Film drainage between small bubbles in pure liquids

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Outline

Industrial context

Bubbly flows are encountered in various IFP Energies Nouvelles processes

- Bubble column reactors (1 mm $\leq a \leq$ 1 cm) : bioreactors, ...
- Froth flotation (500 μm $\leq a \leq$ 1 cm) : separation of particles or droplets
- Dissolved air flotation (1 μm $\leq a \leq$ 100 μm) : separation of microplastics

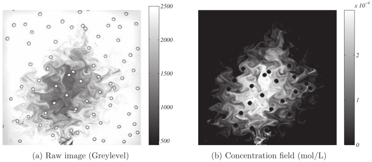


Figure: 2D bubble column from Almeras et al. (2018)

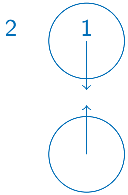
Accurate population balance modeling of such facilities requires knowledge of the "coalescence kernel" whose form depends on the drainage dynamics

We consider small bubbles with weak deformations ($a \leq 200\mu$m)

Bibliography

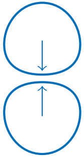
Phenomelogical description of normal fluid particle collision:

Deformations negligible | Deformations & long-time thinning



$t \nearrow$ $\longrightarrow$

$\lambda = \frac{\mu_1}{\mu_2}, Re = \frac{\rho_2 V a}{\mu}, We = \frac{\rho_2 V^2 a}{\gamma}$

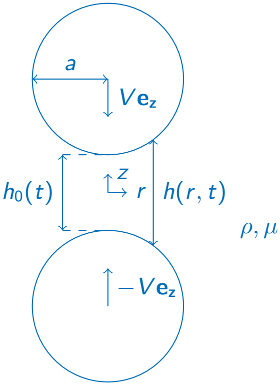


Authors	λ	Re	We	Methodology
Chesters & Hoffman (1982)	0	arbitrary	$\ll 1$	Numerical
Davis et al. (1989)	arbitrary	$\ll 1$	$\ll 1$	Theoretical
Howell (1999)	0	$\ll 1$	$\ll 1$	Theoretical
Pigeonneau & Sellier (2011)	0	$\ll 1$	$0 - 10$	Numerical

What are the forces when the deformations are negligible ? Does the viscosity ratio matter ?

Outline

Problem definition & lubrication assumption



- Bubble velocity: $V(t) = \dot{h}_0/2$.
- Thin film limit: $h_0/a = \epsilon \ll 1$
- Film thickness: $h(r,t) \sim h_0(t) + r^2/a$
- Radial length-scale: $r \sim \sqrt{a h_0}$
- Shear-free boundary conditions: $\lambda \ll \epsilon^{1/2}$
- Negligible deformations: $Ca \ll \epsilon$ or $We \ll \epsilon$

Governing equations

Mass and momentum conservation equations (Chesters & Hoffman 1982, Savva & Bush 2009):

$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(urh) = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} + 2\mu\left(\frac{\partial u^2}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{h}\frac{\partial h}{\partial r}\left(2\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right). \tag{2}$$

From (1) we get

$$u = -\frac{\dot{h}_0}{2}\frac{r}{h}, \tag{3}$$

and from (2) we obtain

$$p = p_\infty - \rho\frac{\dot{h}_0^2}{4}\left(-\frac{a}{h} + \frac{1}{2}\frac{r^2}{h^2}\right) + \rho\frac{\ddot{h}_0}{2}\frac{r}{h} - \frac{\mu\dot{h}_0}{h}. \tag{4}$$

Viscous and inertial lubrication forces

The hydrodynamic force on the bubble reads

$$F = 2\pi \int_0^{R_\infty} \left(p - 2\mu \frac{\partial w}{\partial z} \right) r dr. \quad (5)$$

Integrating the pressure and the viscous stress we obtain

$$\boxed{F = -2\pi\mu V a \log\left(\frac{a}{h_0}\right) + \frac{\pi}{2}\rho V^2 a^2 \log\left(\frac{a}{h_0}\right)} \quad (6)$$

where $R_\infty \propto a$ (to be validated *a posteriori*).

The force is valid for arbitrary Reynolds number !

Comparison with exact solutions

Stokes flow

Kim & Karrila (1991) solution based on the bispherical coordinate solution of Haber et al. (1973):

$$F = -2\pi\mu a V \left[\log\left(\frac{a}{h_0}\right) + A\right] \tag{7}$$

where $A = 2(\gamma + \log 2)$ and γ is the Euler's constant.

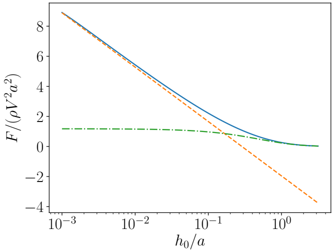
Potential flow

The flow in the film is a plug flow $\mathbf{u} \approx u(r,t)\mathbf{e}_r$, hence $\nabla \times \mathbf{u} \approx \mathbf{0}$.
Miloh (1977):

$$F = 4\pi\rho a^2 V^2 \sum_{n=0}^{\infty} (n+2) A_n A_{n+1},$$

where the coefficients A_n obeys the following equation

$$A_n = -\frac{1}{2}\delta(n-1) + \frac{n}{(n+1)!}\sum_{m=1}^{\infty} A_m \frac{(m+n)!}{m!}\left(\frac{1}{2+h_0/a}\right)^{m+n+1}.$$



$$F = \frac{\pi}{2}\rho V^2 a^2 \left[\log\left(\frac{a}{h_0}\right) + B\right] \tag{8}$$

where $B = -1.24$.

Film drainage time scale of small translating bubbles

Vakarelski et al. (2018) experiments

- Bubble ($a \approx 100\mu$m) rising toward a liquid-gas interface : $Re \approx 0.08$, $We \ll 1$, $\lambda \approx 0.001$
- Film drainage time scale t_c is approximately 3.6 ms.

Force balance $F = F_{\text{Archimedes}}$

$$(2\gamma + \ln 2 + 1) h_0^* - h_0^* \log(h_0^*) = (2\gamma + \ln 2 + 1) h_0^*(0) - h_0^*(0) \log(h_0^*(0)) - \frac{2}{3} t^* \quad (9)$$

where $h_0 = a h_0^*$ and $t = \mu/(\rho a g) t^*$. Film rupture when $h_0 = 0$.

$$t_c^* = \frac{3}{2} h_0^*(0) \left(2\gamma + \ln 2 + 1 - \log(h_0^*(0))\right). \quad (10)$$

leads to $1.6\text{ms} \leq t_c \leq 6\text{ms}$

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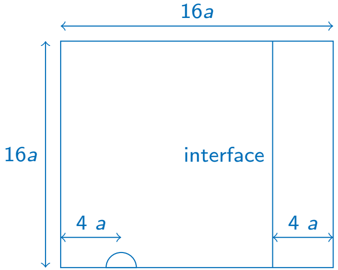
Outline

Numerical methodology

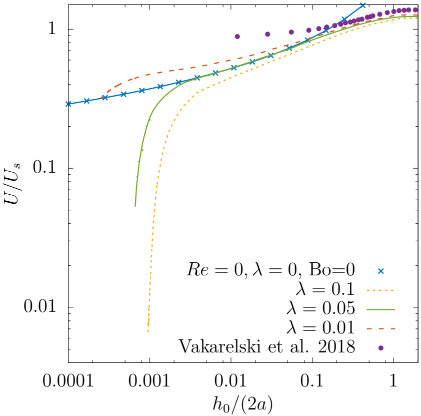
Vakarelski et al. (2018) with $a \approx 260\mu$m: $Re \approx 1.3$, $We \approx 0.01$, $\lambda \approx 0.001$



- Axisymmetric computations with adaptative mesh refinement (up to Level 16)
- Runs up to thickness $h_0/a \approx 0.0005$: the calculations are running during weeks !
- Computations $0.01 \leq \lambda \leq 0.1$

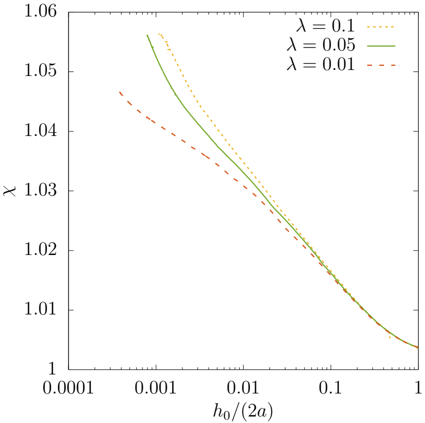
The main constraint is not the mesh but the capillary time step ($\Delta t_\gamma \sim \sqrt{\rho \Delta x^3/\gamma}$) !

Bubble velocity



- Important effect of the viscosity ratio when $h_0/a \ll 1$
- For air bubble in water $\lambda \approx 0.02$ the shear-free boundary condition is most probably inaccurate
- The Stokes solution ($\lambda = 0$) does not match the experimental results
- The initiation of the quasi-steady regime depends on the viscosity ratio
- Balance of shear stress : $\mu_1 \partial u_t/\partial n|_1 \sim \mu_2 \partial u_t/\partial n|_2$. Hence the shear stress within the bubble must be considered for $\lambda \sim (h_0/a)^{1/2}$

Bubble deformation



- No significant effect of the viscosity ratio on the bubble deformation for $h_0/a \geq 0.01$
- Little effect for smaller thickness

Outline

Conclusion

- Forces on bubbles are singular when they are in close proximity
- Accurate predictions of the forces require $\mathcal{O}(1)$ terms which cannot be obtained by lubrication alone
- The liquid-gas viscosity ratio influences the bubble dynamics (except for extremely small viscosity ratio)

Future works

Negligible deformations

- One may readily include radius variations in the lubrication framework (Pierson, 2023 to be submitted)
- For $Re \sim 1$, the $\mathcal{O}(1)$ contribution to the forces may be obtained using DNS.
- $Re \gg 1$ (potential flow limit) : include other forces (Added mass, Levich dissipation force, ...)
- Include the effect of a finite shear stress (i.e. viscosity ratio) in the lubrication equations

Finite deformations

- Revisit Chesters & Hoffman (1982) study by considering a constant force problem.
- What is the interplay between the deformation and the viscosity ratio ?
- Consider the effect of surfactants