Three fluids Simulations

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July 5, 2023

General Context

2 fluids model

$$\begin{cases} \rho_i(\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}) = -\vec{\nabla}P + \nabla(2\mu_i\mathbb{D}) + \rho_i\vec{g} \\ \nabla.\vec{u} = 0 \\ [\underline{\underline{\sigma}}.\vec{n}] = \gamma\kappa\vec{n} \end{cases}$$

This system can be transcript in the 1-fluid formulation

$$\begin{cases} \rho^*(\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}) = -\vec{\nabla}P + \nabla(2\mu^*\mathbb{D}) + \rho^*\vec{g} + \gamma\kappa\delta_s\vec{n} \\ \frac{\partial \rho^* u}{\partial t} + \nabla.(\rho^*\vec{u}) = 0 \\ \nabla.\vec{u} = 0 \end{cases}$$

With $\rho^* = \rho_i * \chi + \rho_j * (1 - \chi)$, (resp μ^*) and χ the caracteristic function.

Let's work with 3 fluids

What's new now if we add a third fluid ?

1. Can we still use the 1-fluid form ?
2. Can we use still one color function ?
3. How solve such a system in *Basilisk* ?

How describe such a system

In *Basilisk* for two fluids we use 1 caracteristic function, we could also use 2, one per fluids : it leads to the same resolution
What about three fluids ?

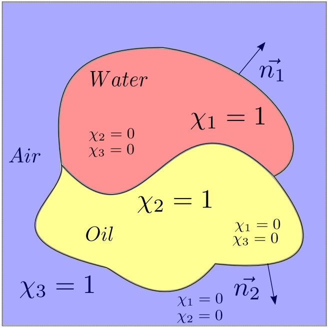


Figure: We chose to define 1 caracteristic function χ_1, χ_2, χ_3 for each fluids

3 fluids 1-fluid formulation

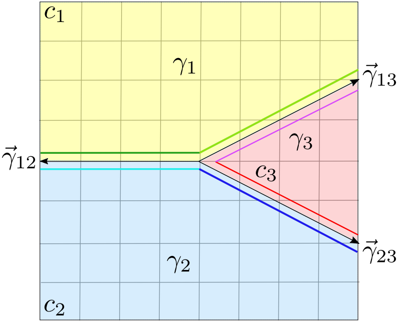
$$\begin{cases} \rho_i(\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}) = -\nabla P + \nabla(2\mu_i\mathbb{D}) + \rho_i\vec{g} \\ \nabla.\vec{u} = 0 \\ [\underline{\underline{\sigma}}_{ij}.\vec{n}] = \gamma_{ij}\kappa\vec{n} \end{cases}$$

This system can be transcript in the 1-fluid formulation

$$\begin{cases} \rho^*(\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}) = -\vec{\nabla} P + \nabla(\mu^*\nabla.\vec{u}) + \rho^*\vec{g} + \gamma^*\kappa\delta_s\vec{n} \\ \frac{\partial \rho^* u}{\partial t} + \nabla.(\rho^*\vec{u}) = 0 \\ \nabla.\vec{u} = 0 \end{cases}$$

With $\rho^* = \rho_w * \chi_1 + \rho_o * \chi_2 + \rho_g * \chi_3$, (resp μ^*) and γ^* is a function of the interfaces.

How are interface represented in *Basilisk*



$$c_k^{ij} = \int_{cell} \chi_k \mathrm{d}v$$

$$c_1 + c_2 + c_3 = 1$$

$$\gamma_{12} = \gamma_1 + \gamma_2$$
$$\gamma_{13} = \gamma_1 + \gamma_3$$
$$\gamma_{23} = \gamma_2 + \gamma_3$$

Figure: There is 3 physical interfaces with each their $\vec{\gamma_{ij}}$ this lead to 6 numerical interfaces

let's make a test

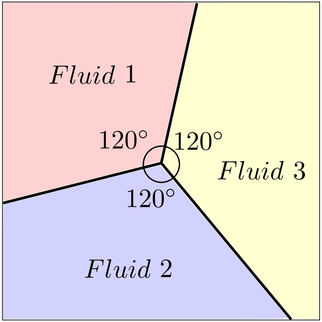


Figure: 1 fluid with 3 differents domain, at equilibrium each angle equals 120°

Let's make a test

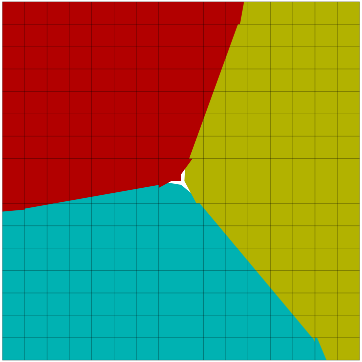


Figure: We solve the NS equation for 3 fluids with exactly the same physical properties, thus the three equilibrium angles equals 120°.

How the triple is reconstructed

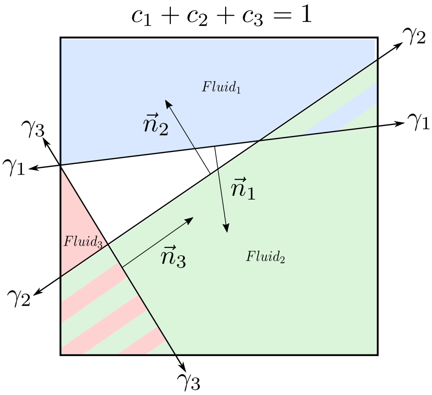


Figure: The VOF method reconstruct one interface per fluids. Here at time t the sum of the color function is conserved. However at $t + dt$, this propertie won't be verify anymore : $\Sigma_i c_i \neq 0$

Test setup

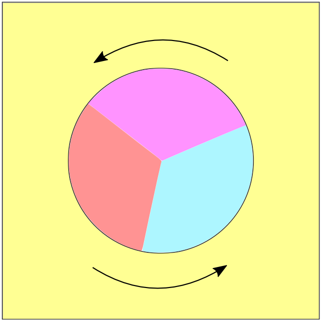


Figure: 4 colors functions are used but only 3 can be at the same time in 1 cell. We apply a solid rotation

Advection and surface tension

We turn of the surface tension term in the solver

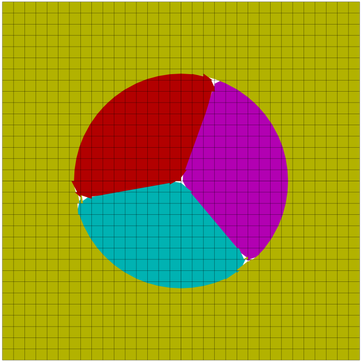
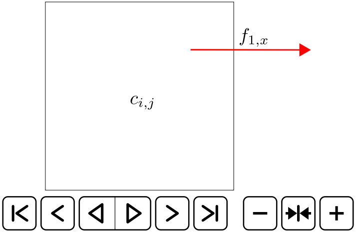
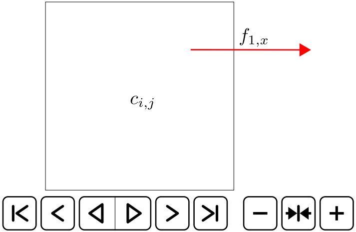


Figure: Even without surface tension term the advection propagate a local mass error.

Can we improve this ?



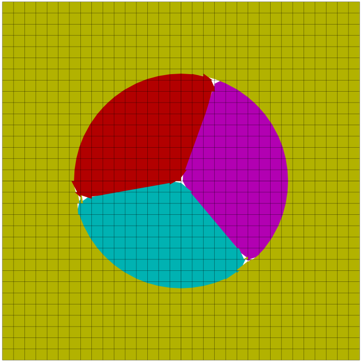
(a) Advection before patch : $\Sigma_k c^k \neq 1$ but $\Sigma_{ij}(\Sigma_k c^k_{ij}) = 1$



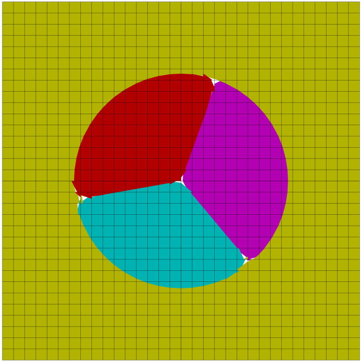
(b) Advection after patch : $\Sigma_k c^k \neq 1$ but $\Sigma_{ij}(\Sigma_k c^k_{ij}) \neq 1$

Comparison between the 2 methods

The advection is tested here : we do not solve the NS equation

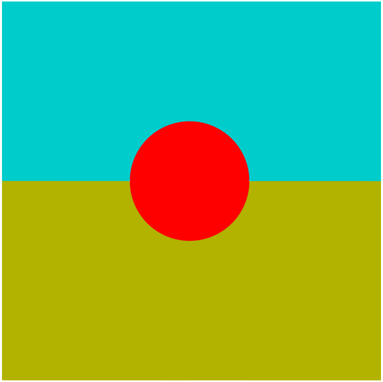


(a) Basic advection scheme

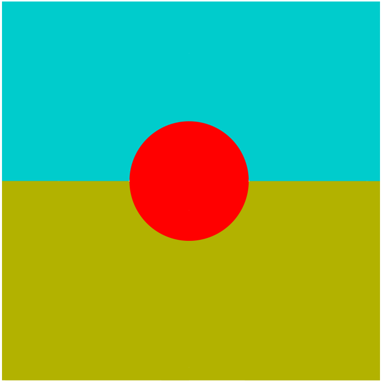


(b) Modified advection scheme

Physical test case on oil lens

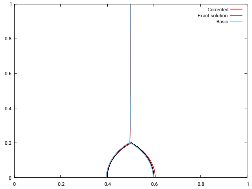


(a) Basic advection scheme

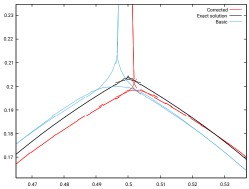


(b) Modified advection scheme

Let's take a look at the triple points



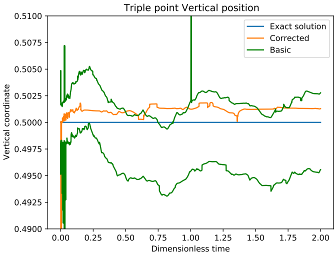
(a) Interfaces of triple points, in black the analytical solution, in blue the original solution and in red the corrected one



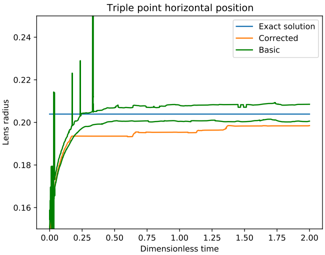
(b) Zoom on the triple points, in black the analytical solution, in blue the original solution and in red the corrected one

Quantitative results

$\frac{\sigma_{13}}{\sigma_{12}}$	$\frac{\sigma_{23}}{\sigma_{12}}$	L_0	L_0^{exact}	$\frac{abs(L_0 - L_0^{exact})}{L_0}$
1.22	1	0.203901	0.198525	2.64 %
1.33	1	0.206748	0.21129	2.15 %
1.22	1.44	0.18086	0.188789	4.2 %



(a) Triple point vertical position



(b) Triple point horizontal position

Exemple of 4 phases use

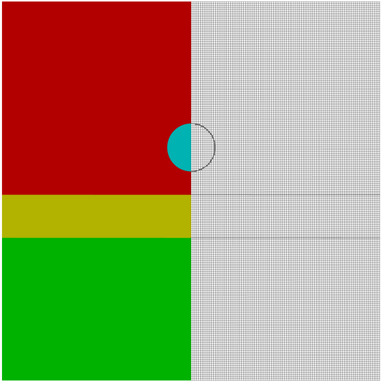


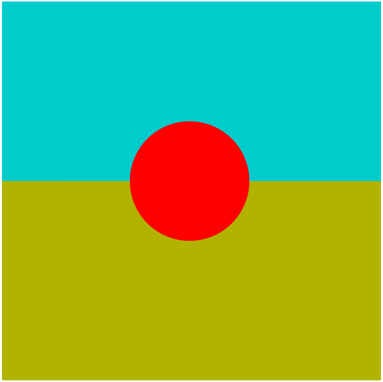
Figure: Impact of water droplet over a oil film, itself over a deep water pool in presence of a gas usig 4 colors functions. Thus the droplet immiscible with the pool.

Conclusion

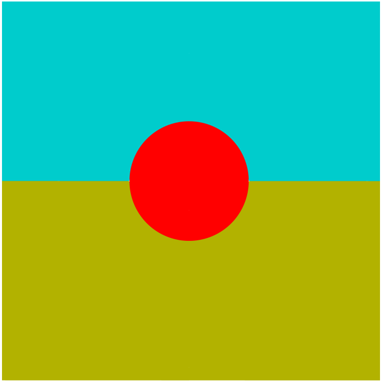
1. We used 3 characteristics functions to describe the 3 fluids problems : This allow to choose independantly the surface tension.
2. We highlight the main issue : resolution of the surface tension term & the growth of the triple point due to the VOF advection.
3. We are currently writing an article characterising the error spreading of the triple point
4. I propose and describe a correction to handle the dissipation of the error.
5. I used this method to perform various physical cases.

Droplet encapsulation

In this simulation the triple point is unstable



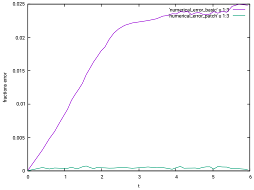
(a) Basic advection scheme



(b) Modified advection scheme

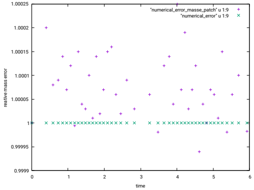
Quantitative measurement

Comparison between the basic version and the corrected one.



(a) Global volume fraction error :

$$\Sigma_{ij}(f_1[] + f_2[] + f_3[] - 1) * Vol_{ij}$$



(b) Relative mass error :

$$\frac{\Sigma_{ij}(f_1[] + f_2[] + f_3[])}{\Sigma_{ij}(f0_1[] + f0_2[] + f0_3[])}$$

- The basic version doesn't not conserve the local volume fractions due to overlap and empty zone propagation.
- The corrected version is adding or substract mass (around 0.0002 % of the global mass)