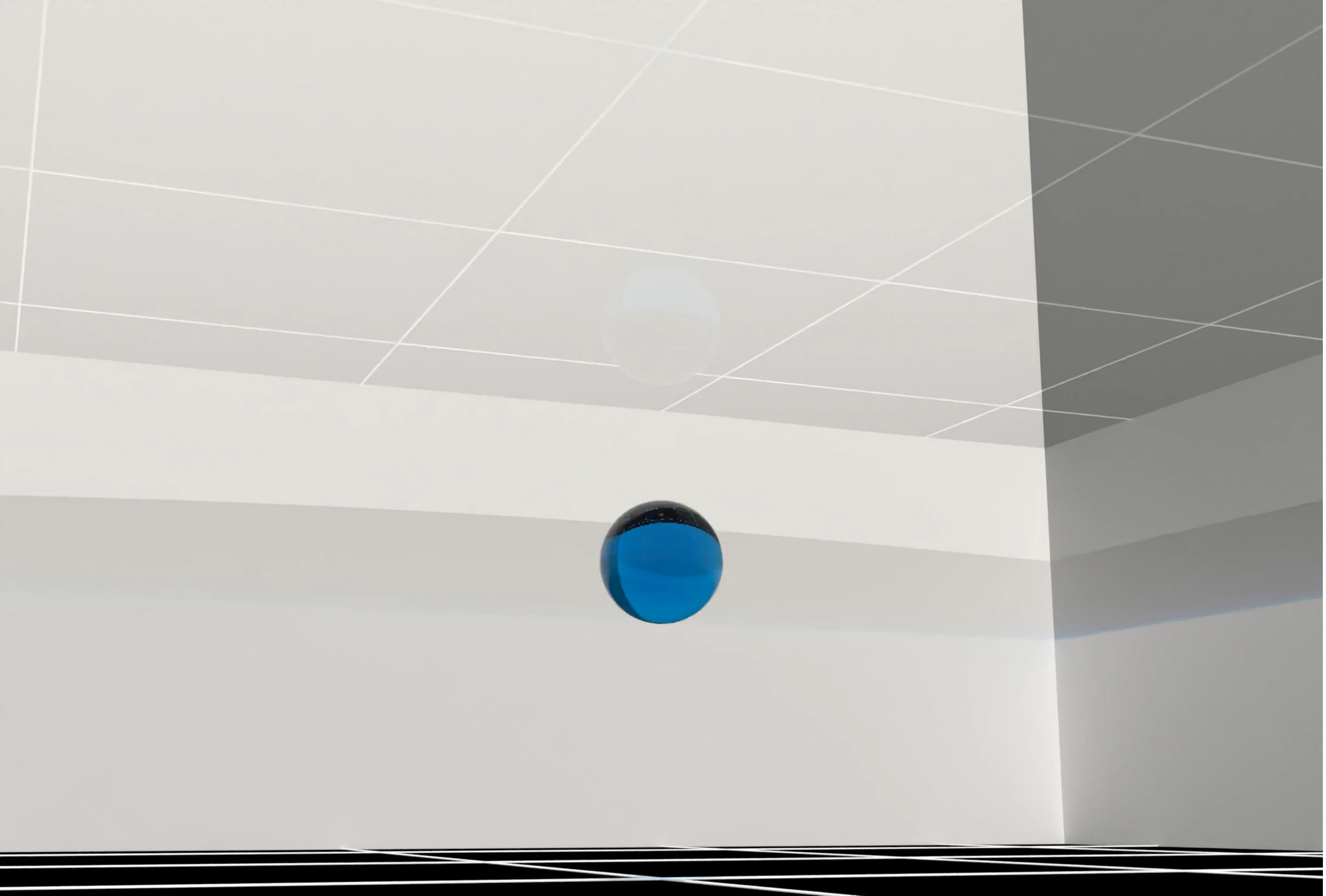
A multigrid solver for the coupled pressure-temperature equations in an all-Mach solver with VoF


Youssef Saade


Detlef Lohse


Daniel Fuster



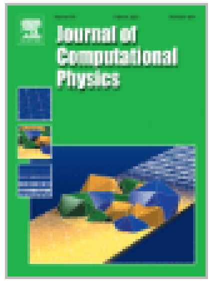
Crown formation from a cavitating bubble close to a free surface

Saade *et al.*,
J. Fluid Mech.,
2021.

ELSEVIER

Journal of Computational Physics

Volume 476, 1 March 2023, 111865



A multigrid solver for the coupled pressure-temperature equations in an all-Mach solver with VoF

Youssef Saade [a], Detlef Lohse [a] [b], Daniel Fuster [c]

ysaade

→ allMach

Governing equations

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \boldsymbol{u}_i) = 0,$$

$$\frac{\partial \overline{\rho\, \boldsymbol{u}}}{\partial t} + \nabla \cdot (\overline{\rho\, \boldsymbol{u}}\; \overline{\boldsymbol{u}}\;) = \nabla \cdot \overline{\boldsymbol{\tau}} \;\; + \sigma \kappa \delta_s \boldsymbol{n},$$

$$\boldsymbol{\tau}_i = -\left(p_i + \frac{2}{3}\mu_i \nabla \cdot \boldsymbol{u}_i\right) \boldsymbol{I} + \mu_i \left(\nabla \boldsymbol{u}_i + \nabla \boldsymbol{u}_i^T\right),$$

$$\frac{\partial}{\partial t}\left[\rho_i \left(e_i + \frac{1}{2}\boldsymbol{u}_i^2\right)\right] + \nabla \cdot \left[\rho_i \left(e_i + \frac{1}{2}\boldsymbol{u}_i^2\right) \boldsymbol{u}_i\right] = \nabla \cdot (\boldsymbol{\sigma}_i \cdot \boldsymbol{u}_i) - \nabla \cdot \boldsymbol{q}_i,$$

$$\boldsymbol{q}_i = -\kappa_i \nabla T_i.$$

Governing equations

$$\rho_i c_{p,i} \frac{DT_i}{Dt} = \beta_i T_i \frac{Dp_i}{Dt} - \nabla \cdot \boldsymbol{q}_i,$$

$$d\rho = \left(\frac{\partial \rho}{\partial p}\right)_T dp + \left(\frac{\partial \rho}{\partial T}\right)_p dT = \frac{\gamma}{c^2} dp - \rho \beta dT,$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \boldsymbol{u}_i) = 0,$$

$$\left(\frac{\gamma_i}{\rho_i c_i^2} - \frac{\beta_i^2 T_i}{\rho_i c_{p,i}}\right) \frac{Dp_i}{Dt} = -\frac{\beta_i}{\rho_i c_{p,i}} \nabla \cdot \boldsymbol{q}_i - \nabla \cdot \boldsymbol{u}_i.$$

Noble-Abel Stiffened Gas EOS

$$\rho_i e_i = \frac{p_i + \Gamma_i \Pi_i}{\Gamma_i - 1}(1 - \rho_i b_i) + \rho_i q_i,$$

$$c_i^2 = \frac{\Gamma_i (p_i + \Pi_i)}{\rho_i (1 - \rho_i b_i)}.$$

Test case I: Epstein-Plesset like problem for temperature

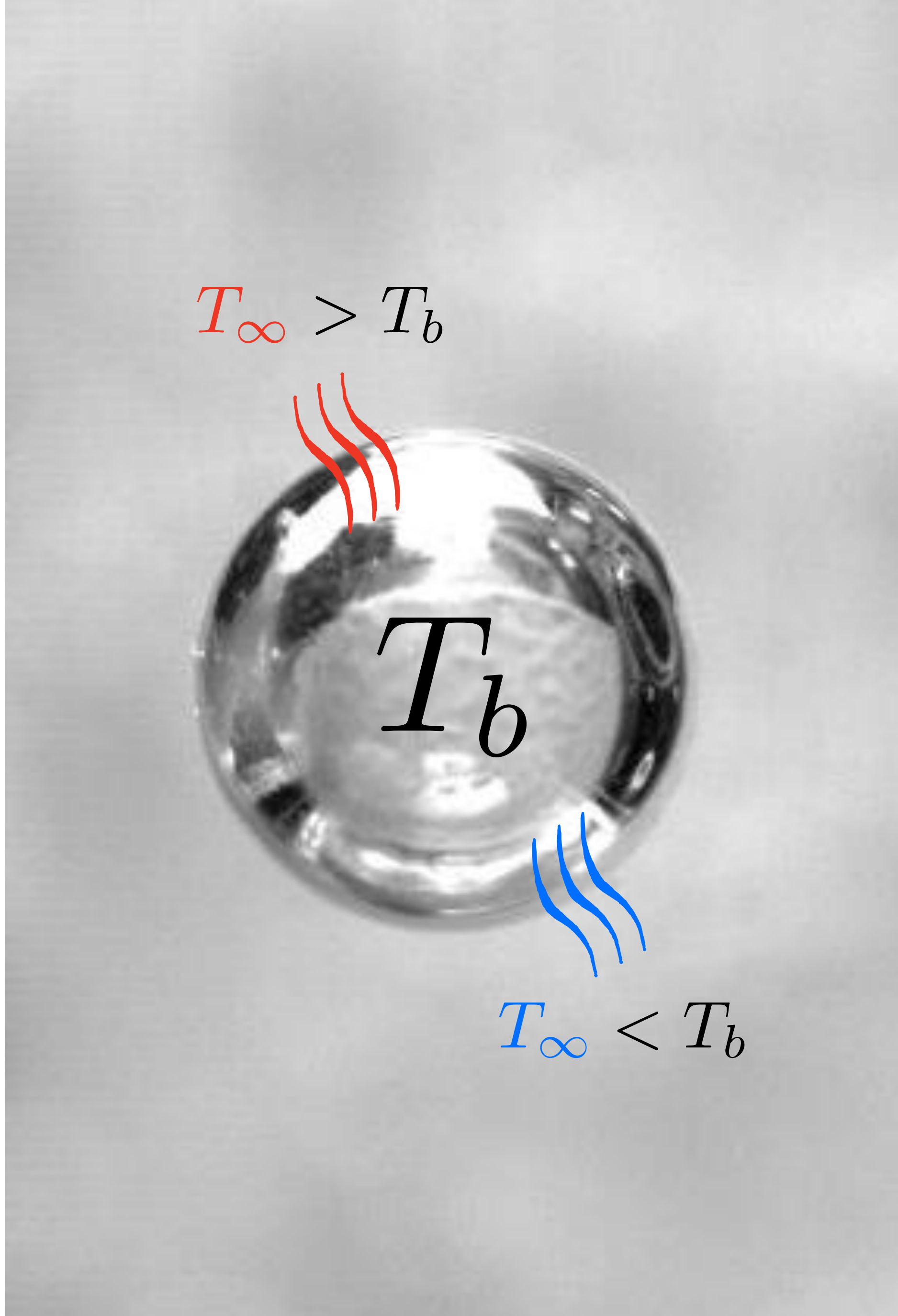
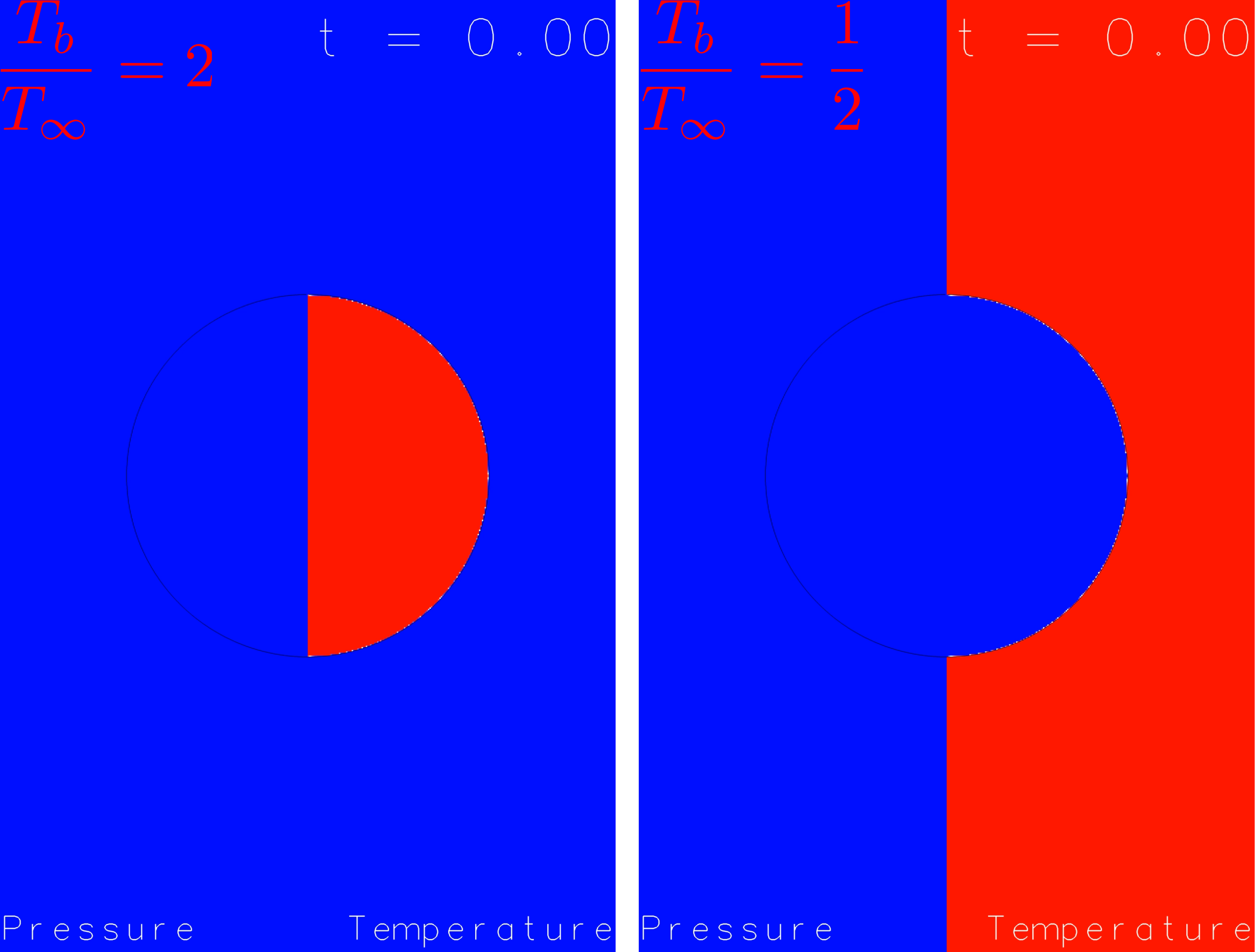


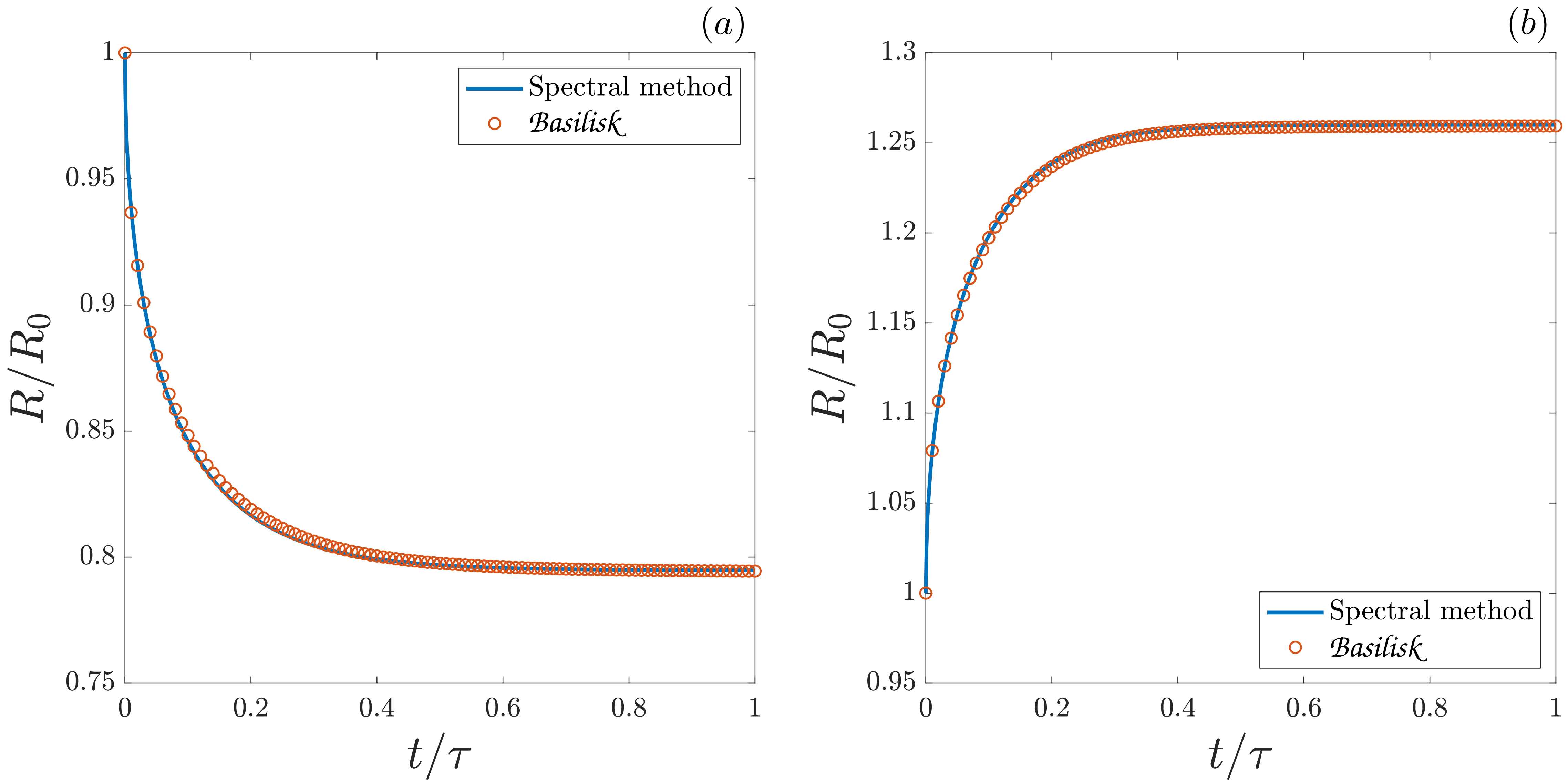
Image credit: Outi Supponen


$\frac{T_b}{T_\infty} = 2$
t = 0.00
Pressure
Temperature
$\frac{T_b}{T_\infty} = \frac{1}{2}$
t = 0.00
Pressure
Temperature

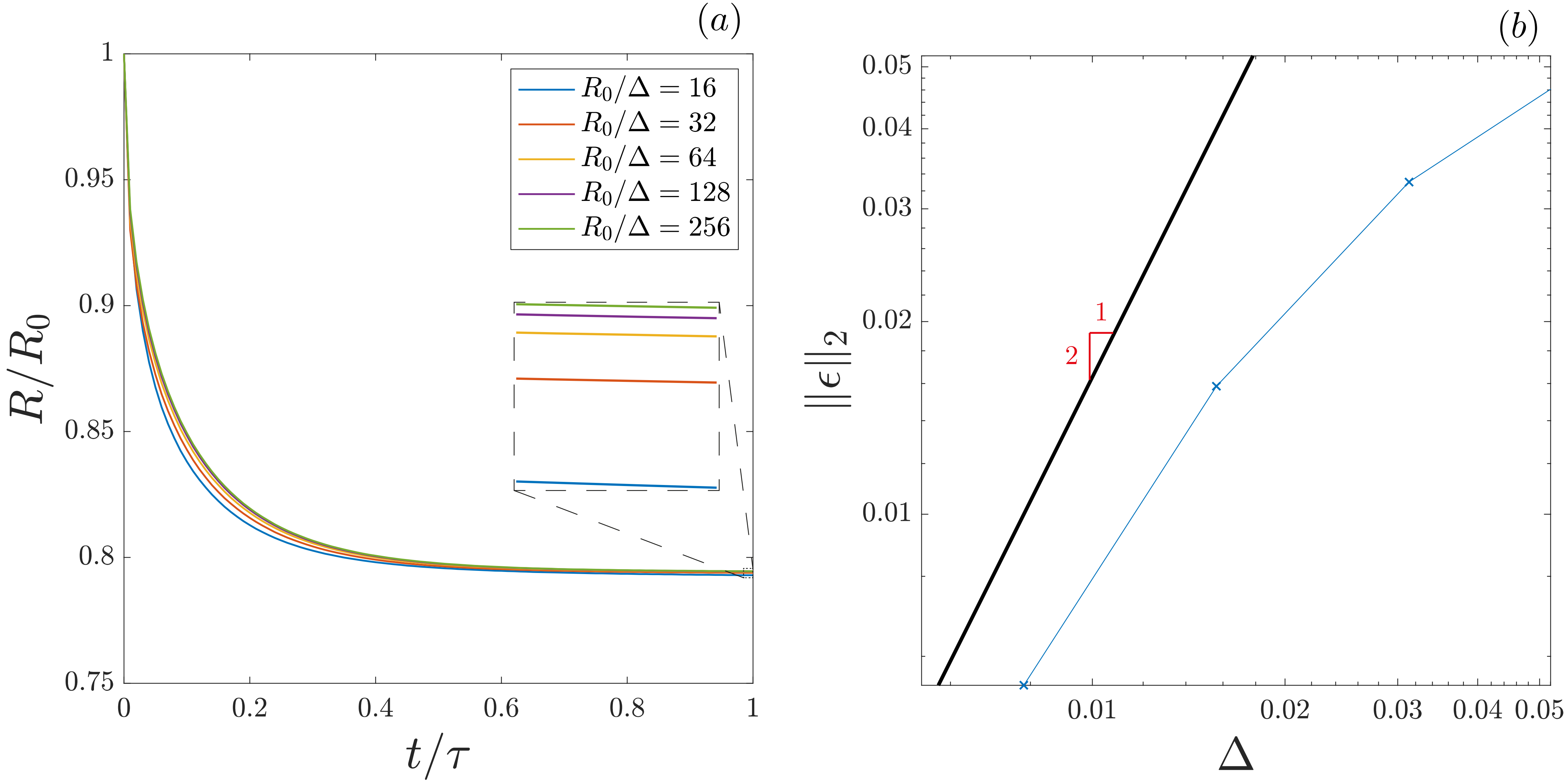
Test Case I

Test case I

Stricker, Prosperetti & Lohse (2011)

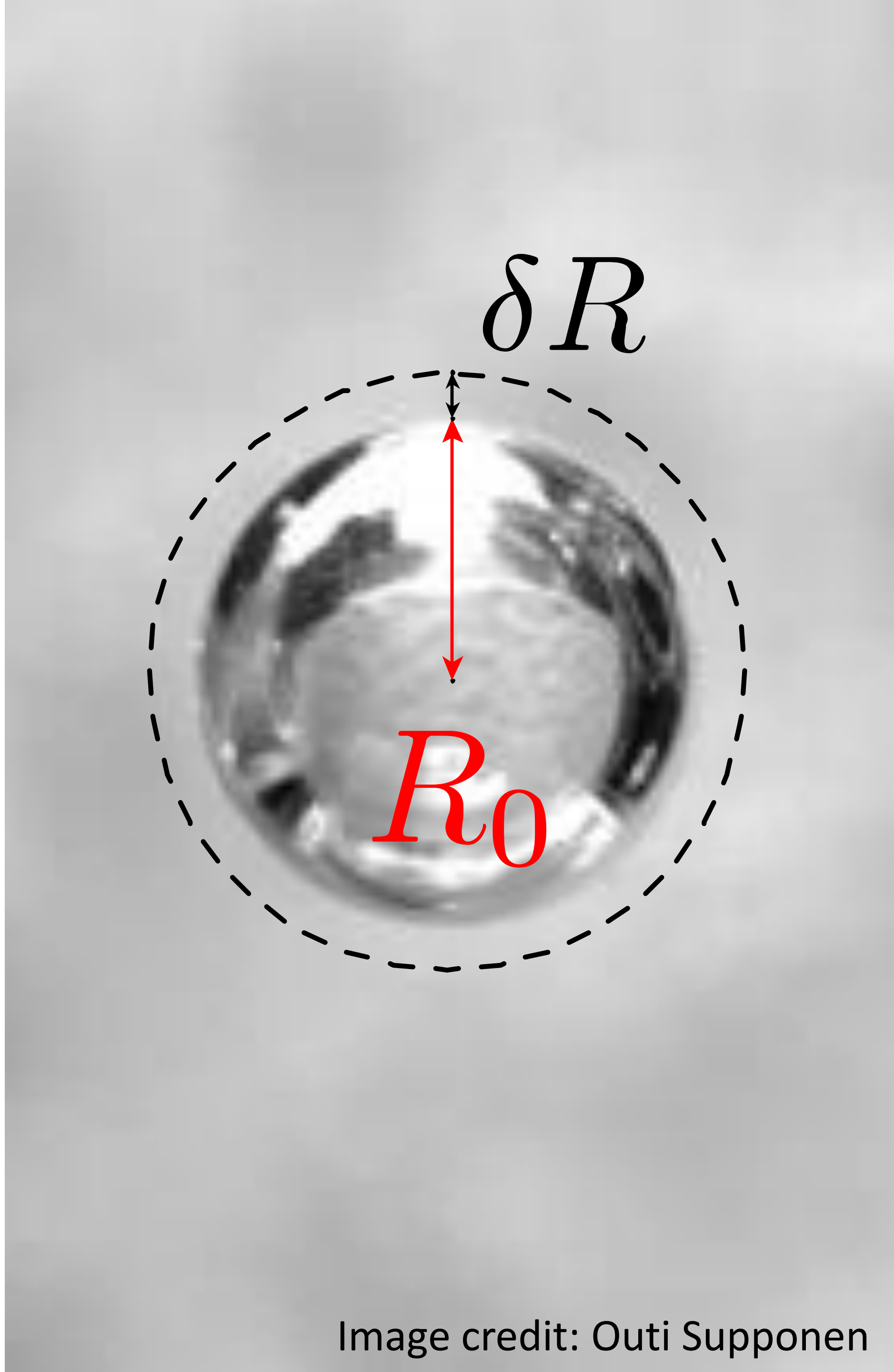


Test case I



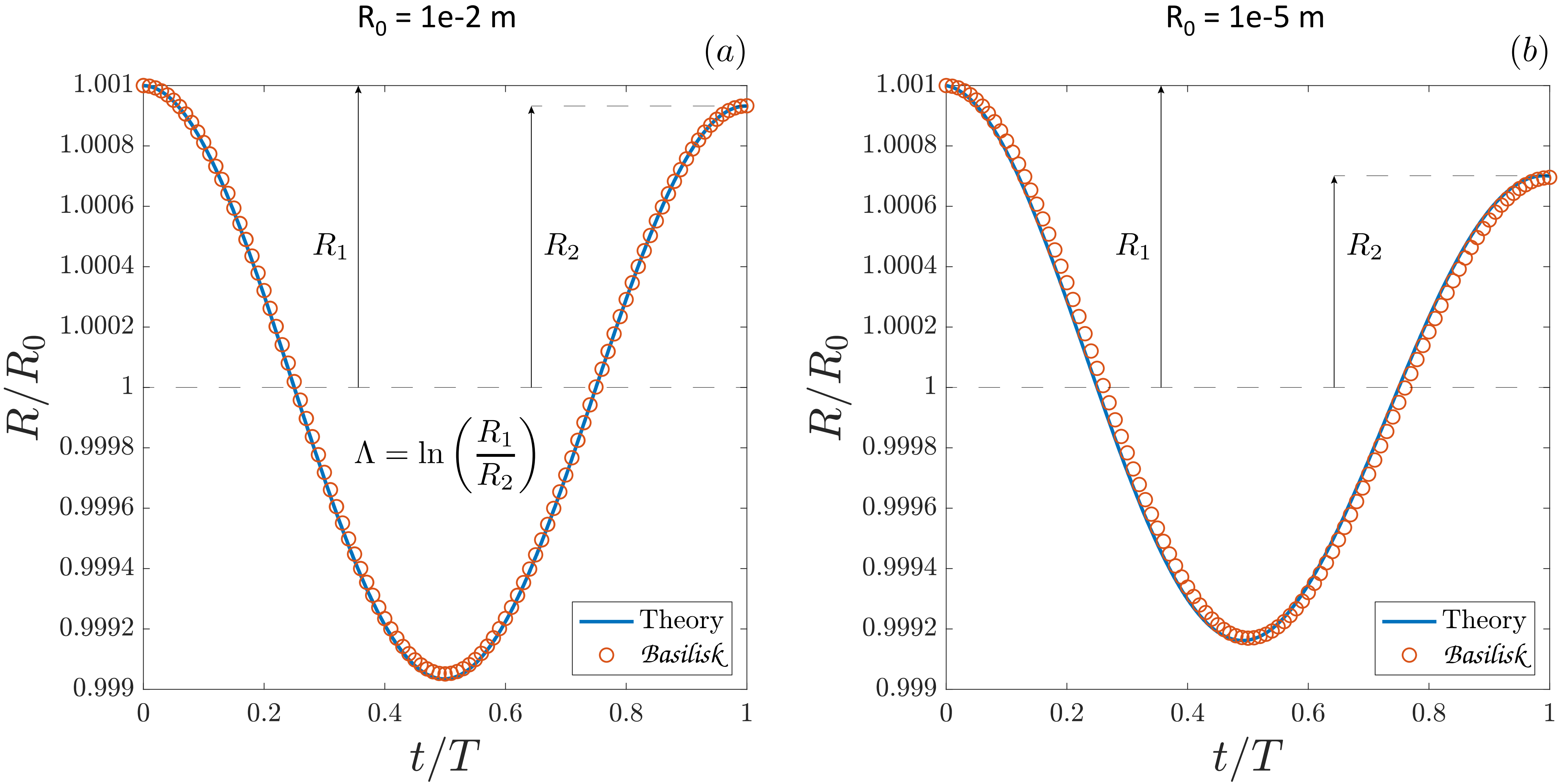
(a)
R/R_0
t/τ
$R_0/\Delta = 16$
$R_0/\Delta = 32$
$R_0/\Delta = 64$
$R_0/\Delta = 128$
$R_0/\Delta = 256$
1
0.95
0.9
0.85
0.8
0.75
0
0.2
0.4
0.6
0.8
1
(b)
$||\epsilon||_2$
Δ
0.05
0.04
0.03
0.02
0.01
0.01
0.02
0.03
0.04
0.05
1
2

Test case II: Free linear oscillations of a gas bubble

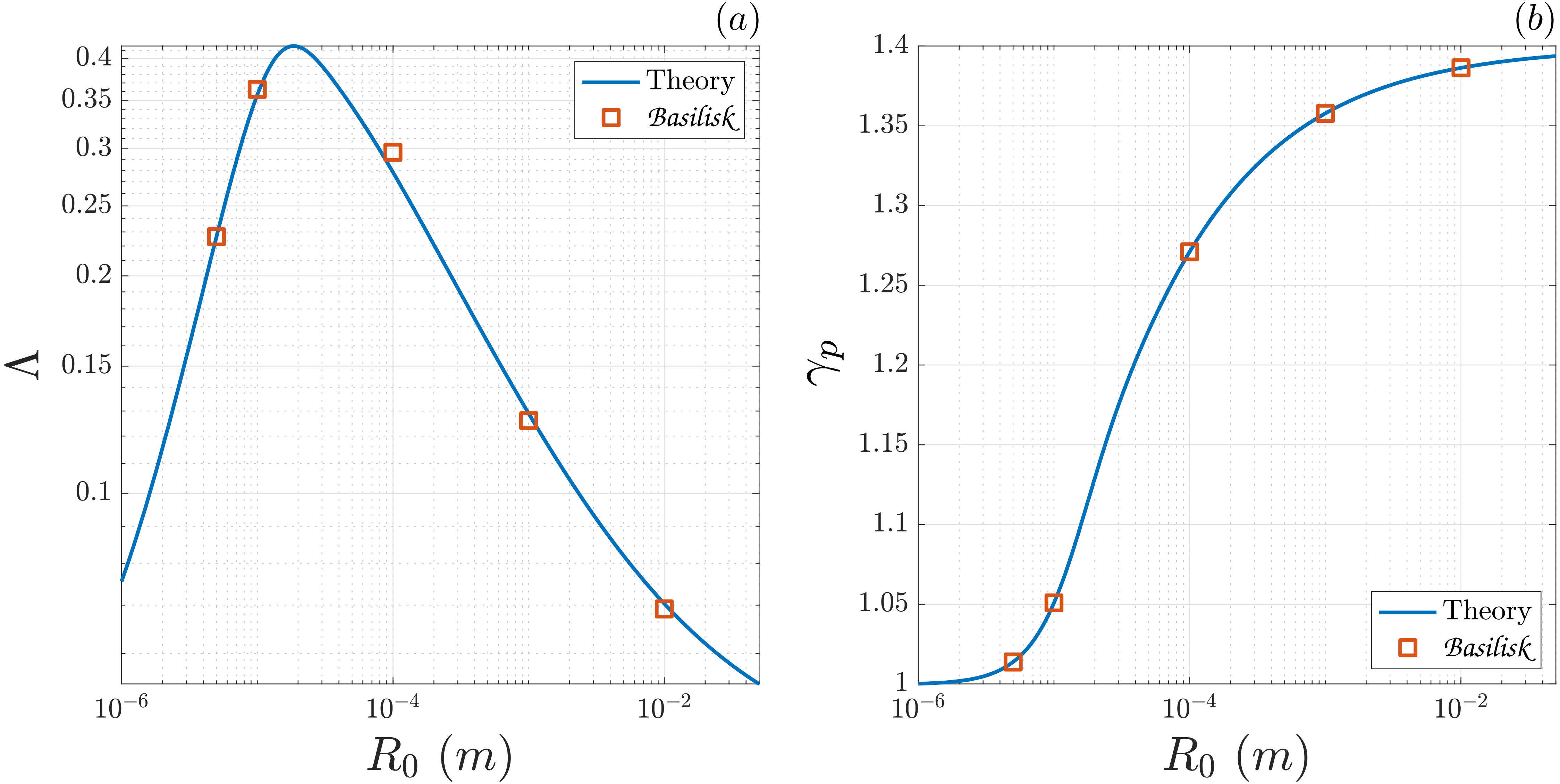


Test case II

$$Pe = \frac{R_0\sqrt{p_\infty/\rho_l}}{\kappa_g}$$



Test case II



(a)
Theory
Basilisk
0.4
0.35
0.3
0.25
0.2
0.15
0.1
Λ
10^{-6}
10^{-4}
10^{-2}
$R_0\ (m)$
(b)
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
γ_p
Theory
Basilisk
10^{-6}
10^{-4}
10^{-2}
$R_0\ (m)$

Test case III: Single bubble sonoluminescence

$$R_0 = 4.5\ \mu m$$
$$P_a = 1.2\ atm$$
$$f = 26.5\ kHz$$

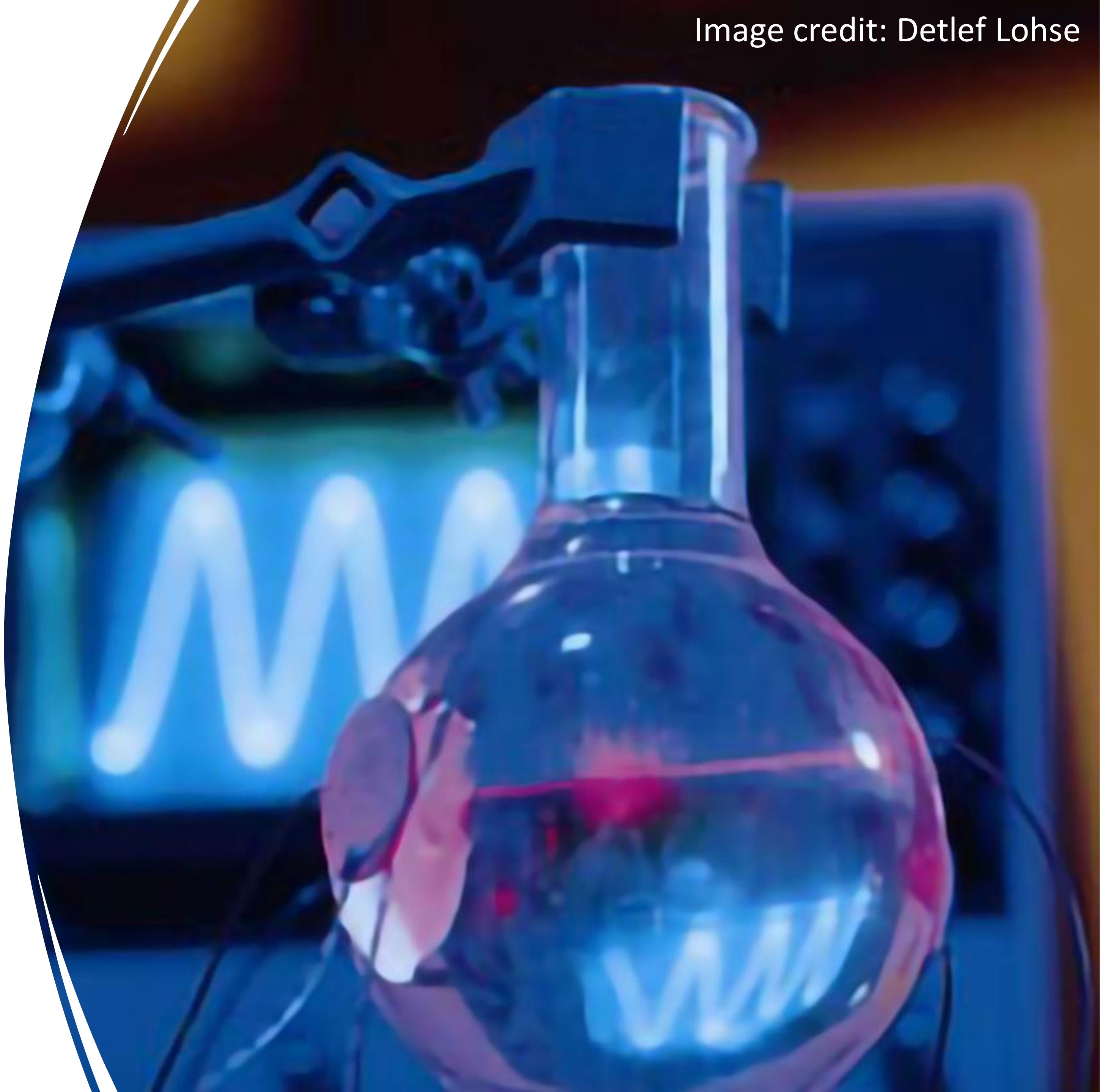
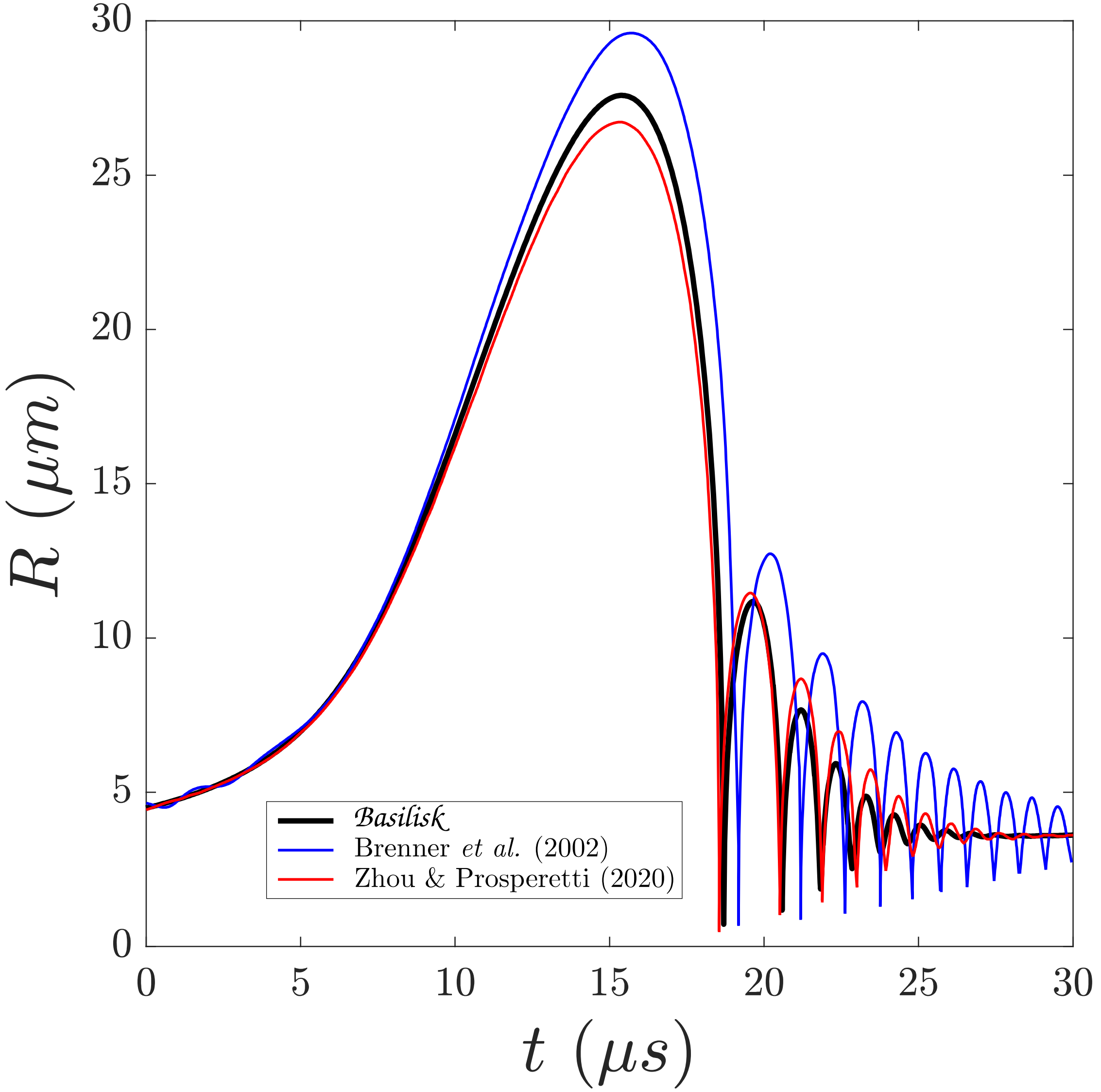
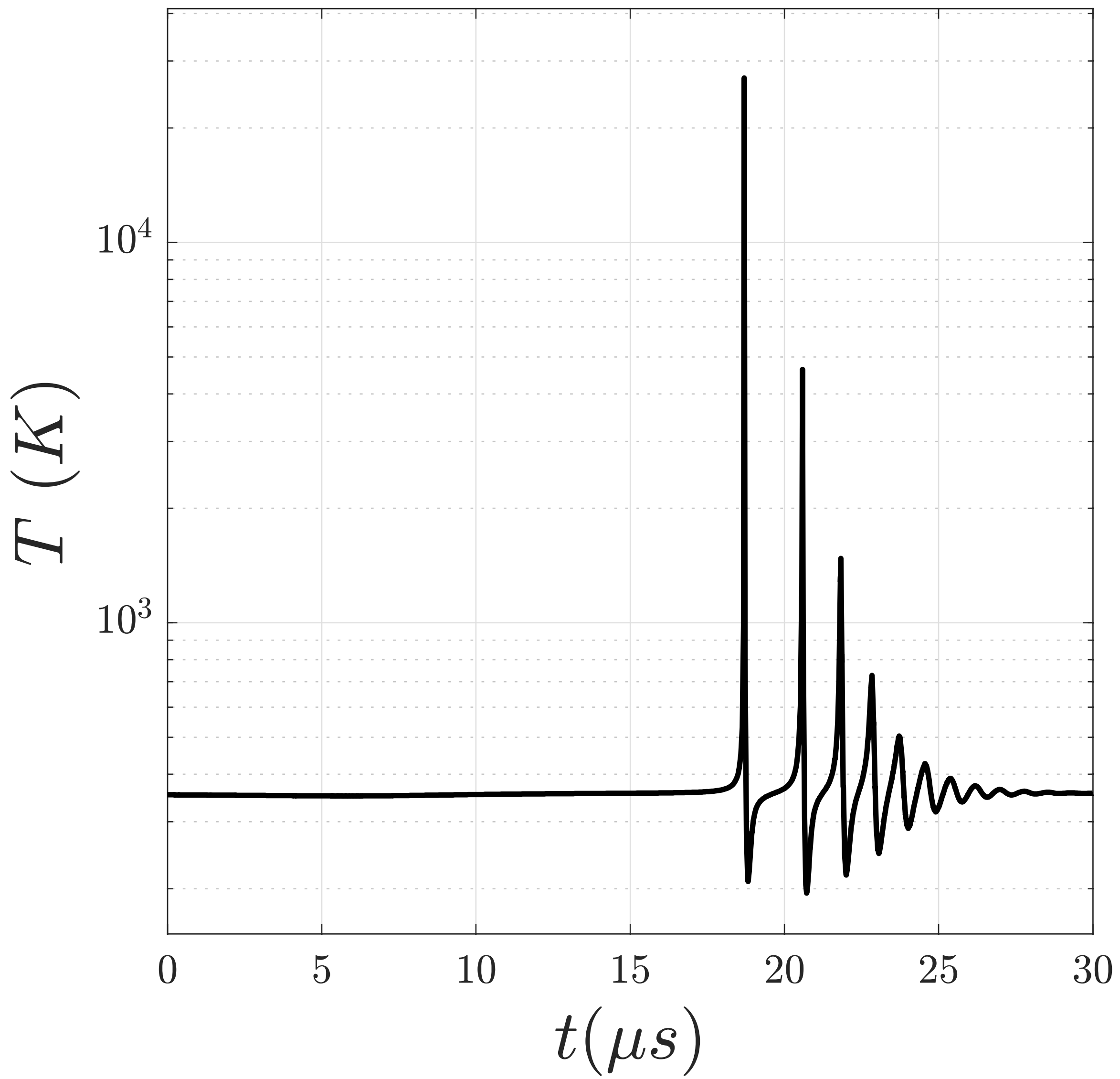


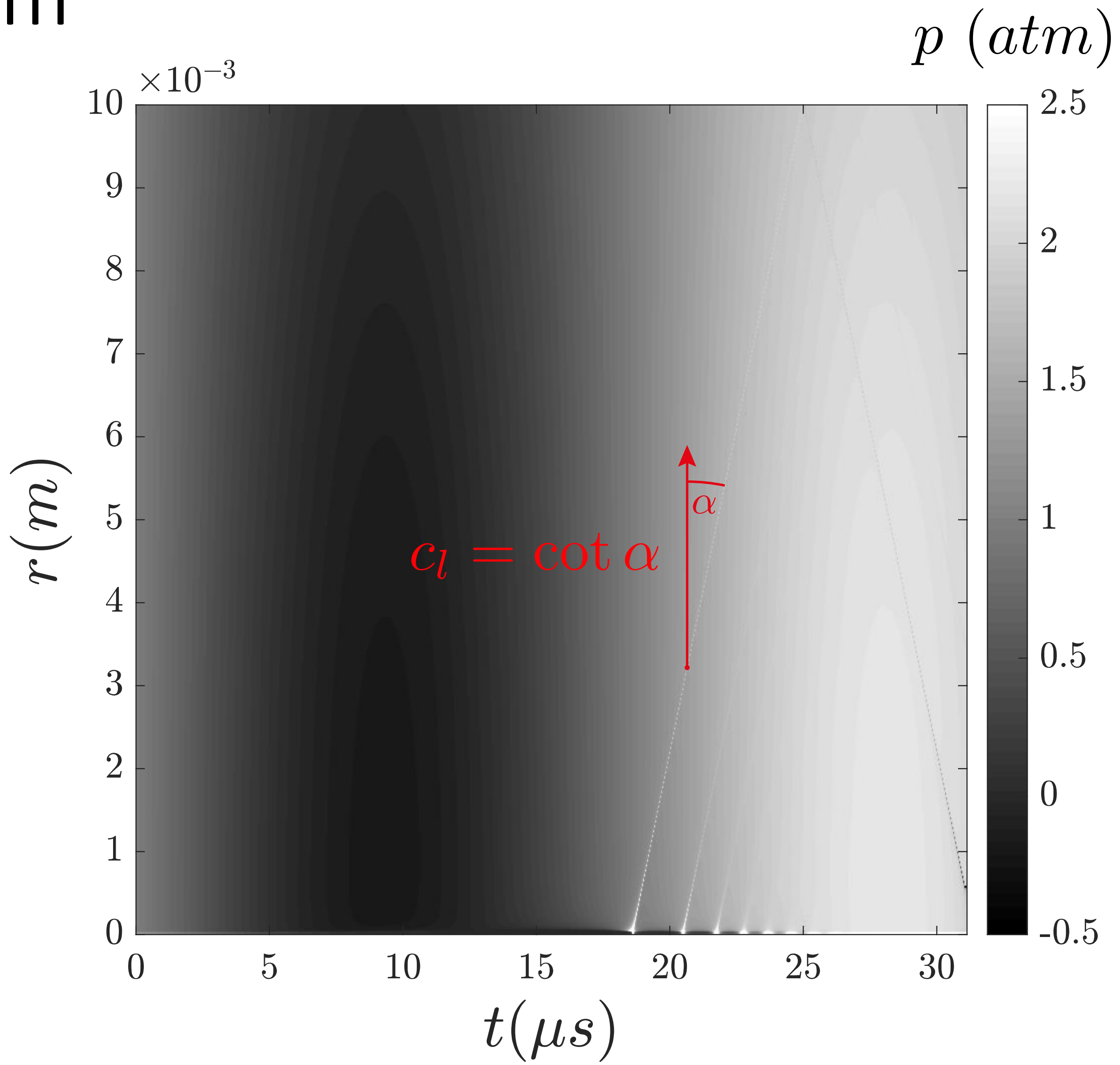
Image credit: Detlef Lohse

Test case III

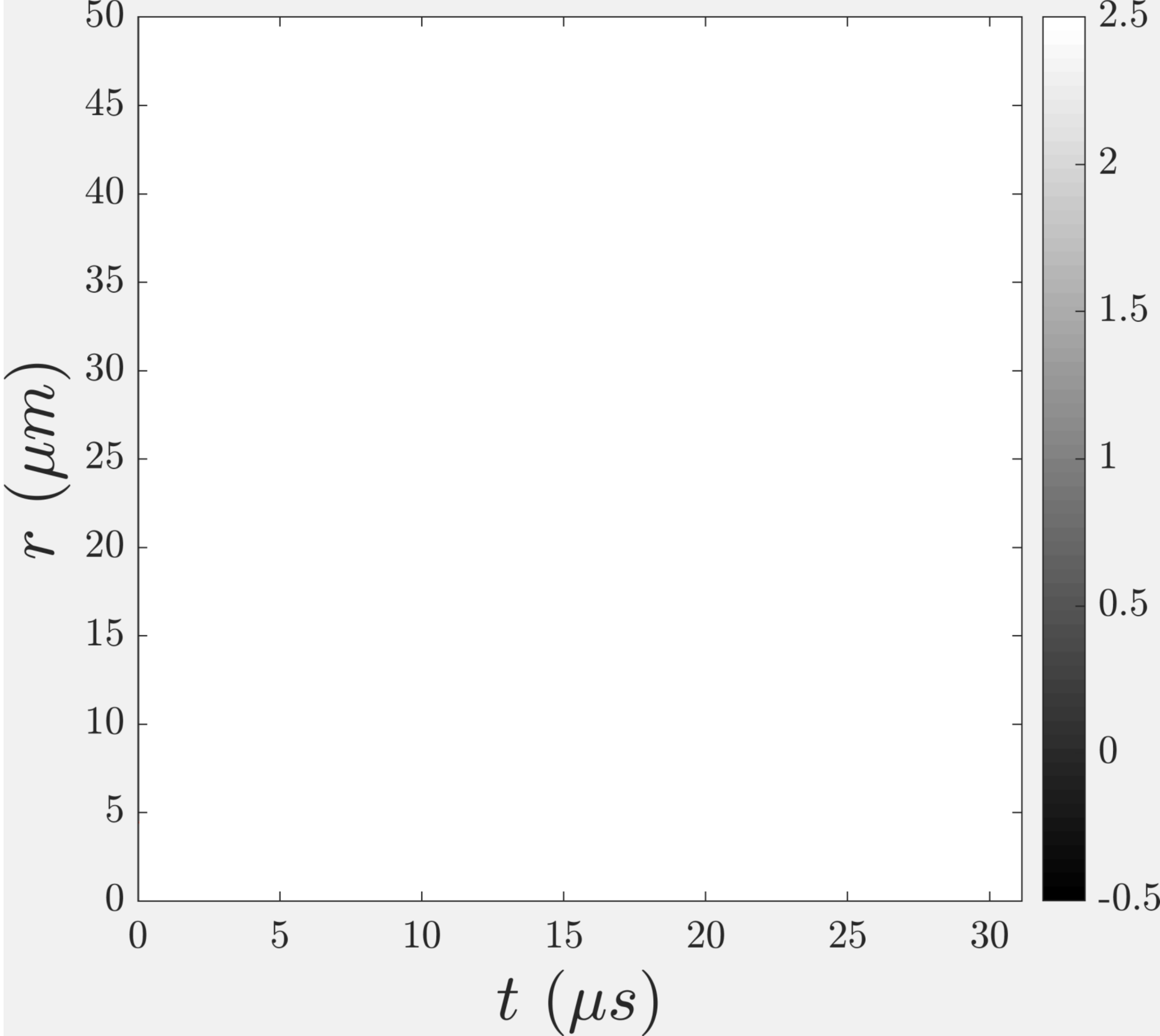




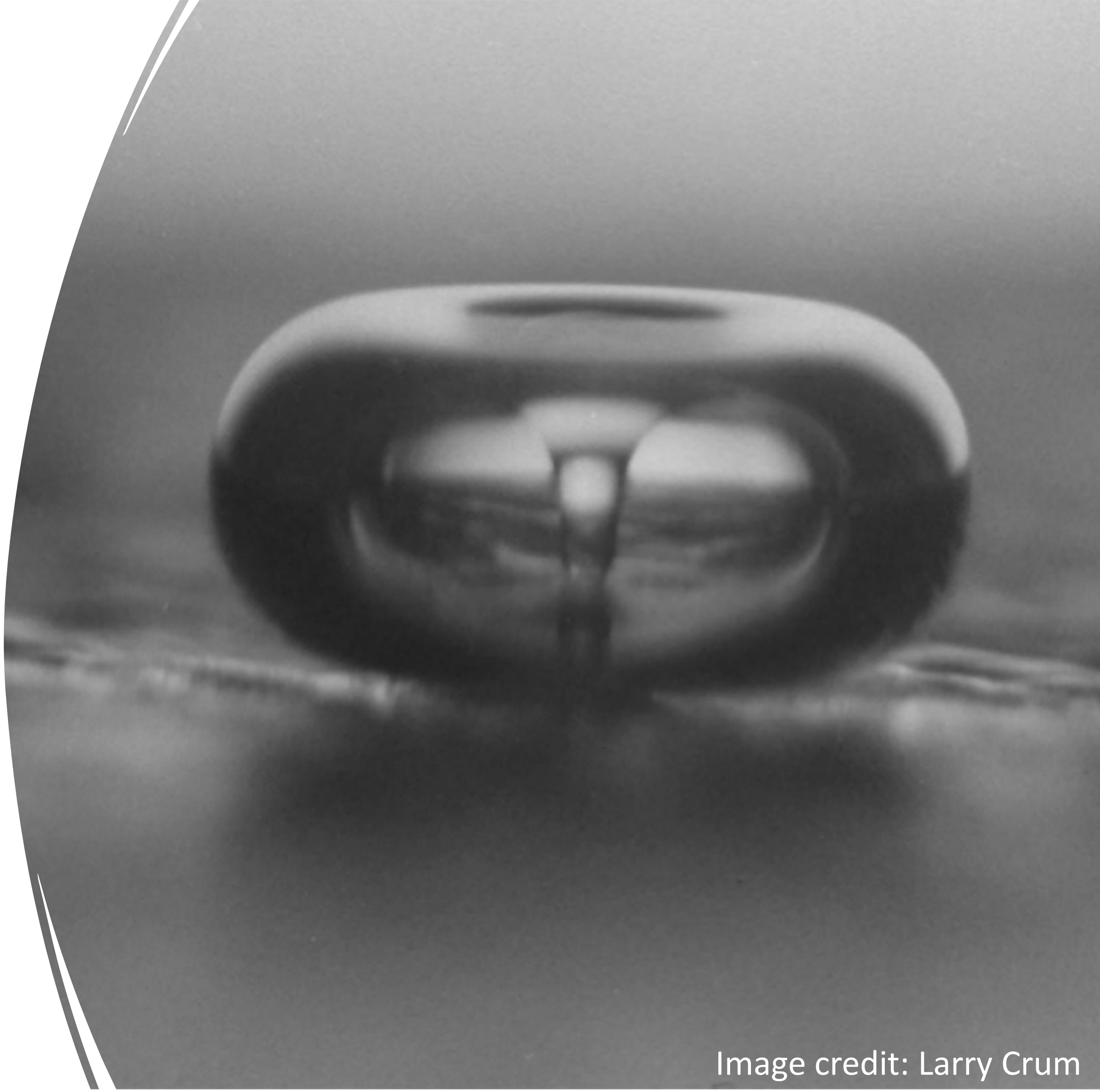
Test case III



Test case III



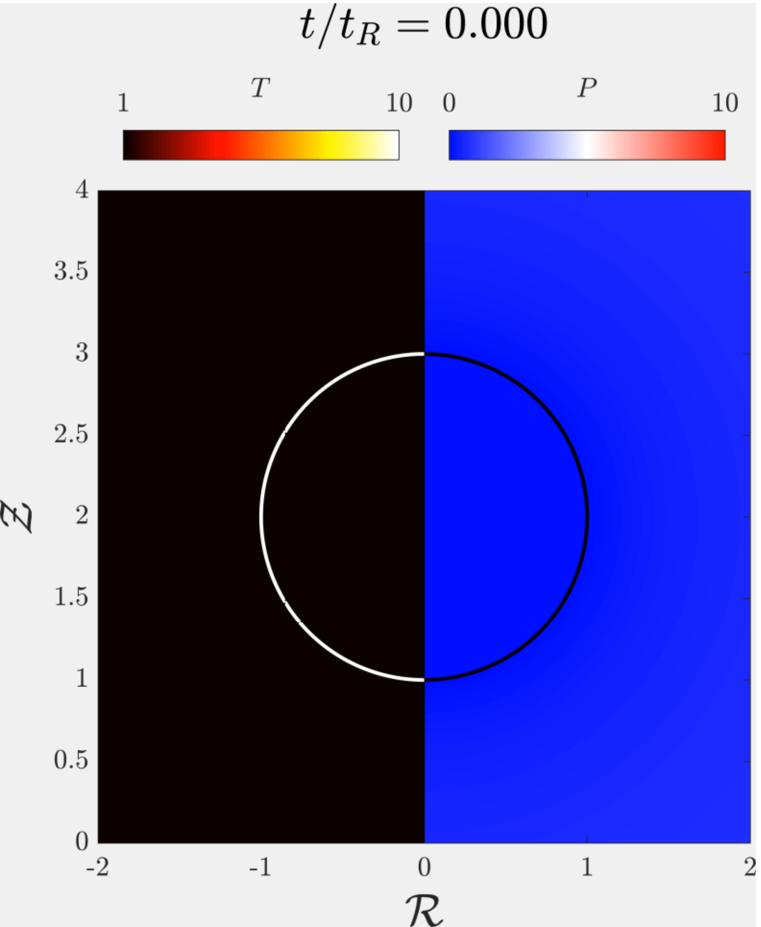
Test case IV: Bubble collapse close to a rigid boundary



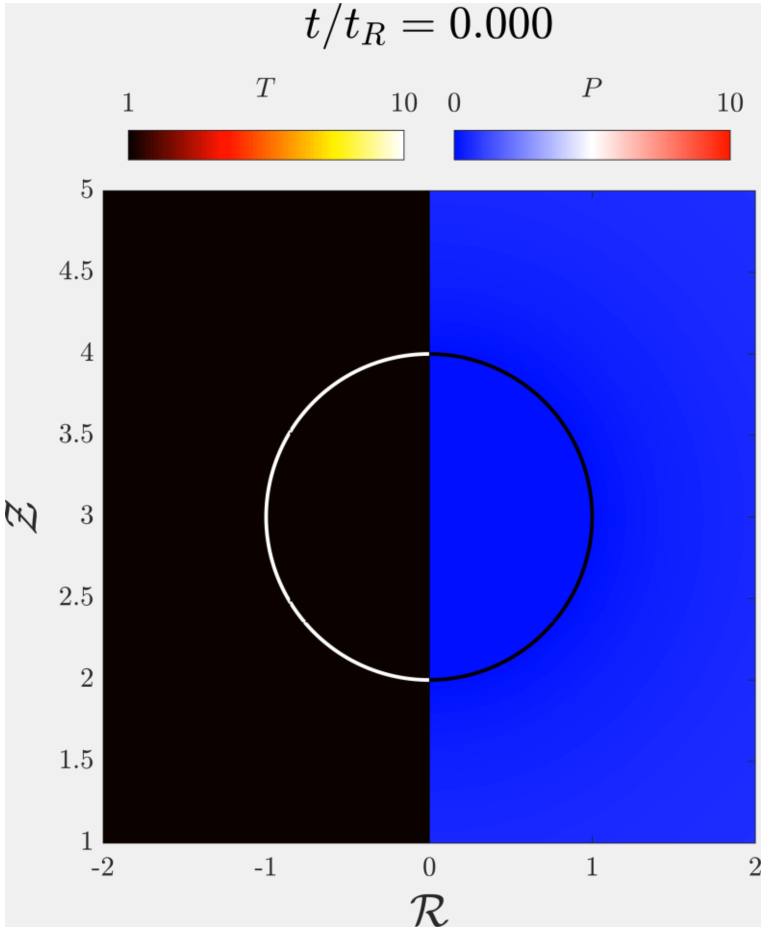
Test case IV

$$\delta = H/R_0$$

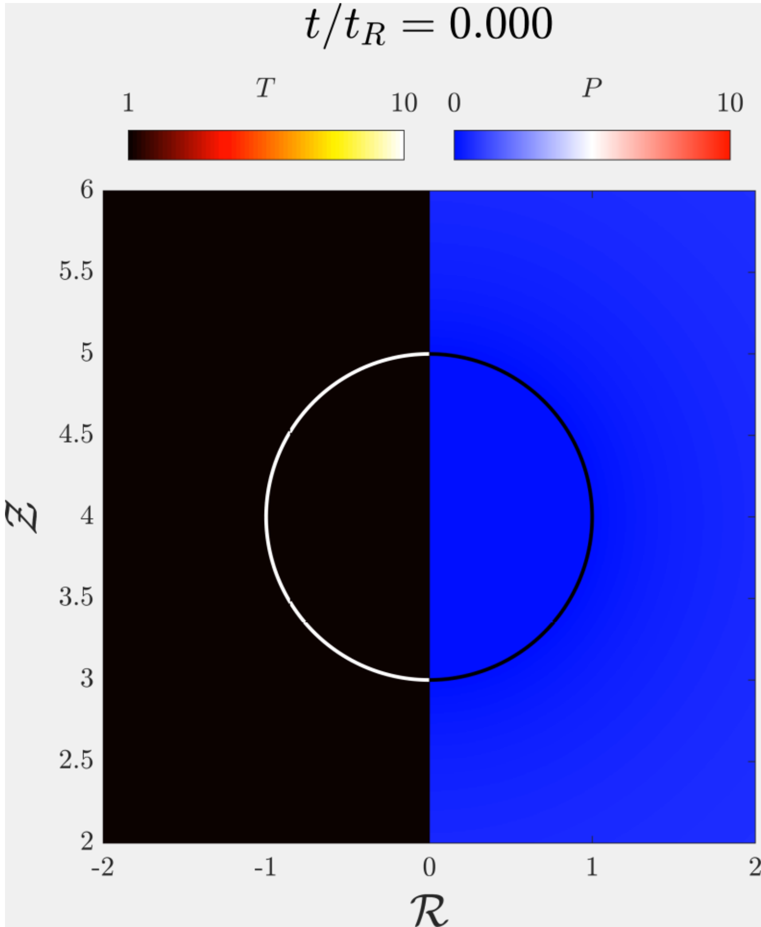
$\delta = 2$



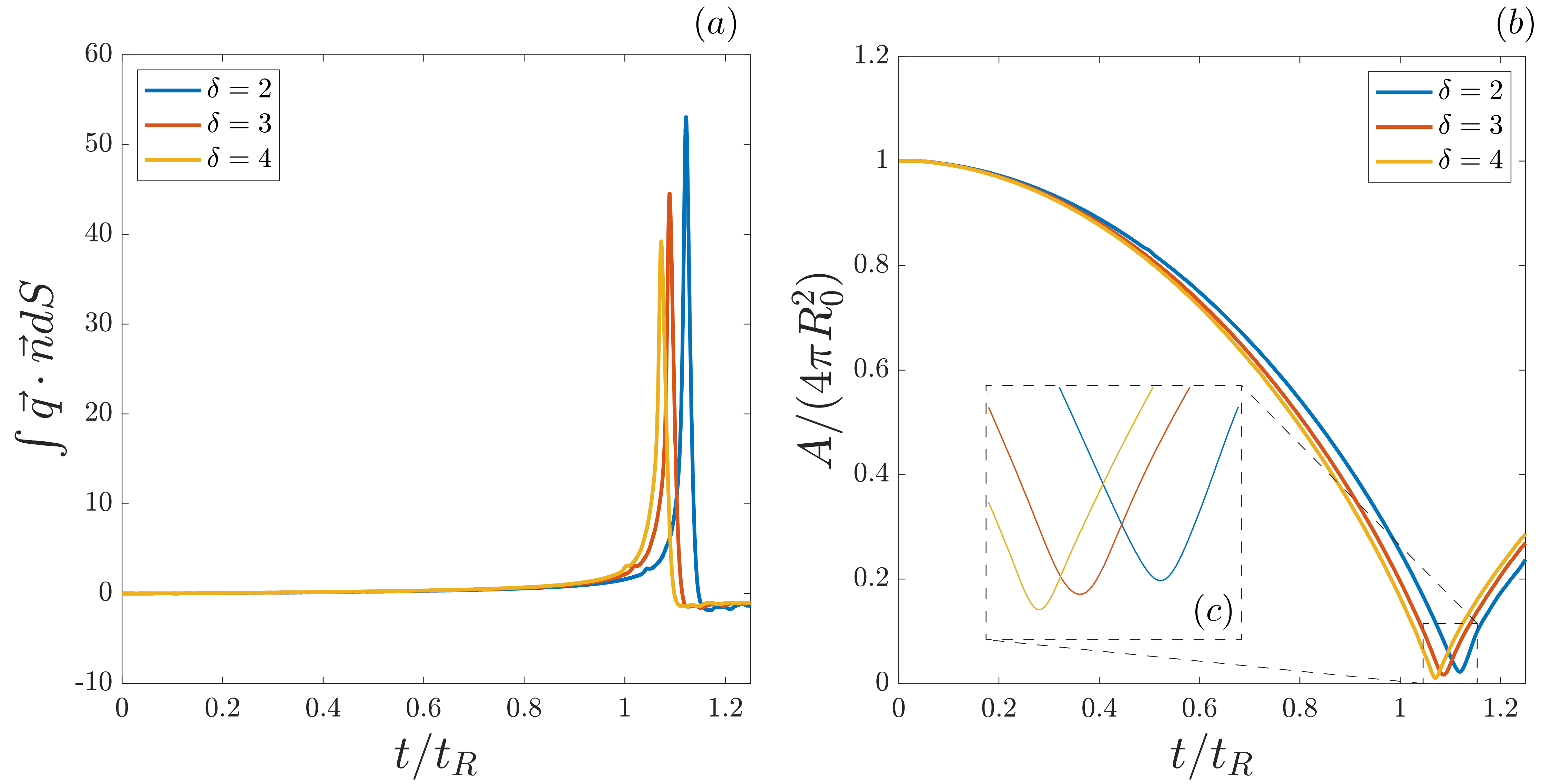
$\delta = 3$



$\delta = 4$



Test case IV


(a)
60
50
40
30
20
10
0
-10
$\delta = 2$
$\delta = 3$
$\delta = 4$
$\int \vec{q} \cdot \vec{n} dS$
0
0.2
0.4
0.6
0.8
1
1.2
t/t_R
(b)
1.2
1
0.8
0.6
0.4
0.2
0
$\delta = 2$
$\delta = 3$
$\delta = 4$
$A/(4\pi R_0^2)$
(c)
0
0.2
0.4
0.6
0.8
1
1.2
t/t_R

Thank you!

y.saade@utwente.nl

youssef.saade@cpp.canon