



Subcritical bubble break-up

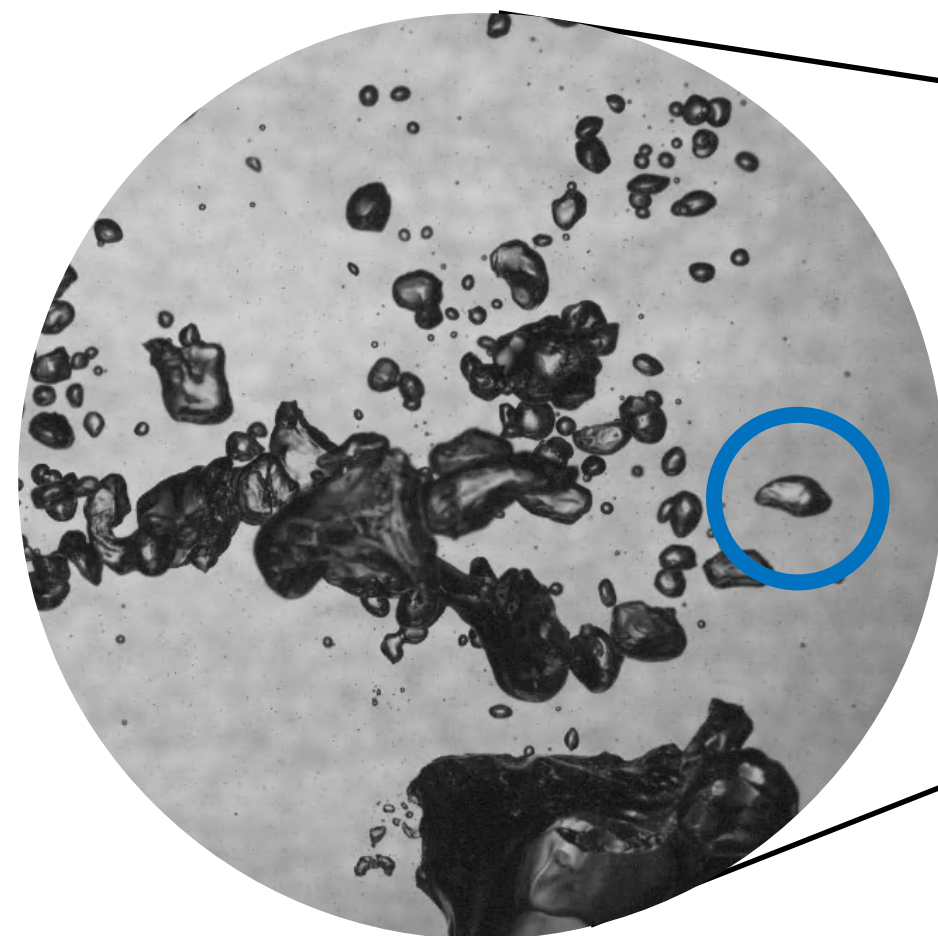
BGUM 2023
Paris, 5th July

Bubbles enhance gas transfers

Bubble mediated gas transfer
at the ocean-atmosphere interface:

40% of the CO₂ transfer

Reichl & Deike (2020)

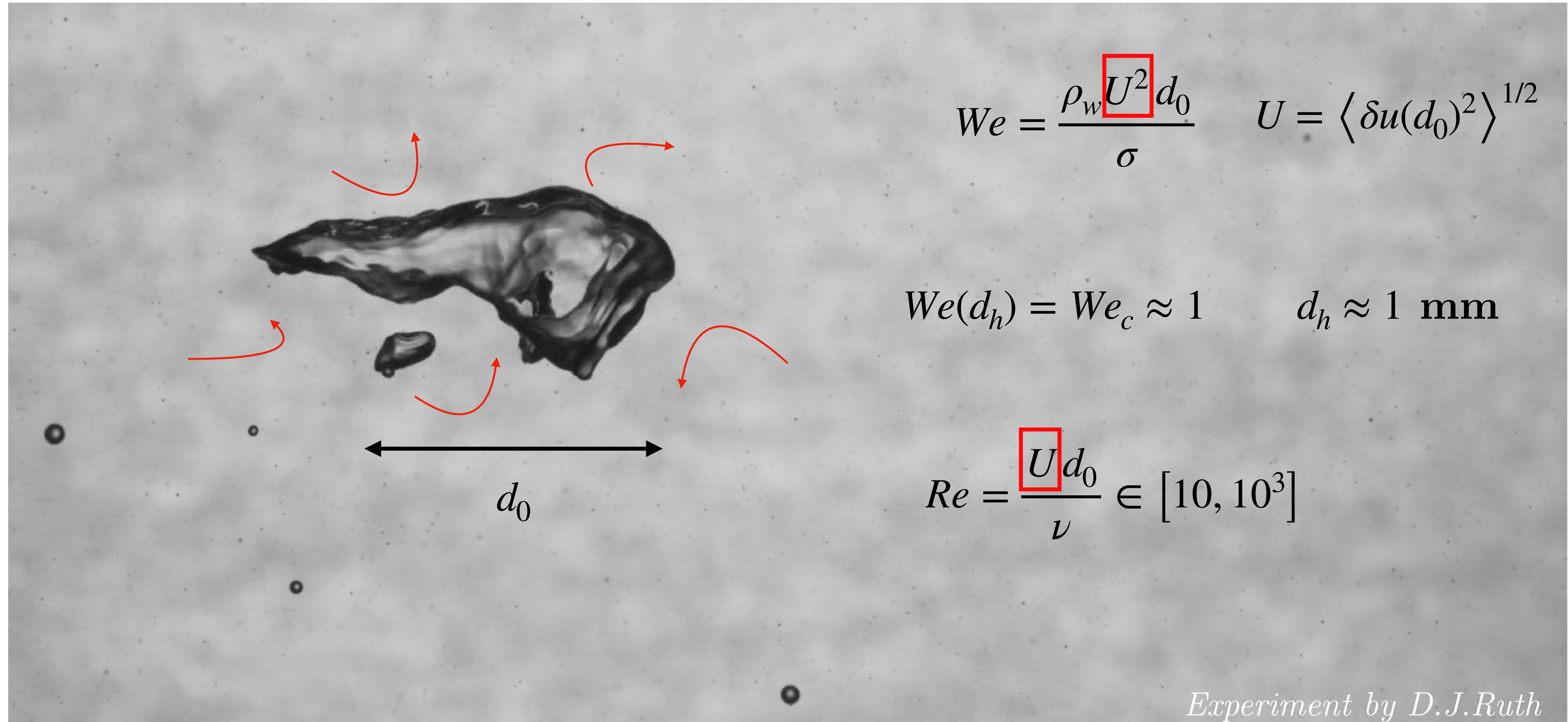


Experiment by D.J.Ruth



courtesy of J.Rivière

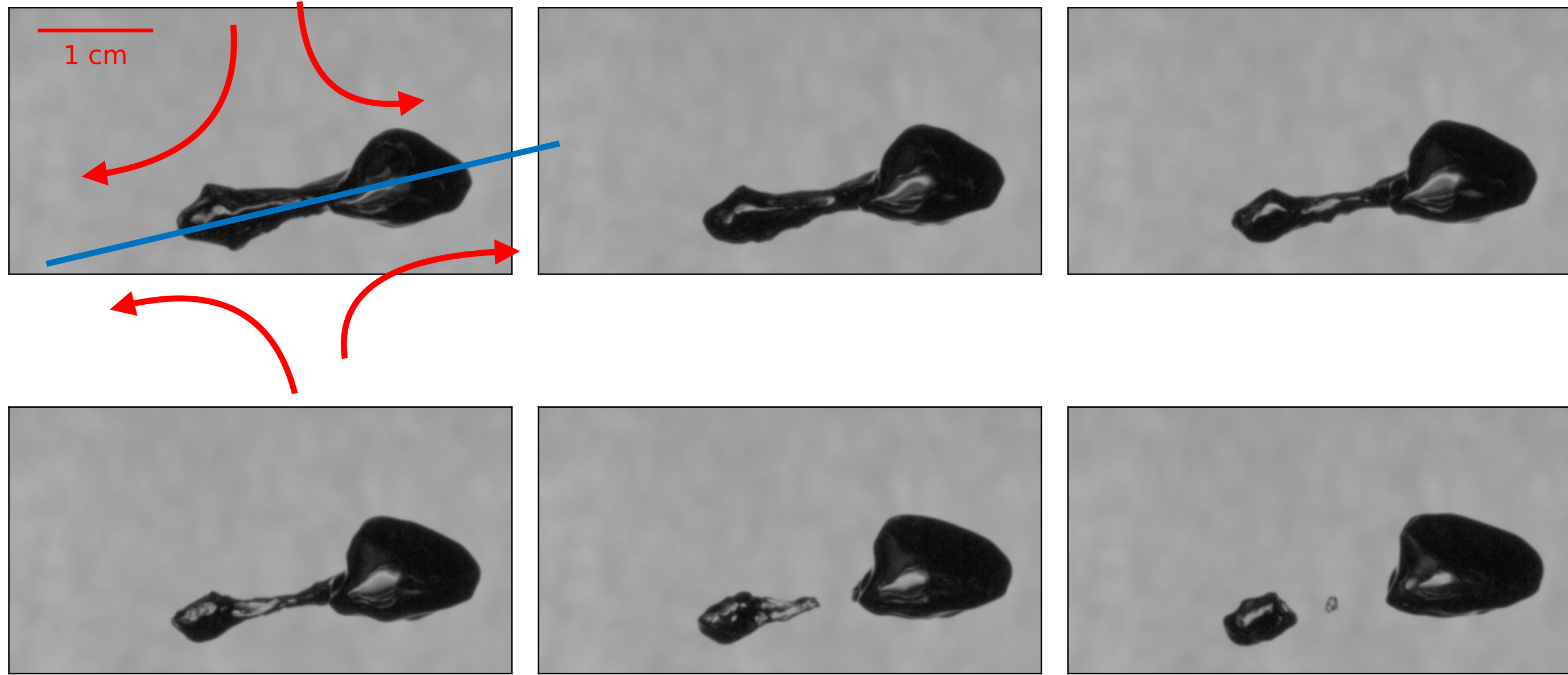
A fluctuating flow



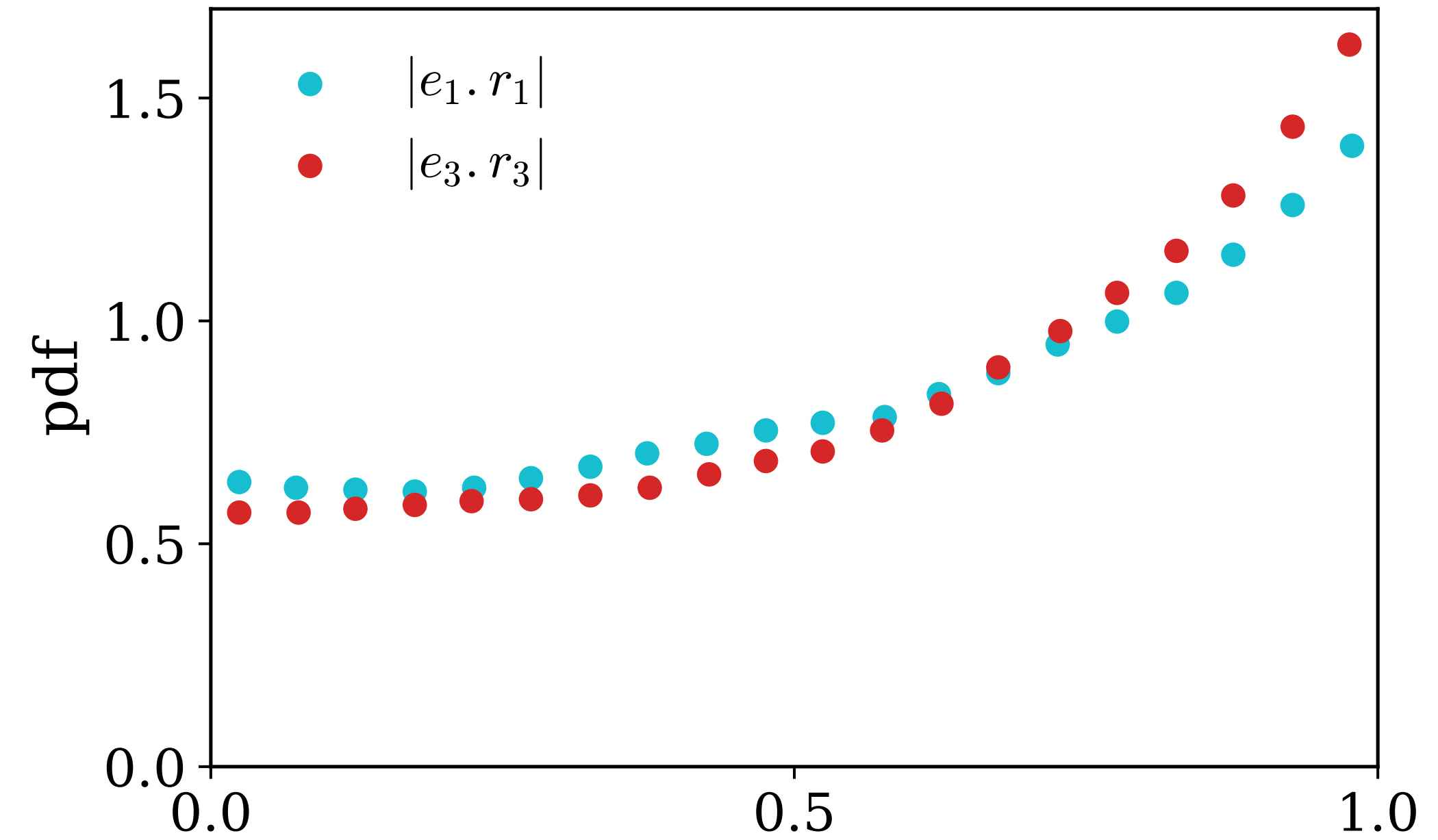
Large distribution of We_c : between 1 and 10. *Risso & Fabre (1998), Martinez-Bazan et al (1999a),
Rivière et al. (2021), Masuk et al. (2021)*

Must take into account the **local flow geometry**

From turbulence to a model flow



Experiment by D.J.Ruth

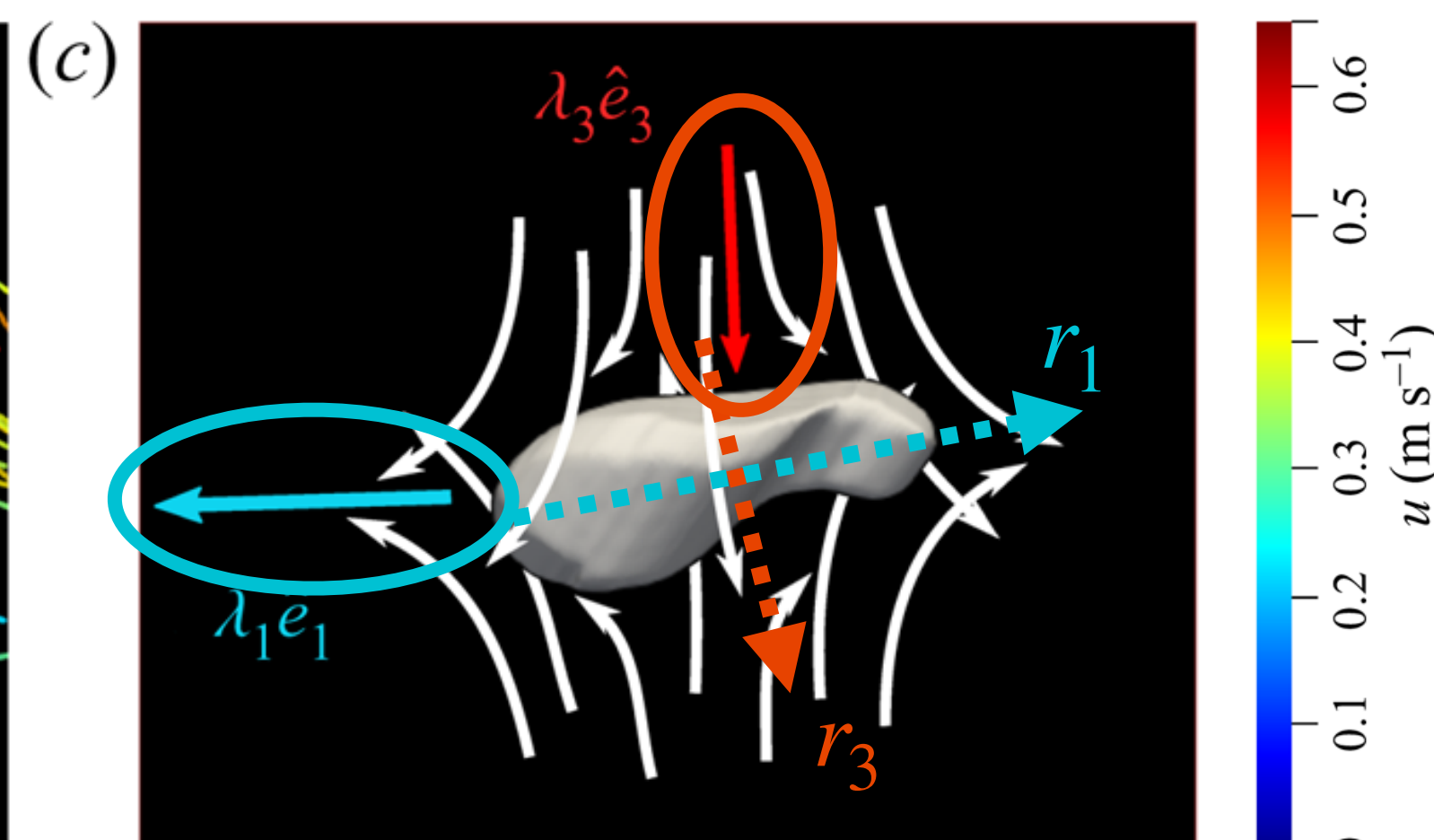
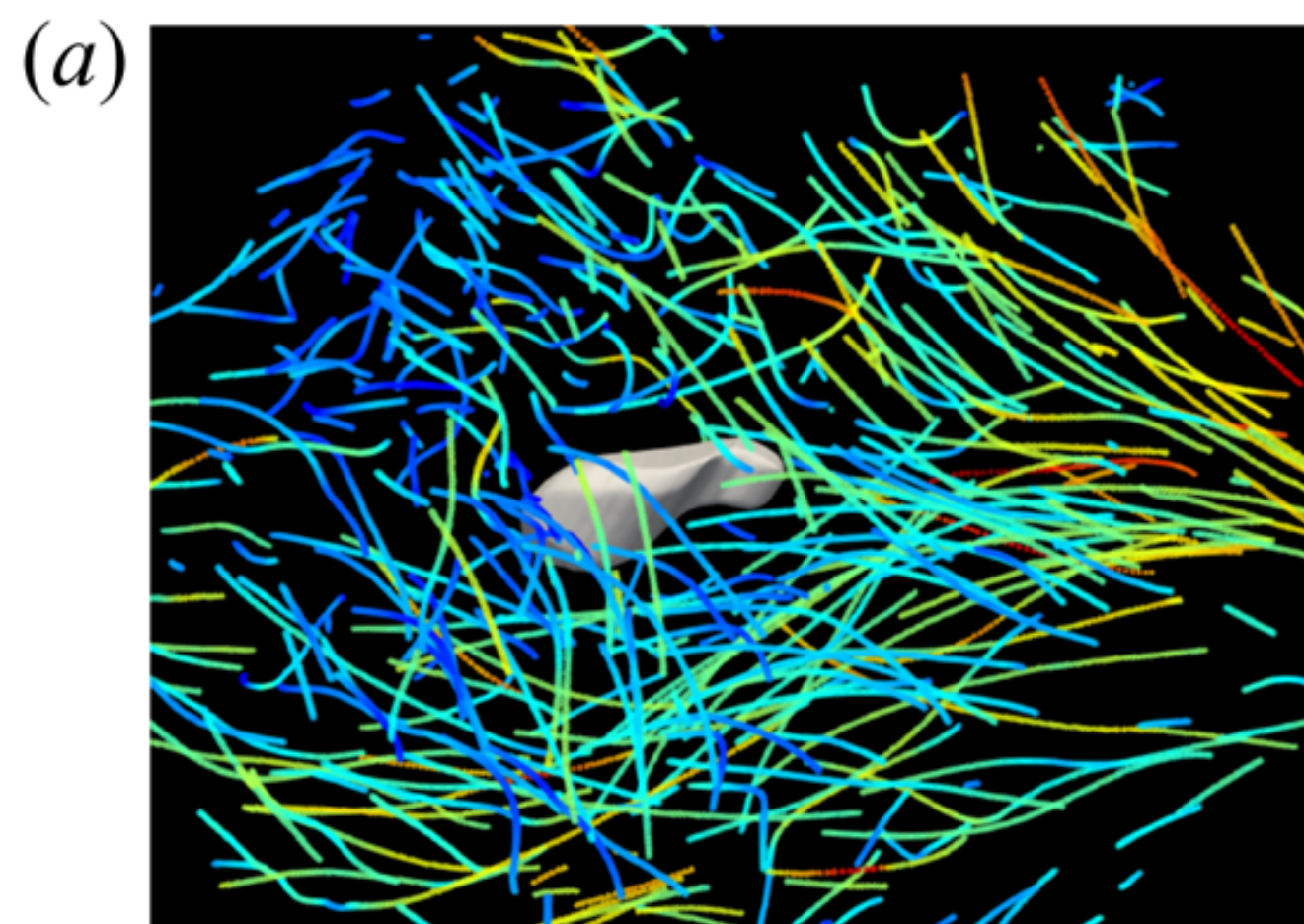


Masuk et al. (2021)

Rodriguez-Rodriguez et al. (2006)

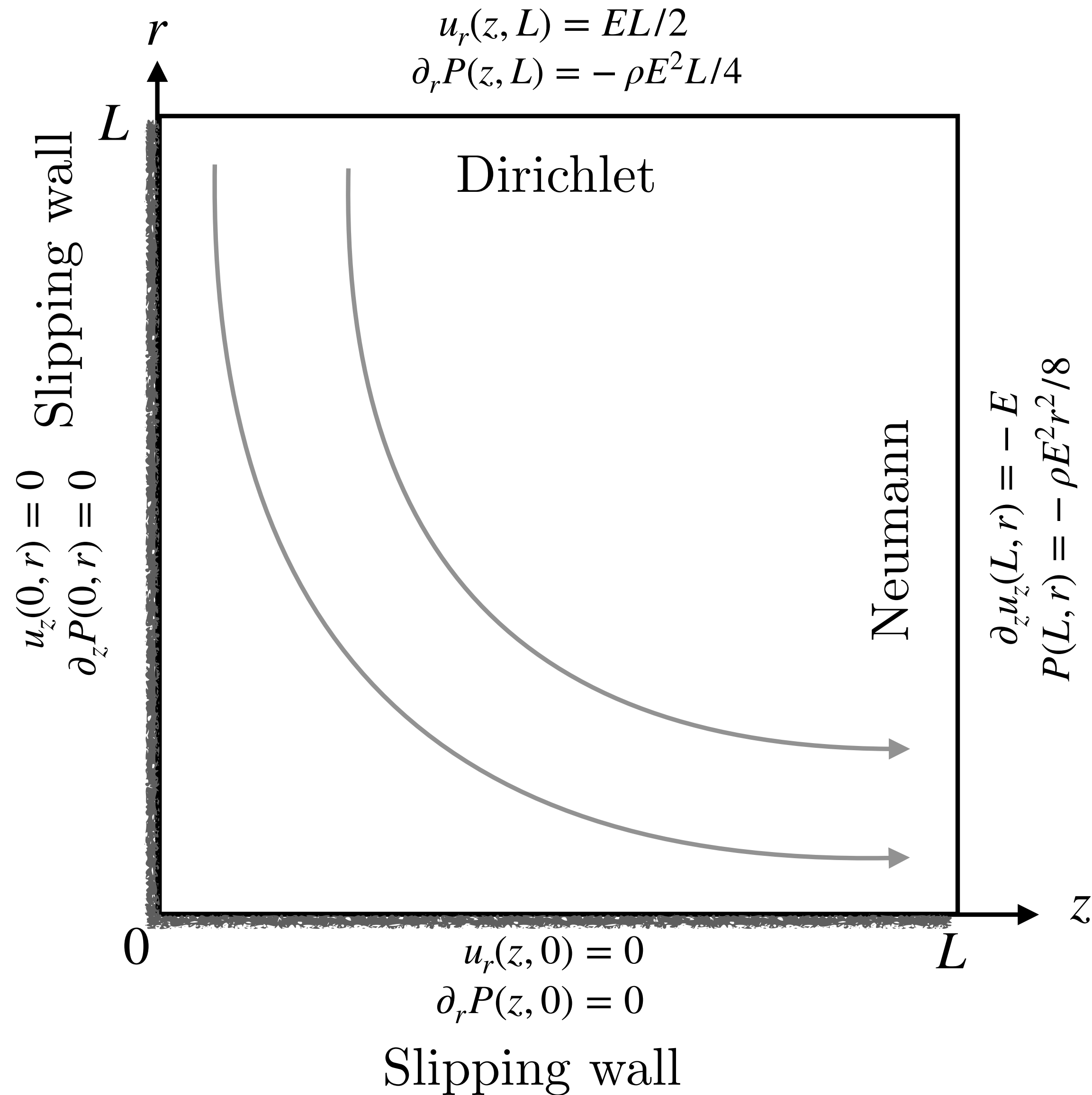
Revuelta (2006)

Masuk et al. (2021a,b,c)



Masuk et al. (2021)

Numerical configuration - Flow creation



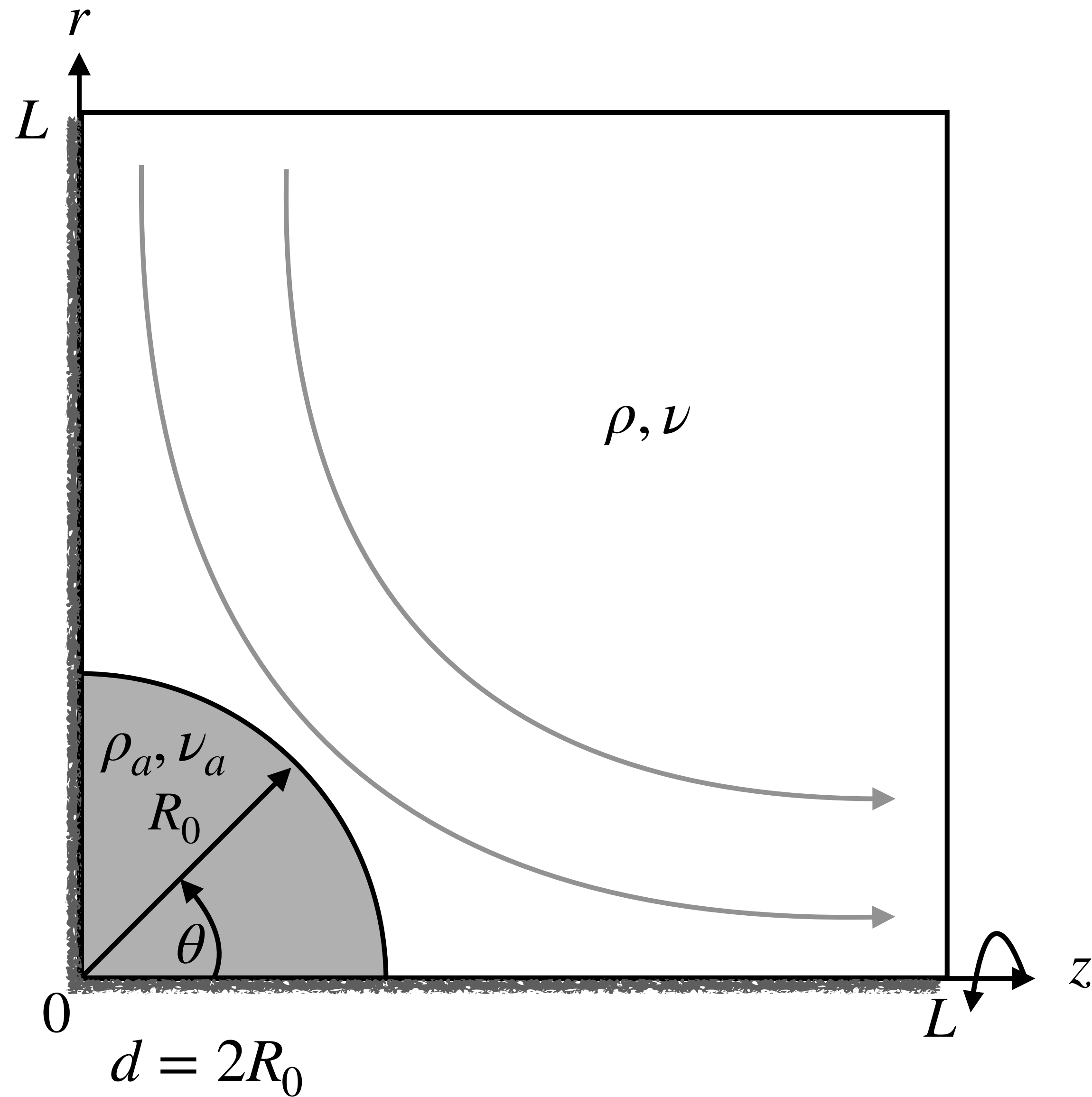
$$\mathbf{v}(z, r) = -\frac{E}{2} r \mathbf{e}_r + E z \mathbf{e}_z$$

Solve axisymmetric NS equations with *Basilisk*:

- ▶ momentum conserving scheme
- ▶ geometric VOF method
- ▶ AMR

<http://basilisk.fr>

Numerical configuration - Bubble injection

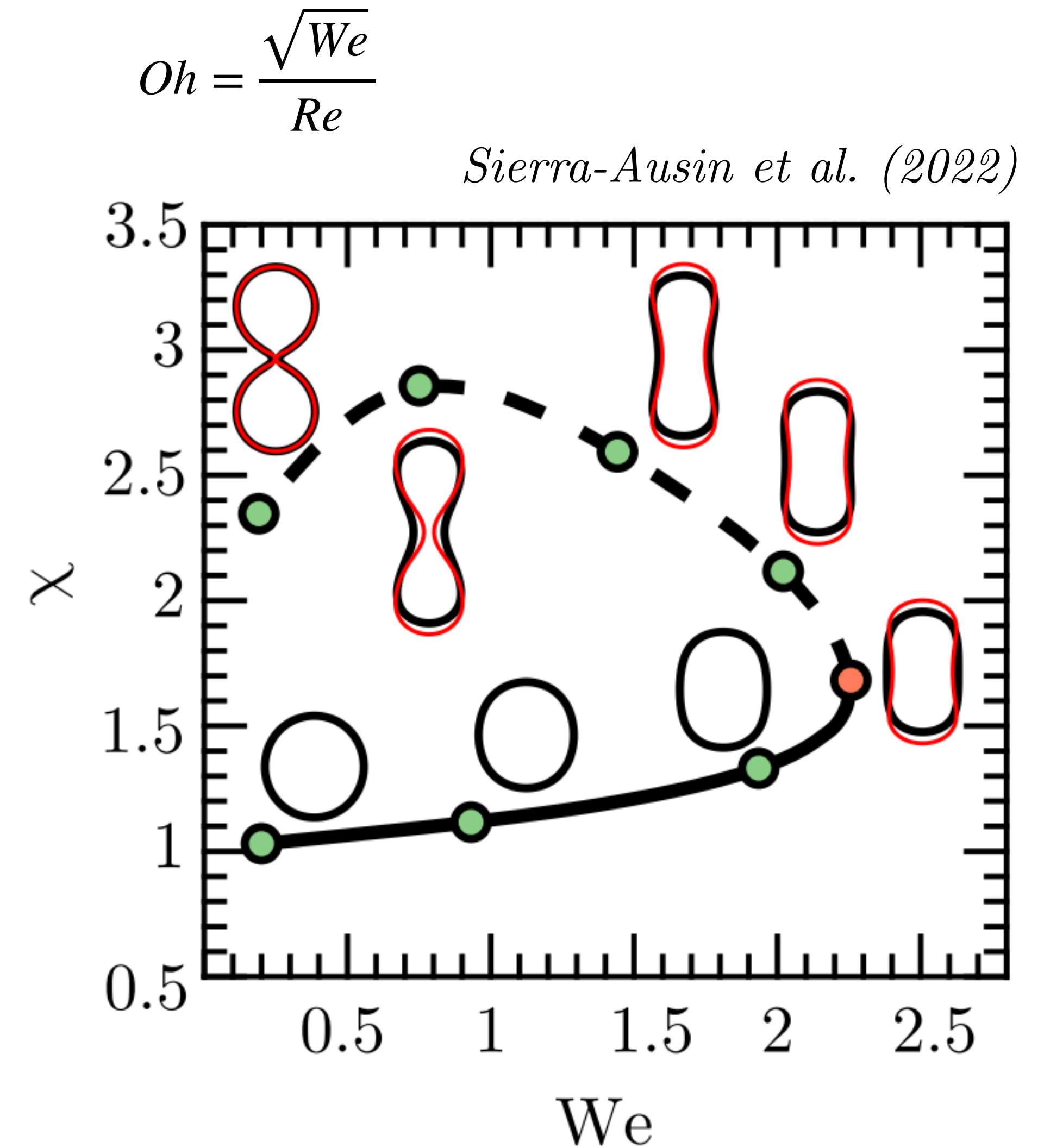
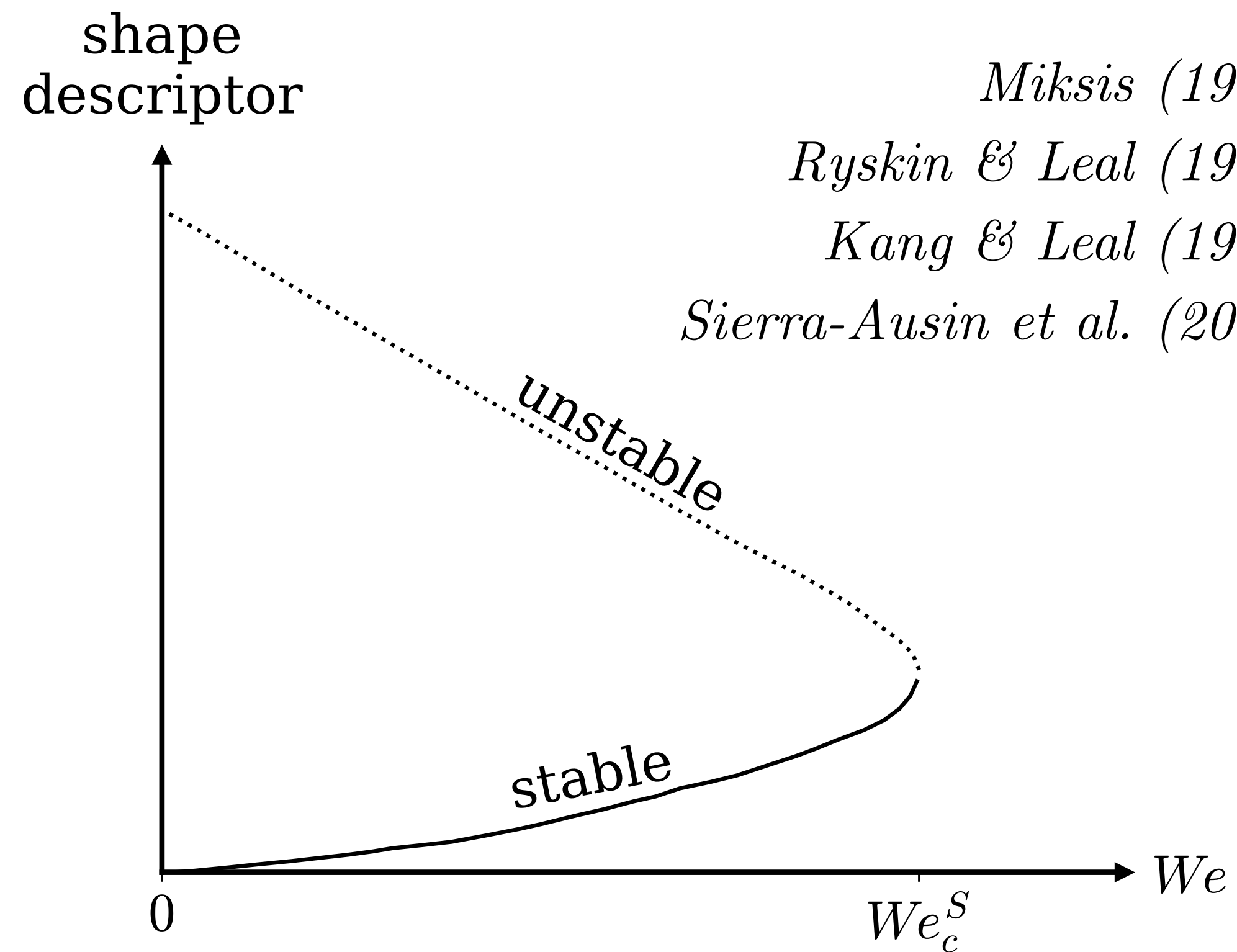


$$\mathbf{v}(z, r) = -\frac{E}{2}r\mathbf{e}_r + Ez\mathbf{e}_z$$

$$We = \frac{\rho U^2 d}{\gamma}$$
$$U = Ed$$
$$Re = \frac{Ud}{\nu} \gg 1$$

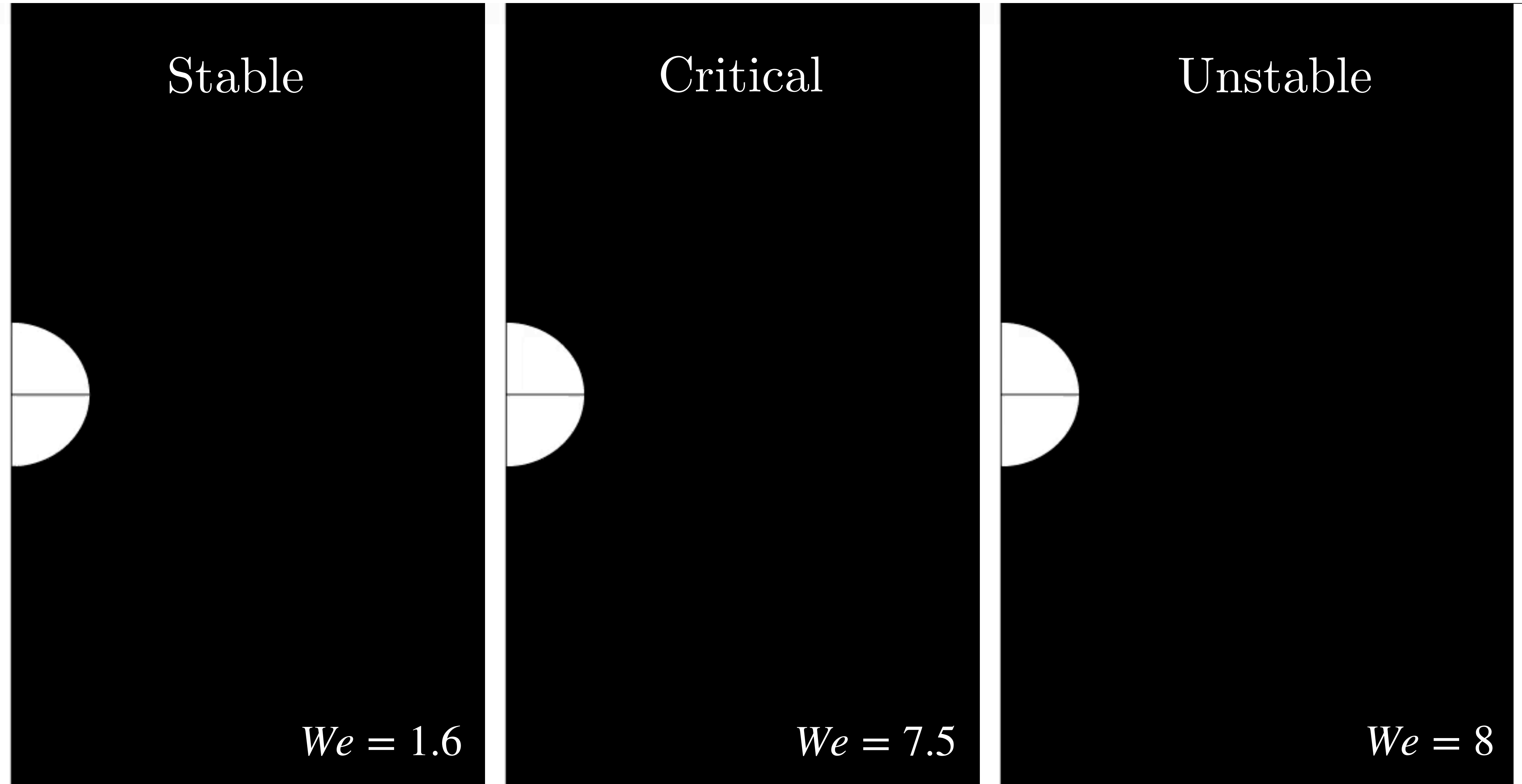
Phase diagram: Equilibrium positions

Saddle-node bifurcation

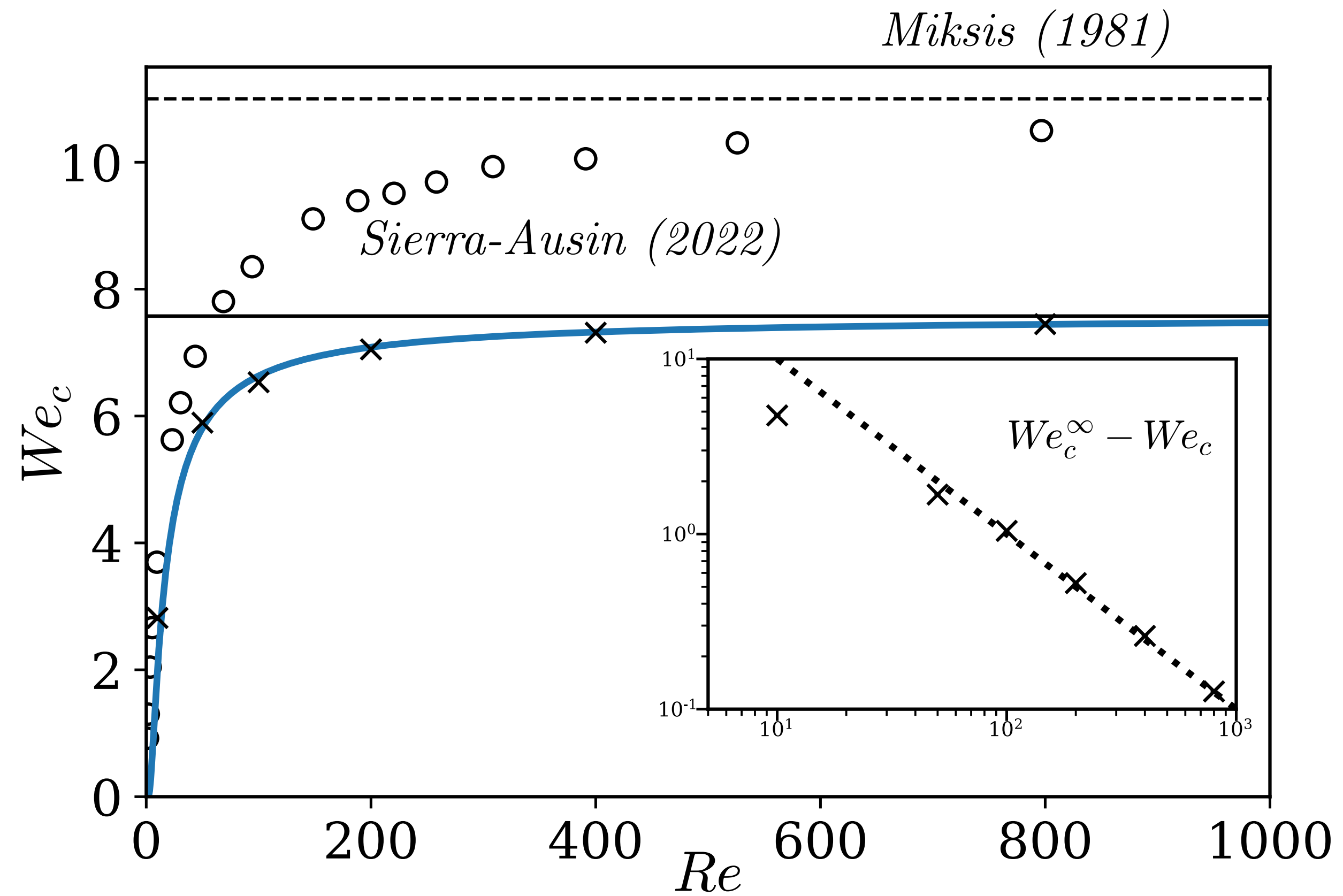


Phase diagram

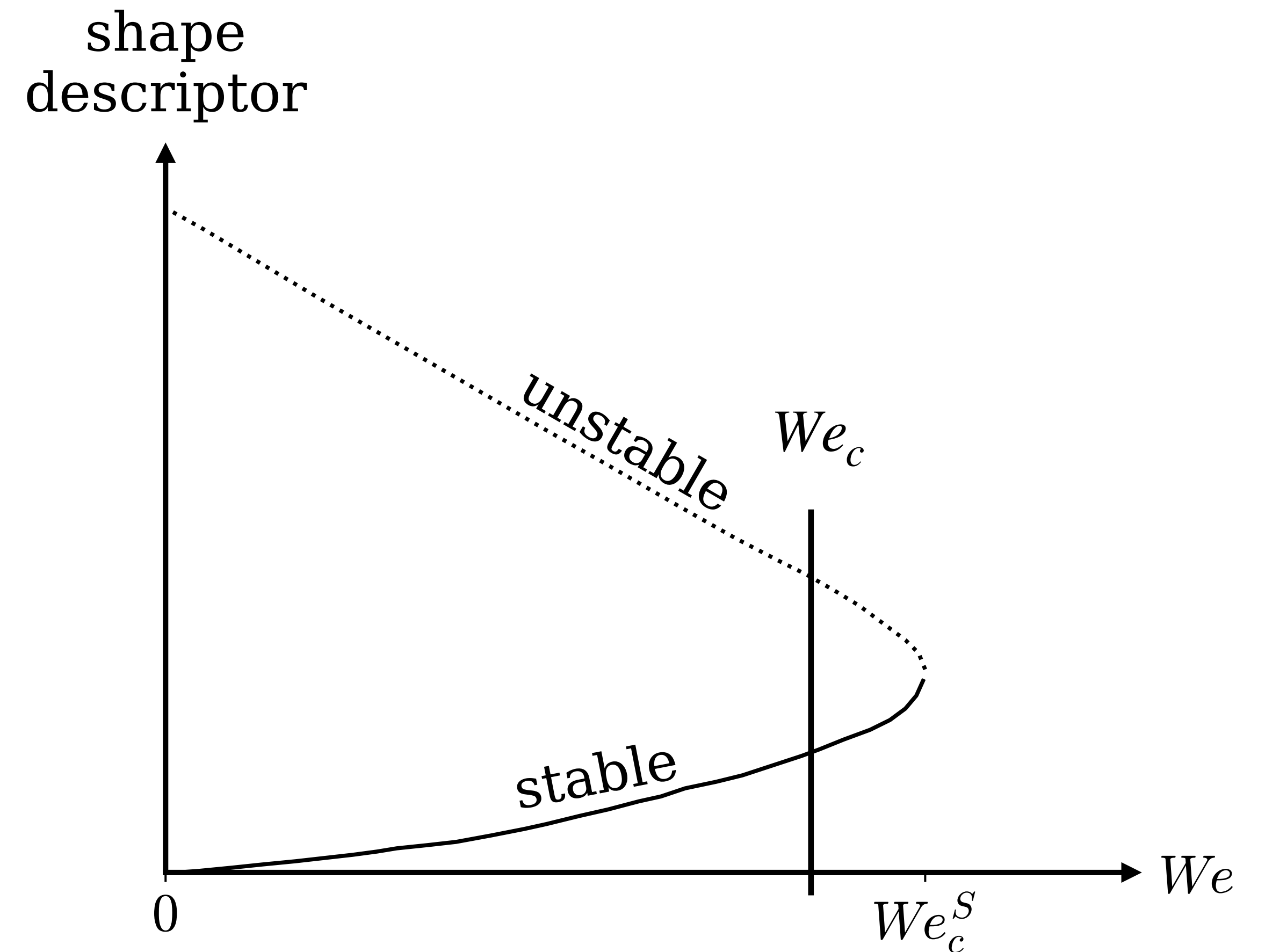
$Re = 400$



Phase diagram: Importance of the IC

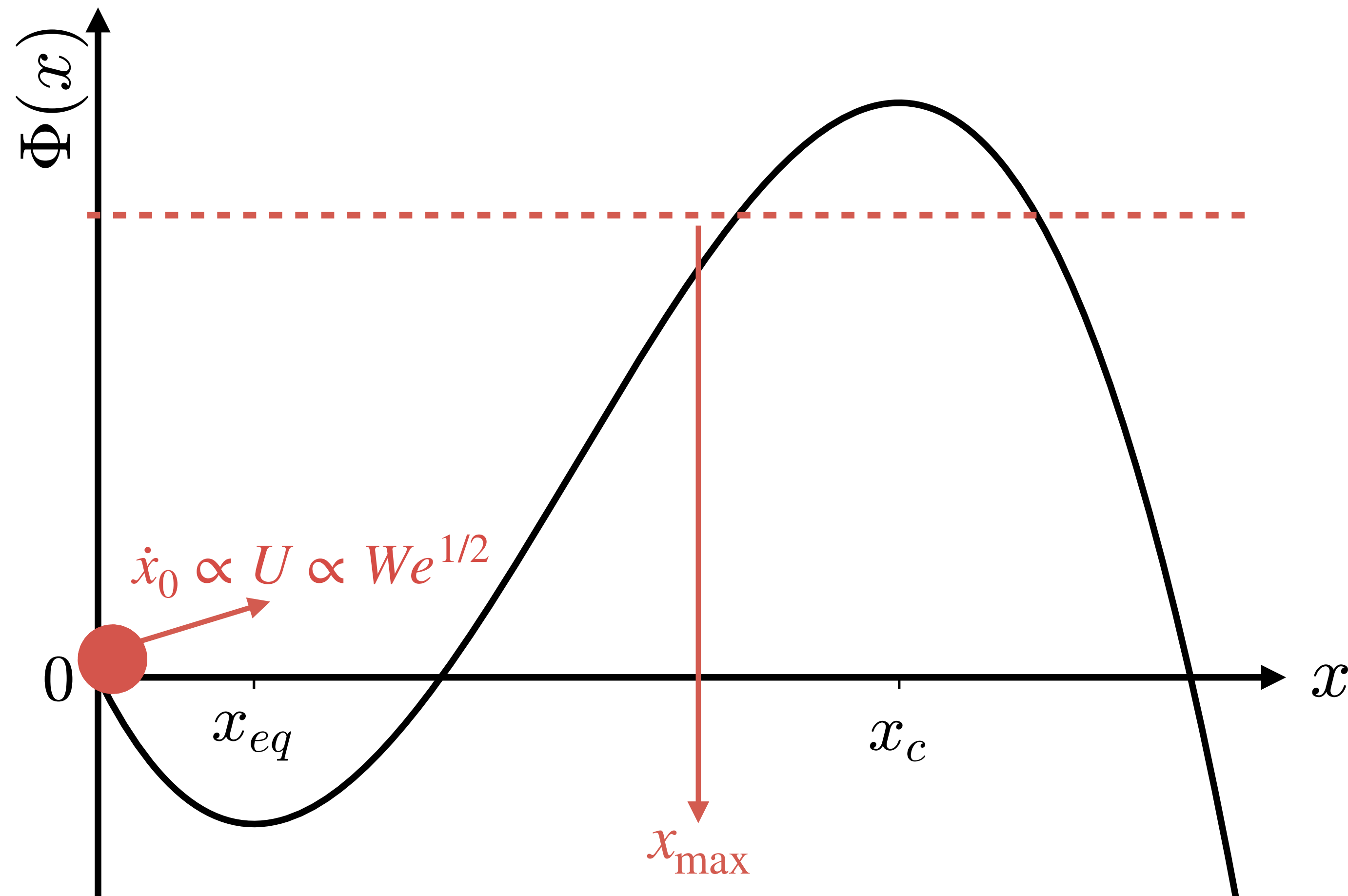


Viscosity is destabilizing



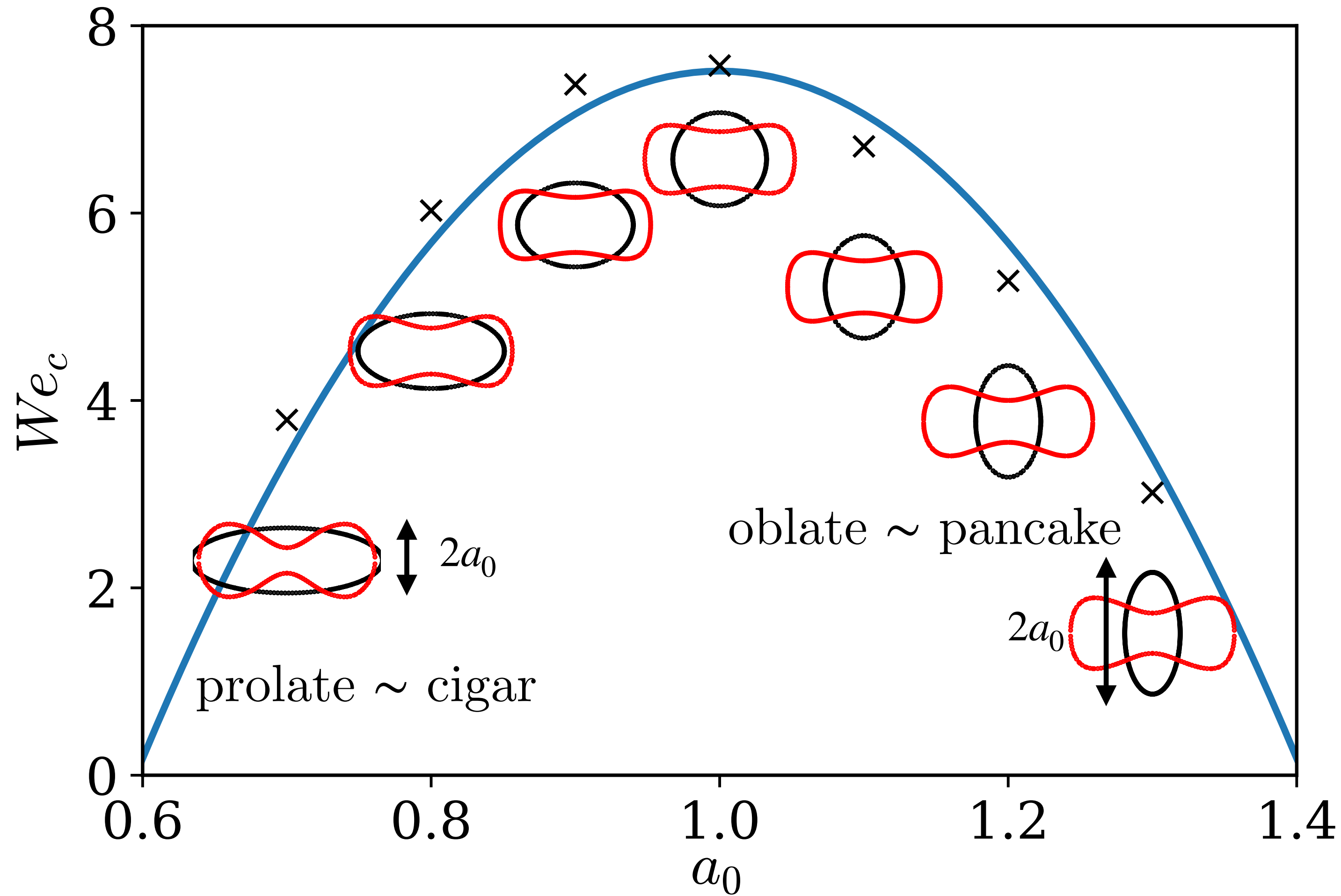
Phase diagram: A bubble as a particle

$$We = \frac{\rho U^2 d}{\gamma} = \frac{\rho E^2 d^3}{\gamma}$$



Sub-critical break-up

Dependance on the IC

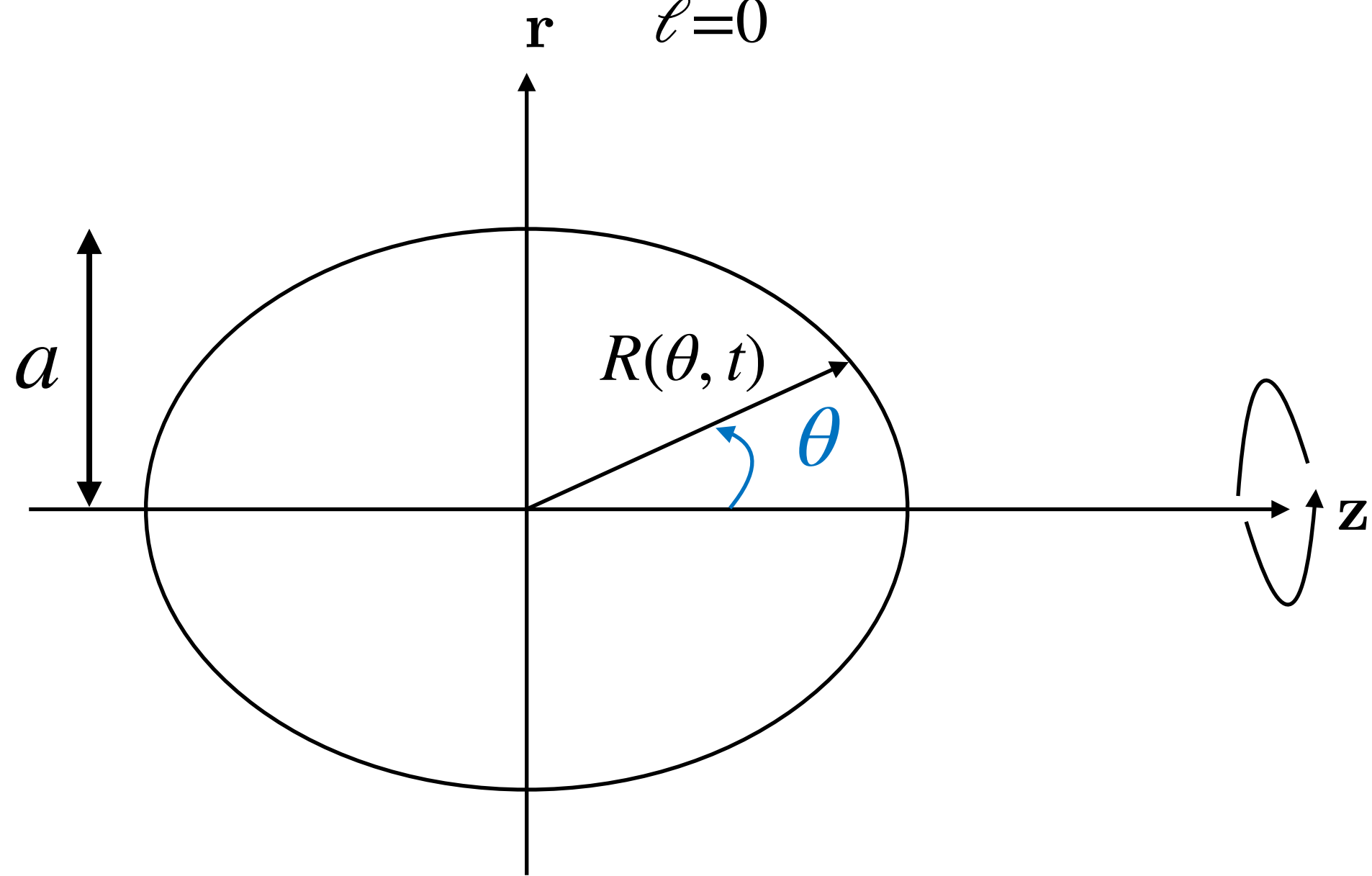


History matters

We must describe the **whole dynamics.**

Deformation quantification

$$R(\theta, t) = R_0 + \sum_{\ell=0}^{\infty} c_{\ell,0}(t) Y_{\ell}^0(\theta)$$

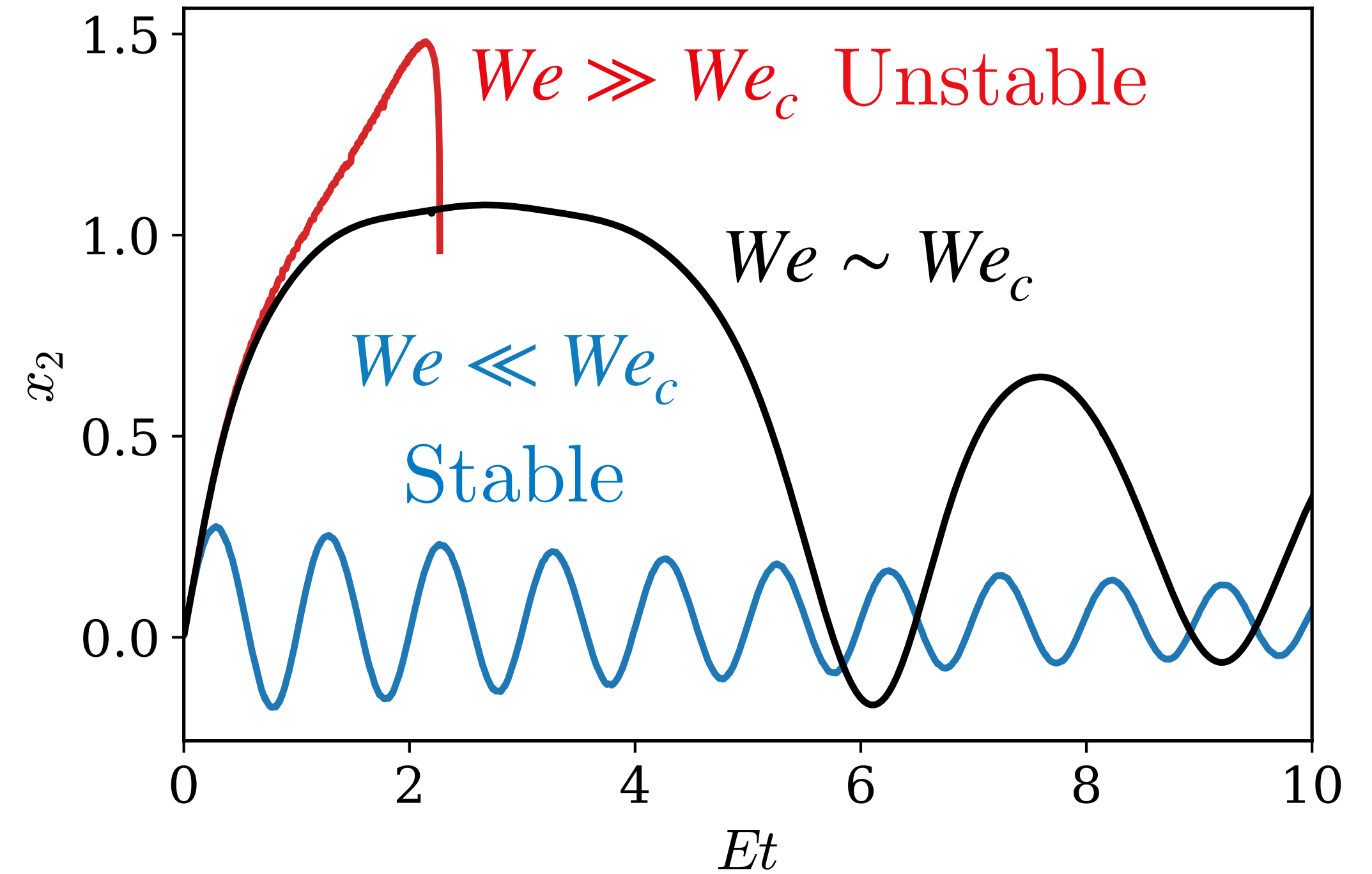


Mode 2 dominates

Risso & Fabre 1998,
Lalanne et al. 2019,
Perrard, Rivière et al. 2021

$$x = \frac{c_{2,0}}{R_0}$$

$Re = 400$



$$\ddot{x} + \Lambda \dot{x} = -\nabla V(x, We, Re, a_0)$$

$$t' = \omega_2 t$$

$$\omega_2 = \sqrt{12} \sqrt{\frac{\gamma}{\rho R_0^3}}$$

Rayleigh (1879)

$$\Lambda = 20 \sqrt{\frac{2}{3}} \frac{\sqrt{We}}{Re}$$

Lamb (1932)

Kang & Leal (1987)

$$\deg(V) = 3$$

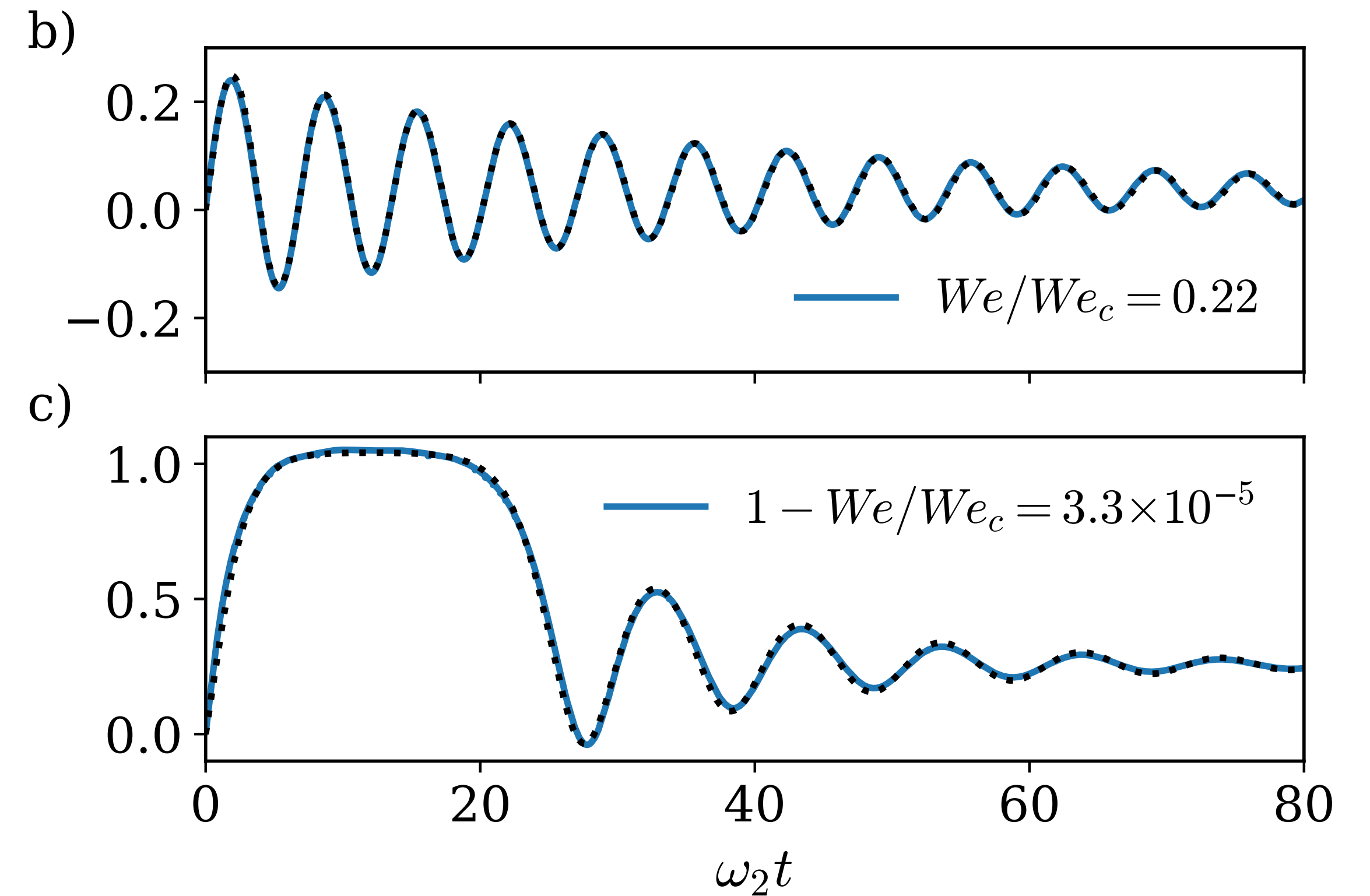
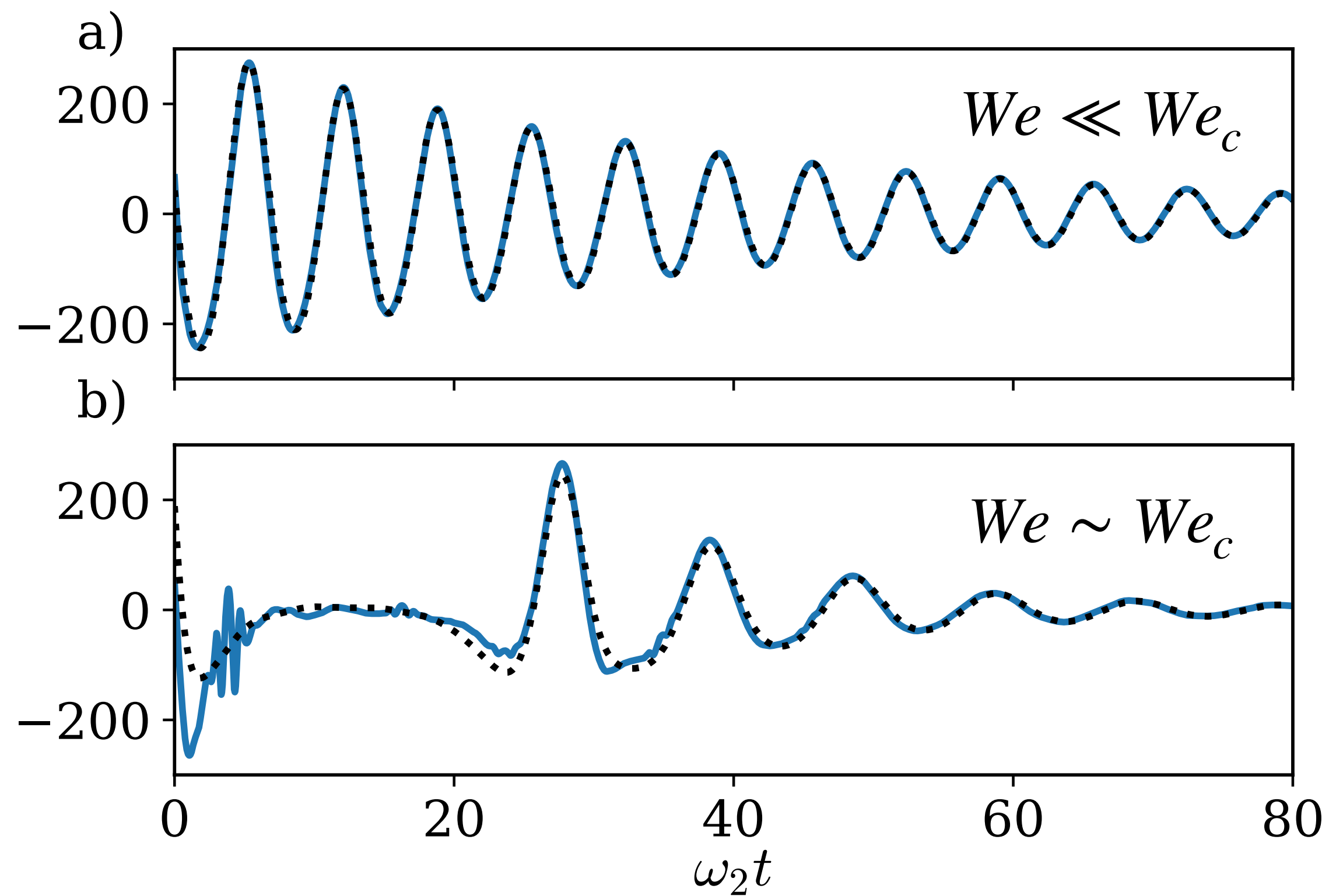
Model fitting

$Re = 400$

$$\ddot{x} + \Lambda \dot{x} = -\nabla V(x, We, Re, a_0)$$

Acceleration \ddot{x}

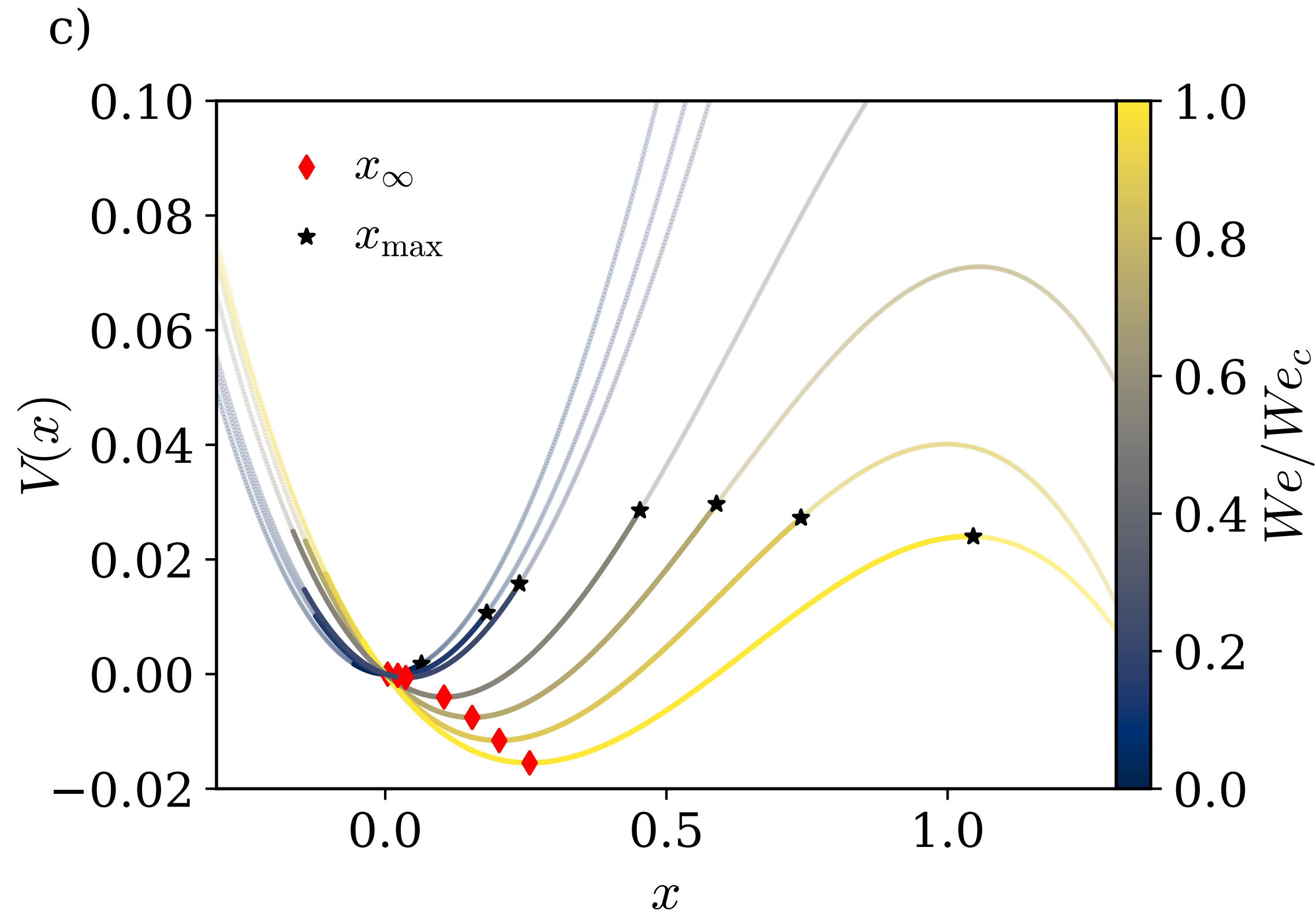
Position x



Effective potential shape - Impact of We

$Re = 400$

$$\ddot{x} + \Lambda \dot{x} = -\nabla V(x, We, Re, a_0)$$

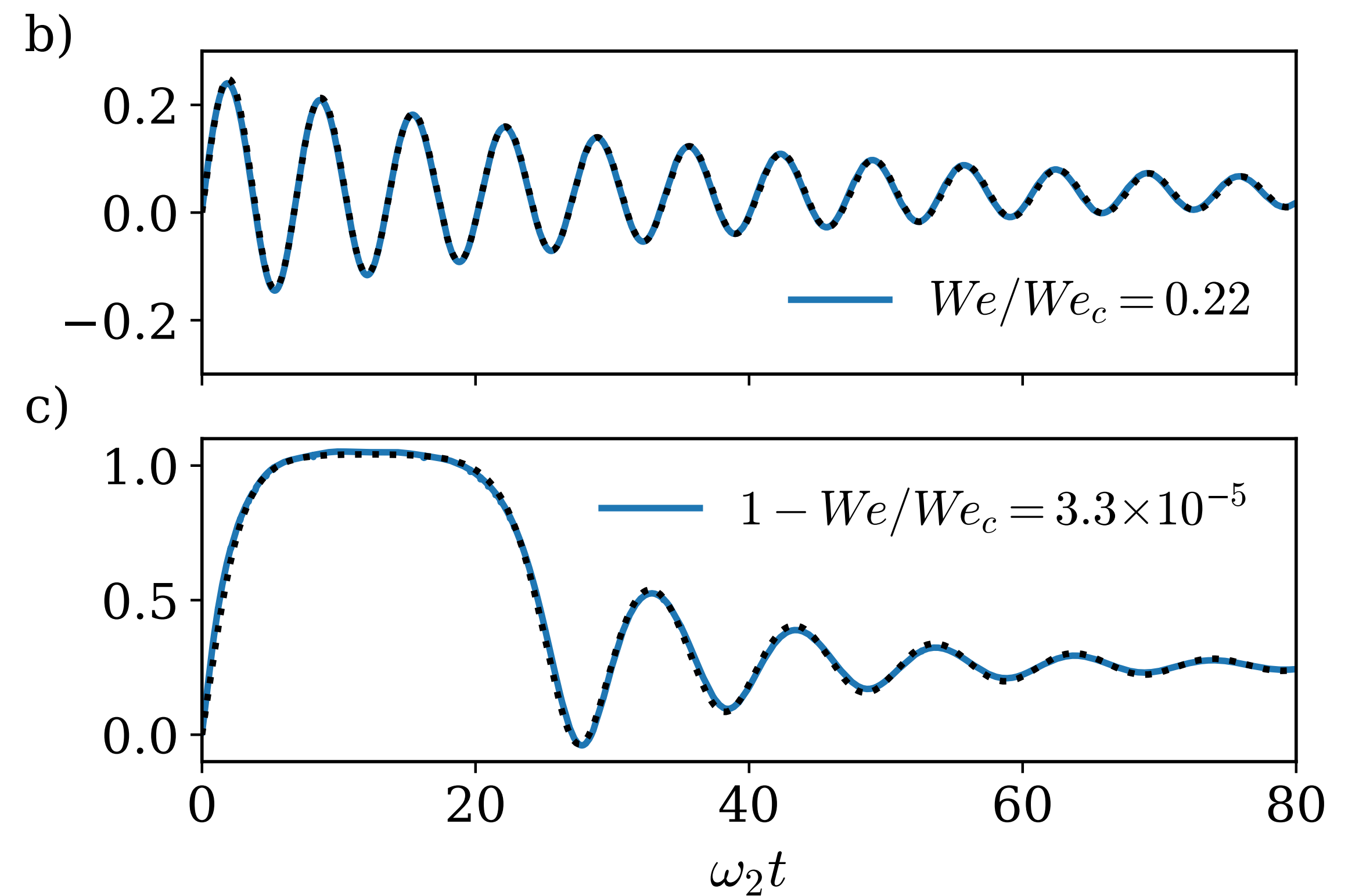


We_c combination of

1. initial velocity
2. potential shape

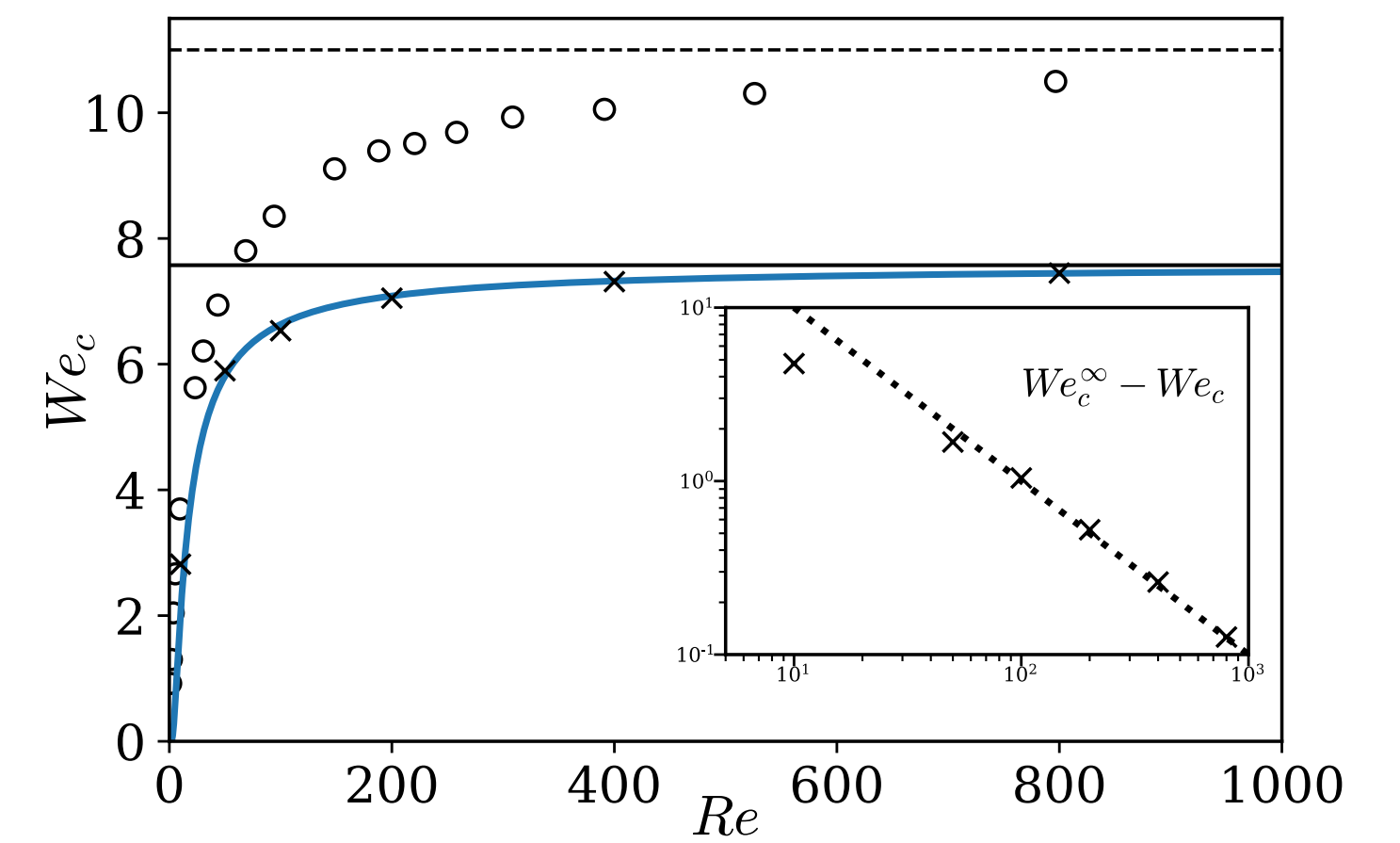
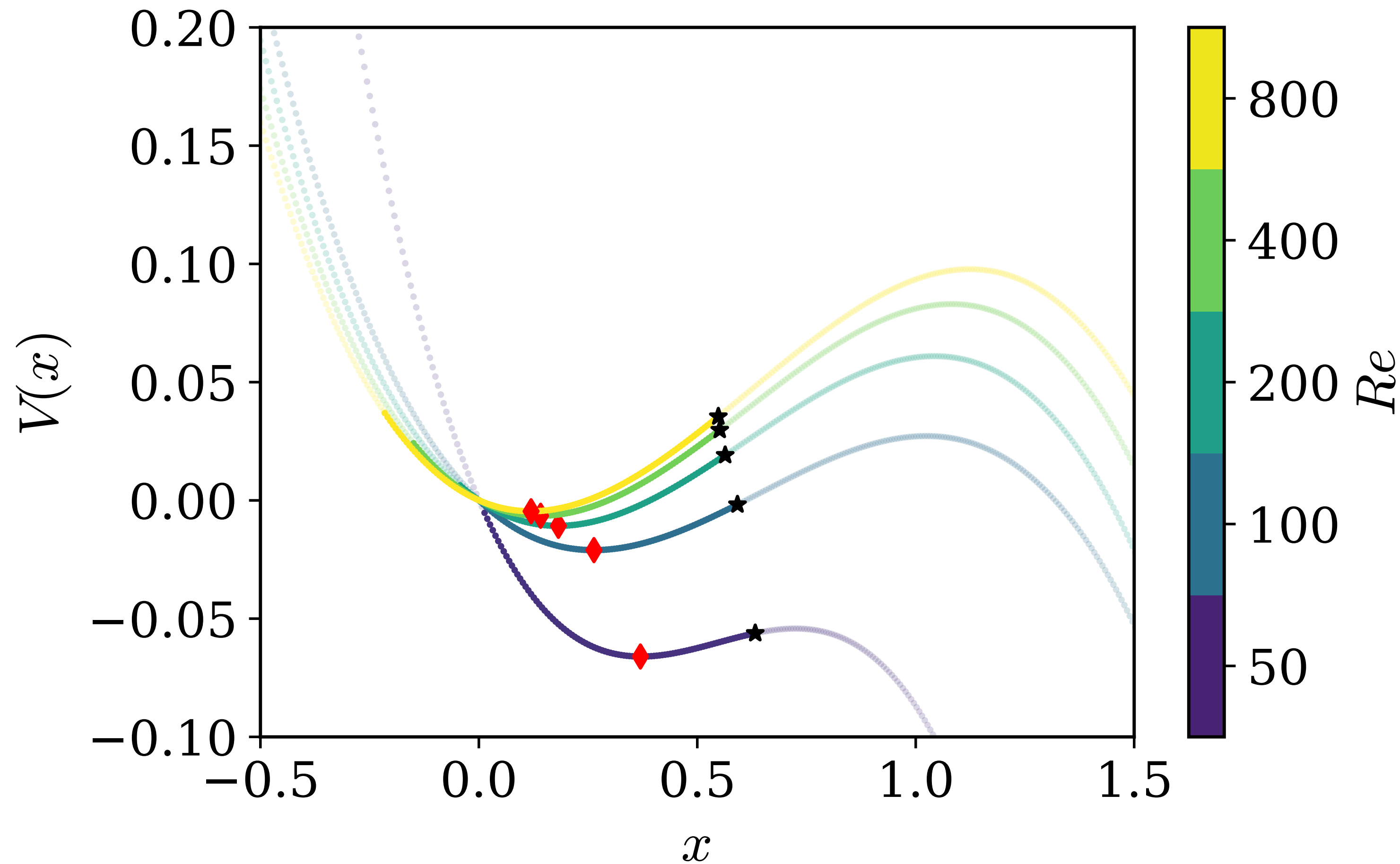
$$\dot{x}_0 \propto \sqrt{We}$$

Position x



Destabilizing effect of viscosity

$We = 5$



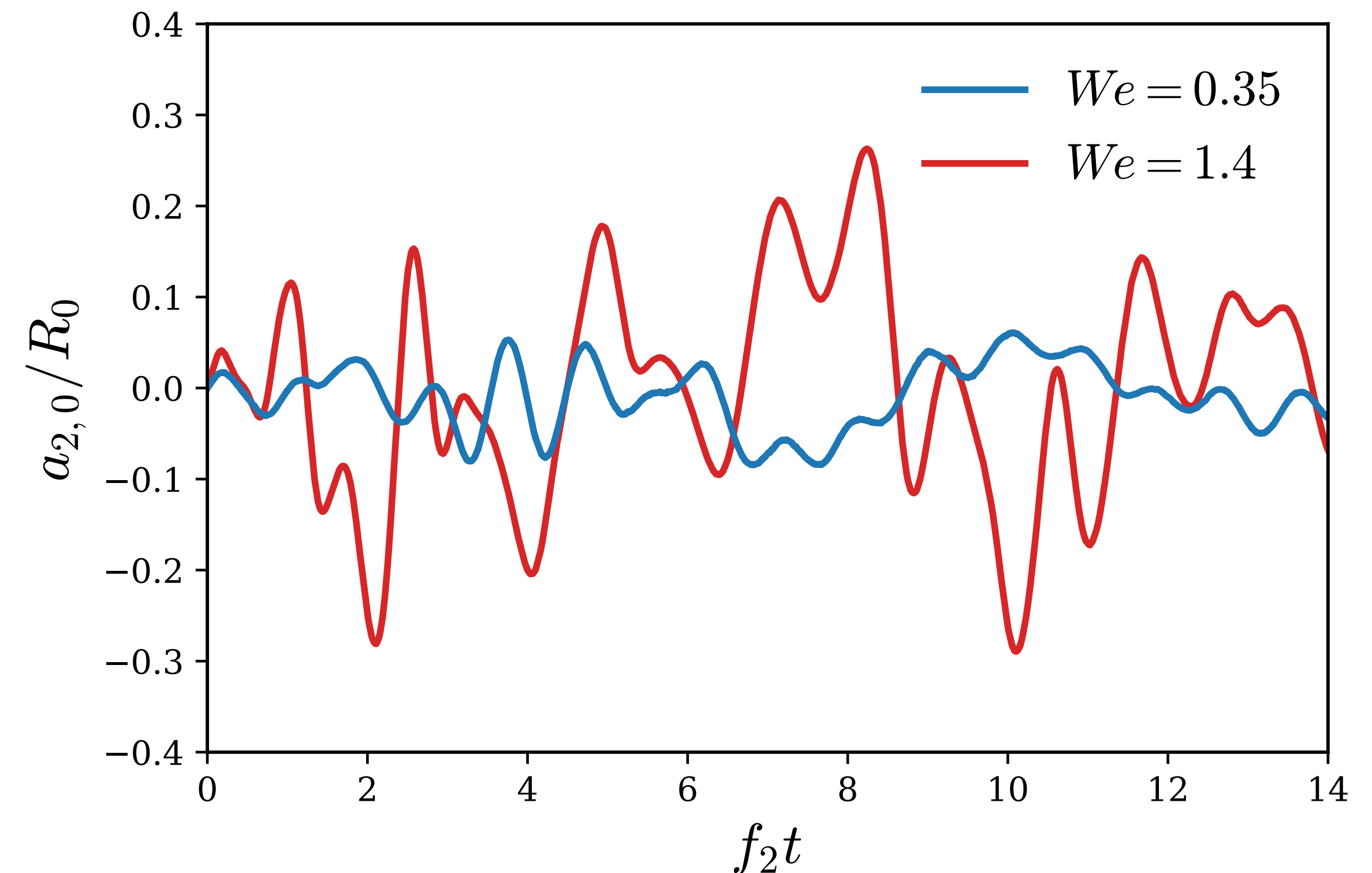
Viscosity is destabilizing

Conclusions & Perspectives

- ▶ Observed transitions are always **subcritical**: call for a **dynamical description**
 - No loss of stability
 - Initial bubble shape matters
- ▶ Quantitative coupling between flow and interface: **1D** non linear oscillator

Rivière & al, Bubble break-up reduced to a 1D non-linear oscillator, *Under review at PRF*

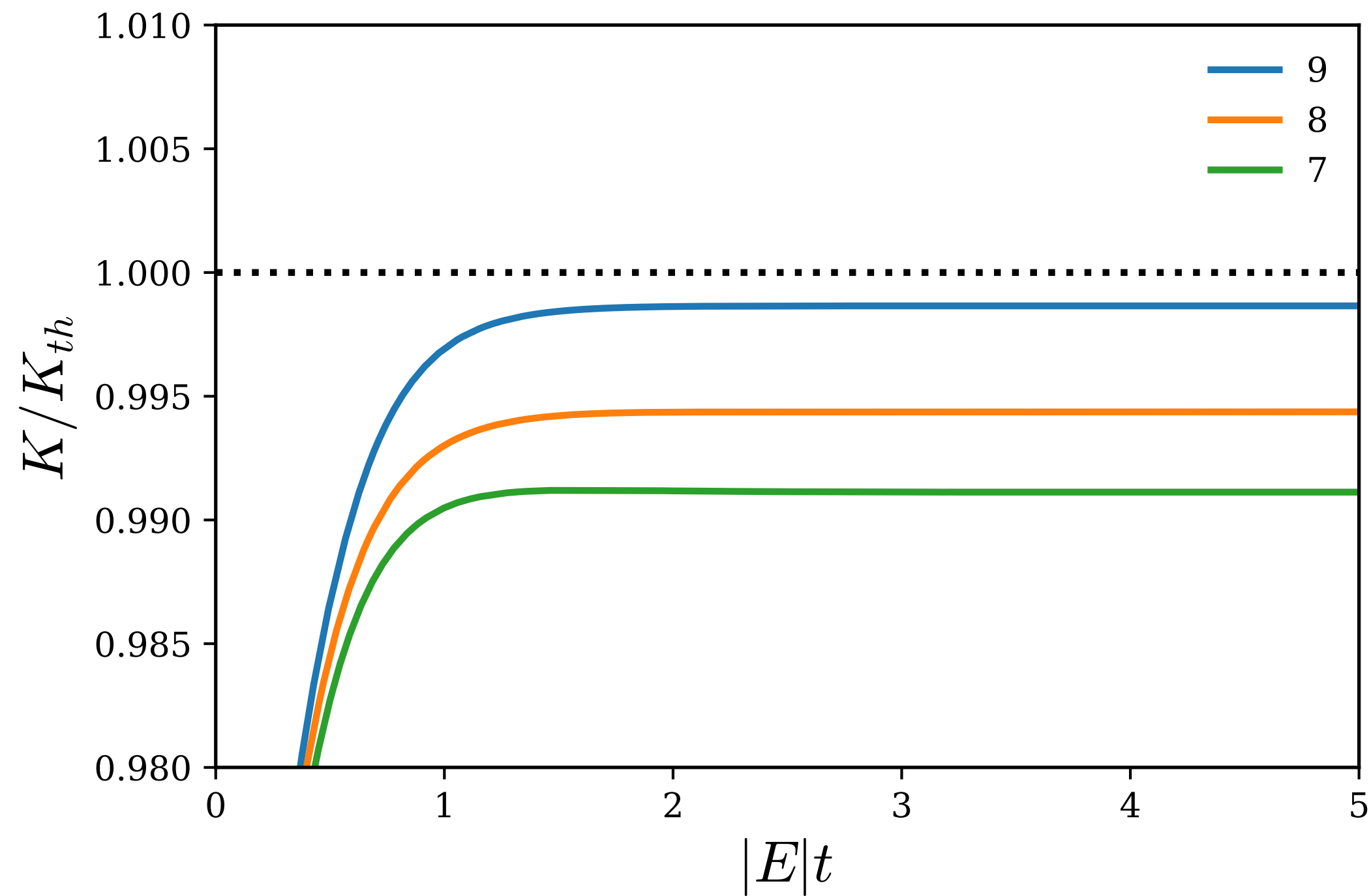
- ▶ Model for bubble in turbulent flows at low We



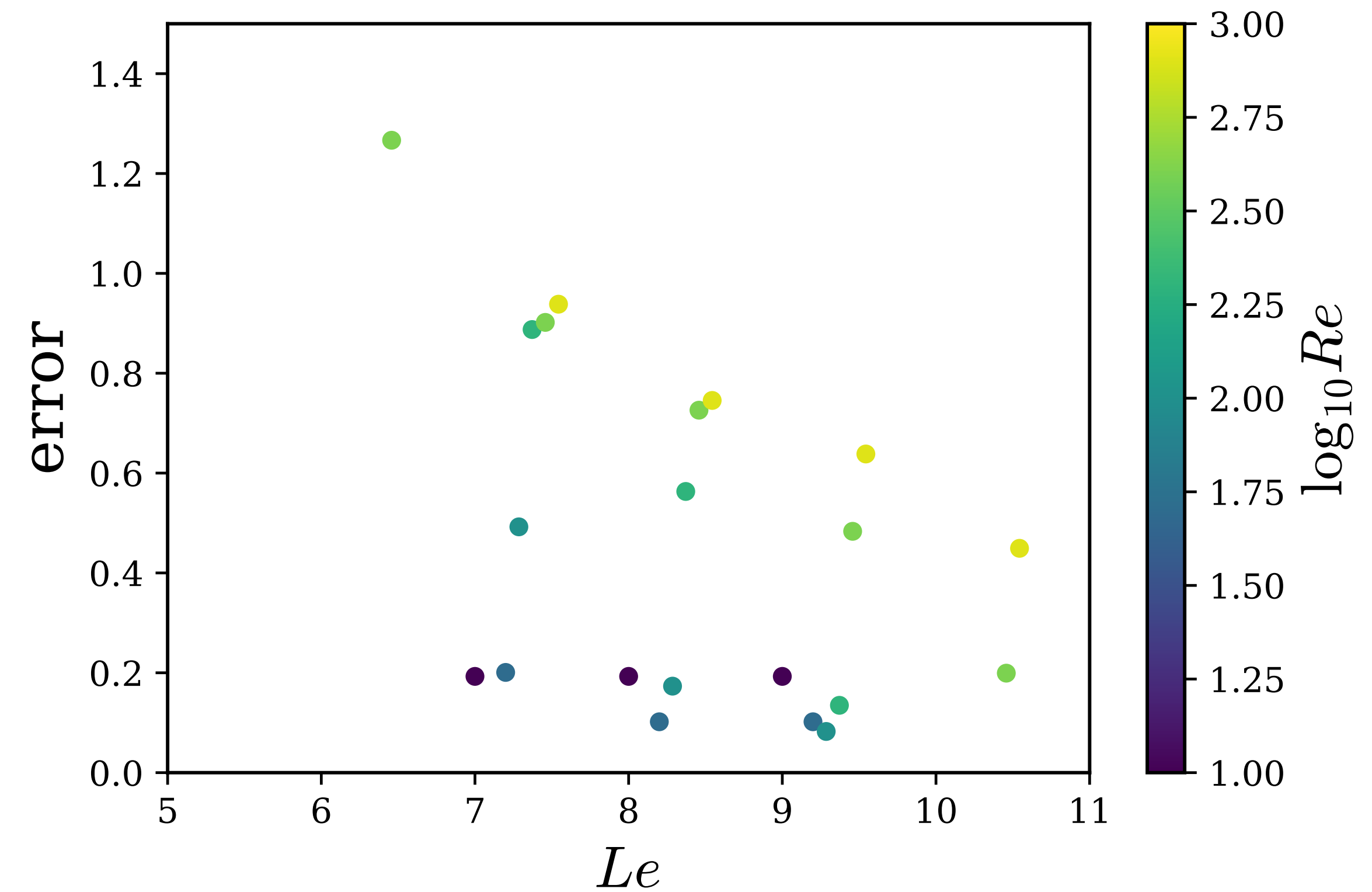
$$\ddot{x} + \Lambda(t)\dot{x} = -\nabla V(x, We(t), Re(t))$$

Convergence study - Flow

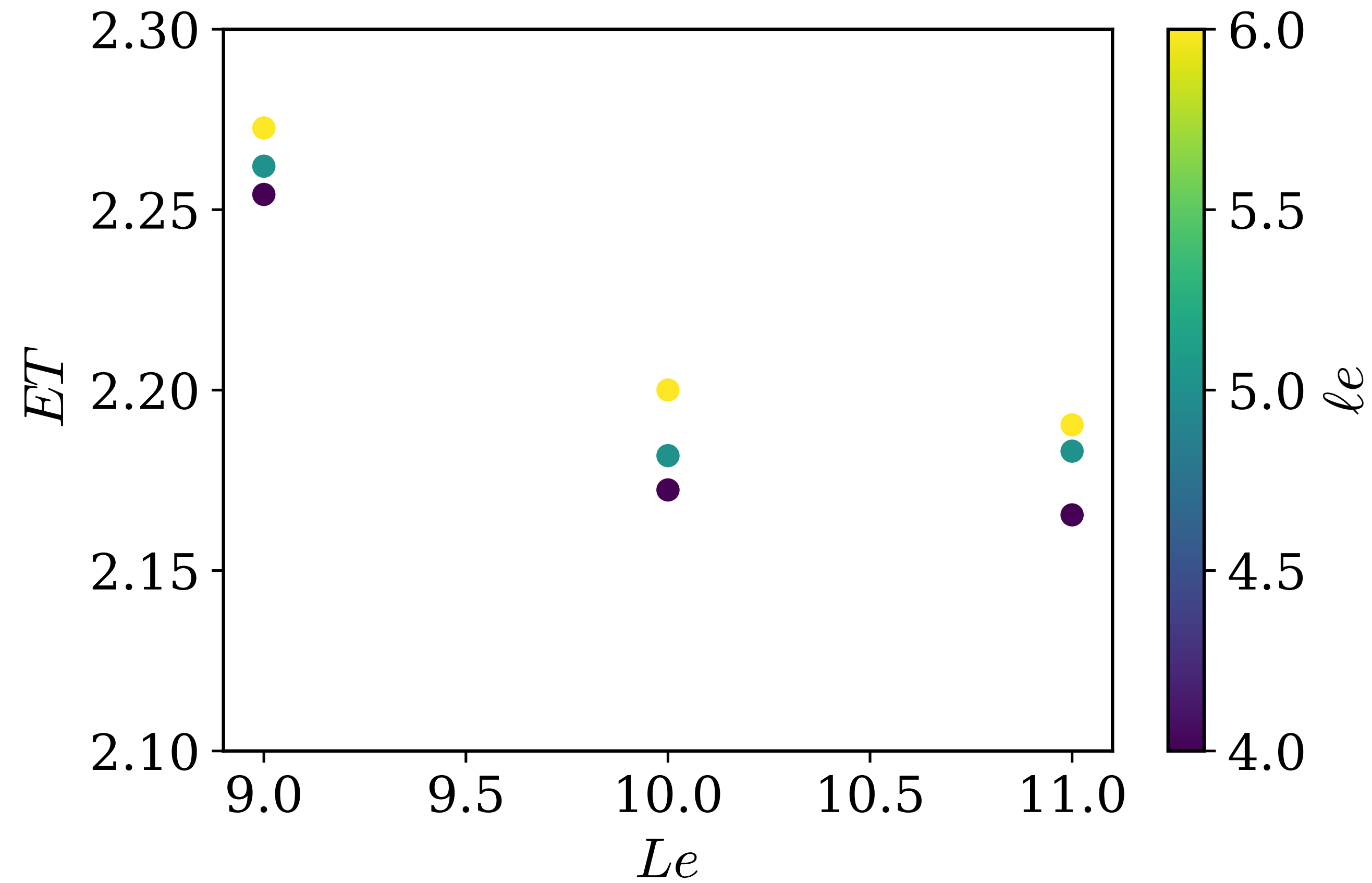
$Re = 200$



$$\text{error} = \frac{|K - K_{th}|}{K_{th}} \cdot 100$$



Convergence study - Bubble



Potential coefficients

$$\ddot{x} + \Lambda \dot{x} = -\nabla V(x, We, Re, a_0)$$

$$\ddot{x} + \Lambda \dot{x} = p_0 + p_1 x + p_2 x^2$$

