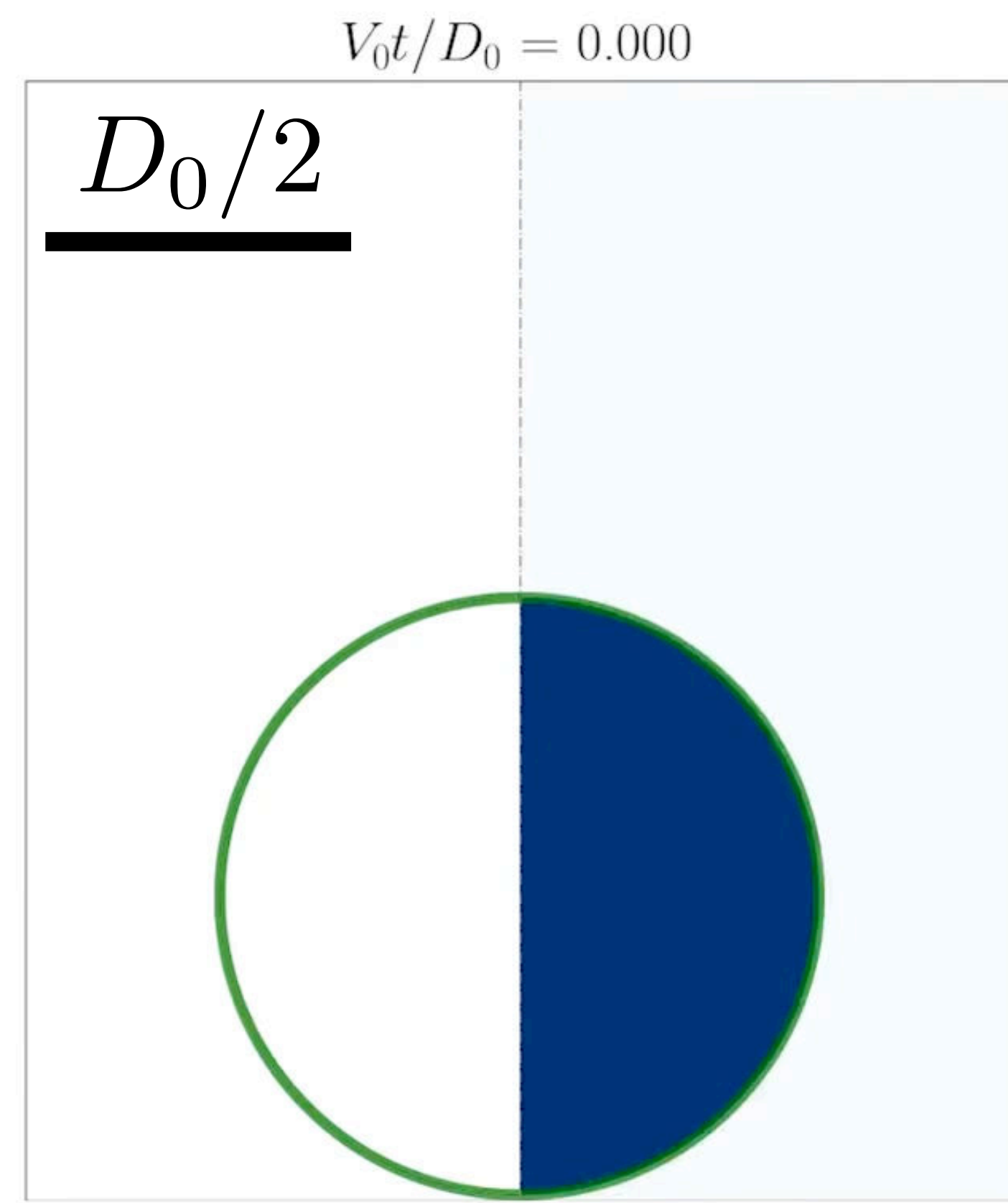
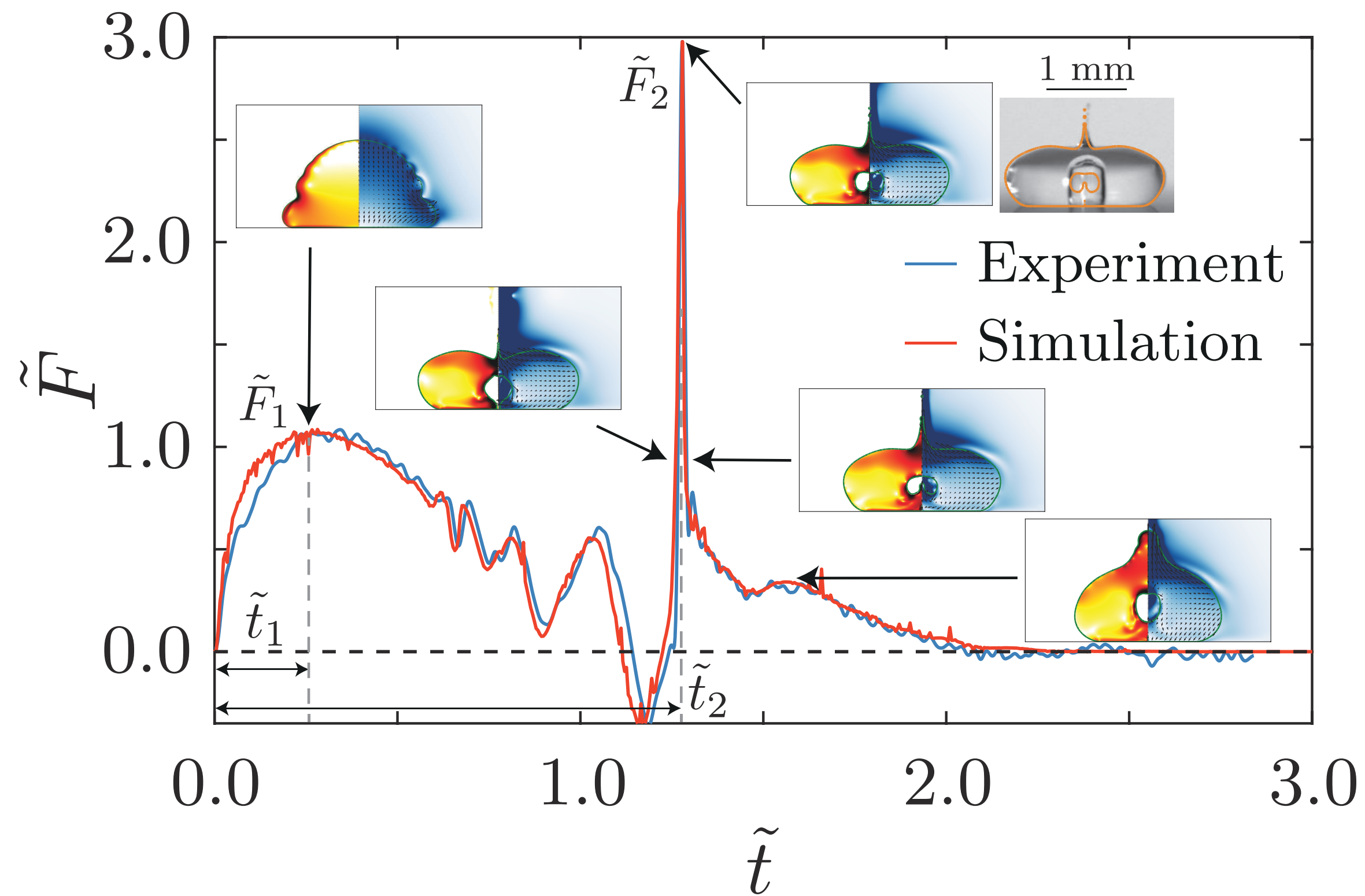
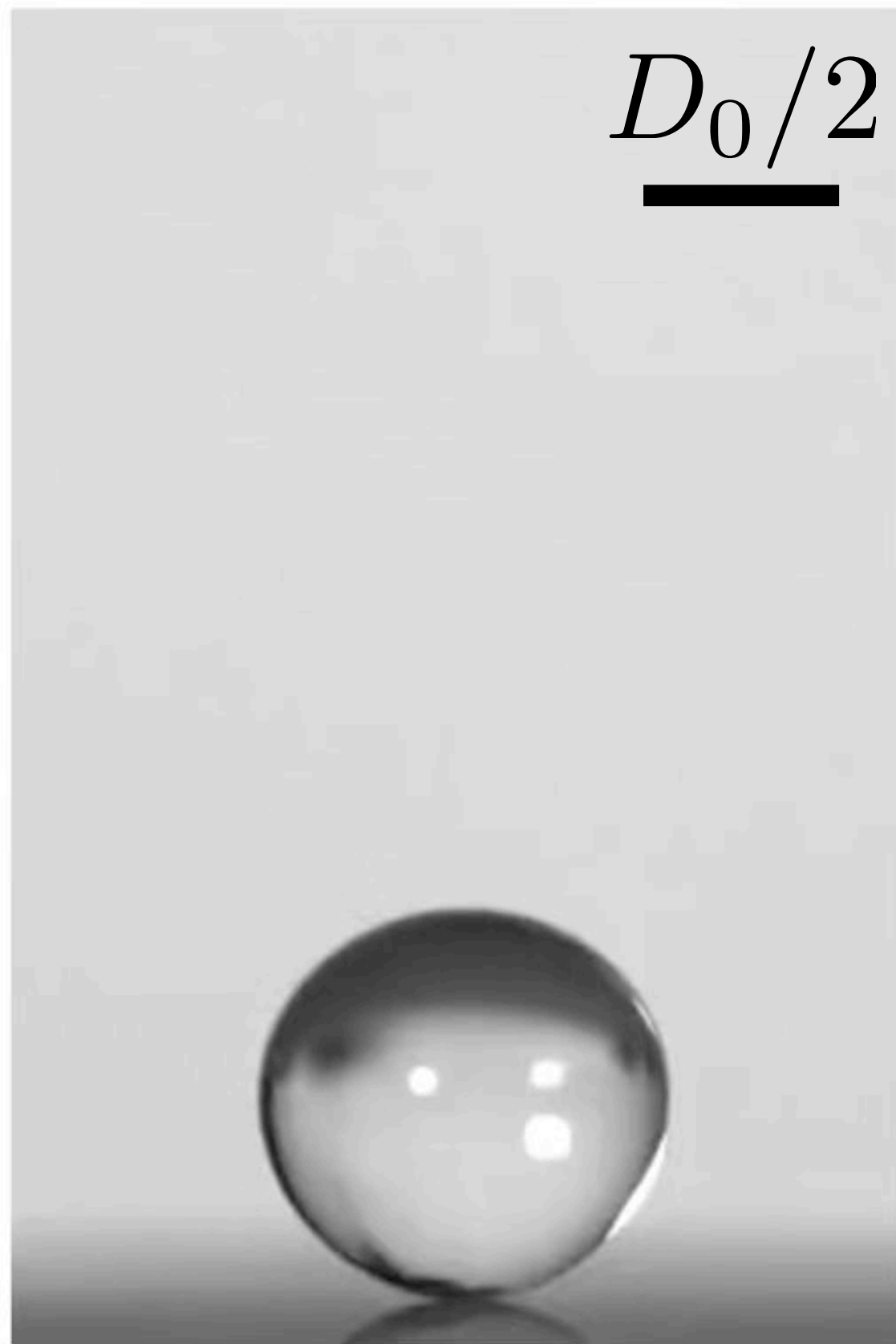
Canon ASML

## Contact lines

Inertial instability







Thank you!

[contact@vatsalsanjay.com](mailto:contact@vatsalsanjay.com)  
[vatsalsanjay.com](http://vatsalsanjay.com)  
[basilisk.fr/sandbox/vatsal/](http://basilisk.fr/sandbox/vatsal/)



Resources

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- **Vincent Bertin**
- Youssef Saade (Canon)\*
- Mandeep Saini\*
- **Pallav Kant (Univ. of Cambridge)**
- Jonathan Pham (Univ. of Kentucky)
- Doris Vollmer (MPI-Mainz)

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**Physics of Fluids**



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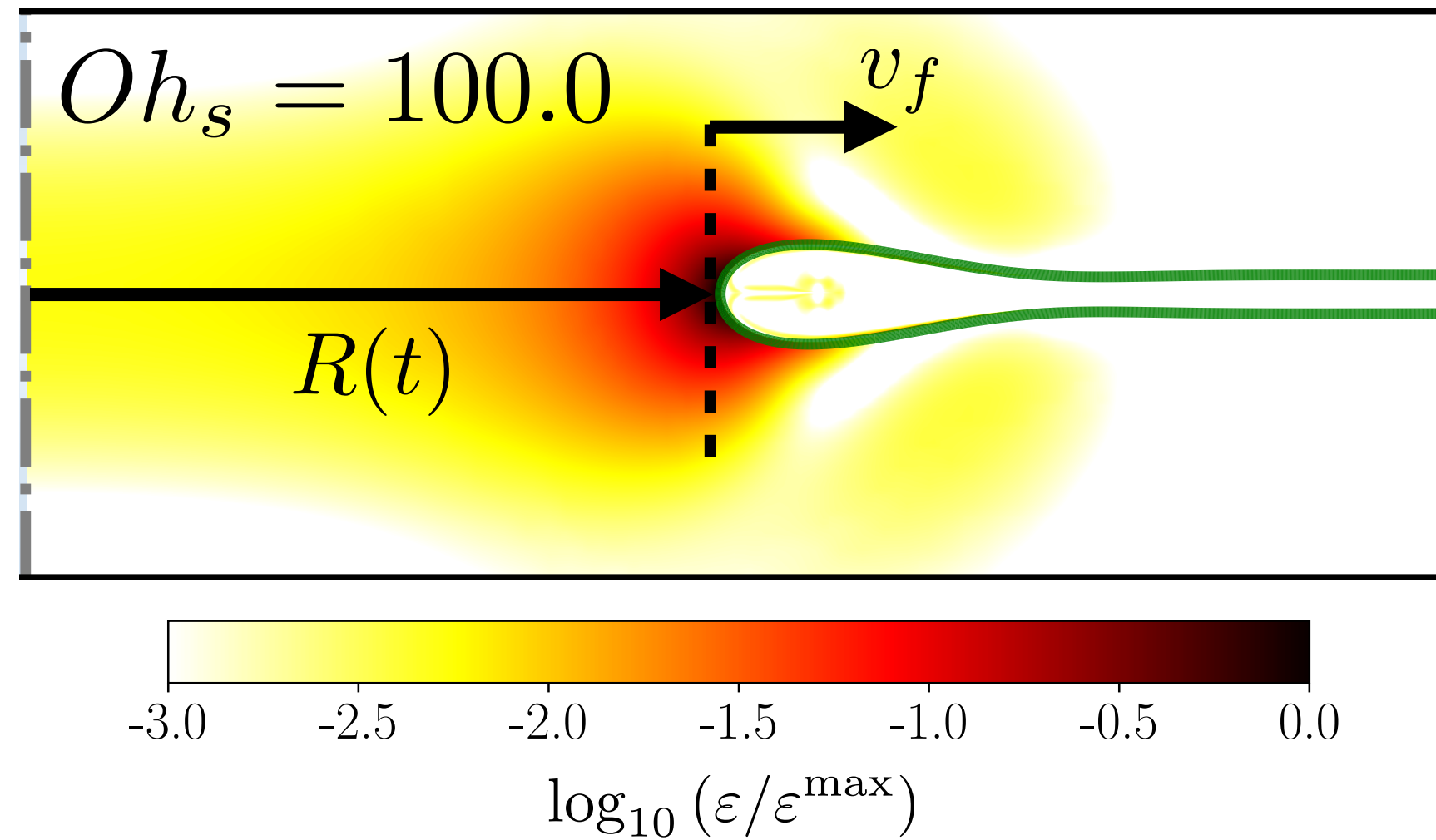


How can we understand the scaling for retraction velocity?

$$We_f \sim Oh_s^{-2}$$

# Dynamics: 2 phase

Capillary force during retraction vs. Viscous resistance



$$F_\gamma \sim F_\eta$$

Stokes flow/Oseen approximation

$$2\gamma_{fs} \sim \eta_s v_f$$

$$Ca_s \sim Oh_s^0$$

$$v_f = v_s$$

$$We_f \sim Oh_s^{-2}$$

$$Oh_s = \frac{\eta_s}{\sqrt{\rho_f (2\gamma_{fs}) h_0}}$$

$$Ca_s = \frac{\eta_s v_s}{2\gamma_{fs}}$$

$$We_f = \frac{\rho_f v_f^2 h_0}{2\gamma_{fs}}$$

Fraaije and Cazabat, JCIS 133, 1989

Reddy et al., PRF 5, 2020

# 2-phase Taylor-Culick retraction

$$Oh_f = \frac{\eta_f}{\sqrt{\rho_f (2\gamma_{fs}) h_0}} \text{ independent}$$

