High-speed impact across scales: a hybrid approach

Basilisk (and Gerris) Users' Meeting

Sorbonne Université, July 6th, 2023

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Introduction

Group updates

Dr. Michael Negus (2022)

Dr. Ben Fudge (2023)

Oscar Holroyd (2025)

Sebastian Dooley (2026)

Some activities in the wider collaborative group:

Drop dynamics
- Bouncing
- Coalescence
- Splashing
- Fluid-structure interaction

Liquid films
- Multi-physics modelling
- Asymptotic analysis
- Control theory
- Equation discovery

Industrial mathematics

Sustainable software

Outreach and art

Acknowledgements:
- **EPSRC** (EP/V051385/1 on liquid film control, EP/W032201/1 on ReproHacks) **and NSF** (EP/W016036/1)
- **UK Fluids Network** (Drop Dynamics and Interfacial Flows SIGs)

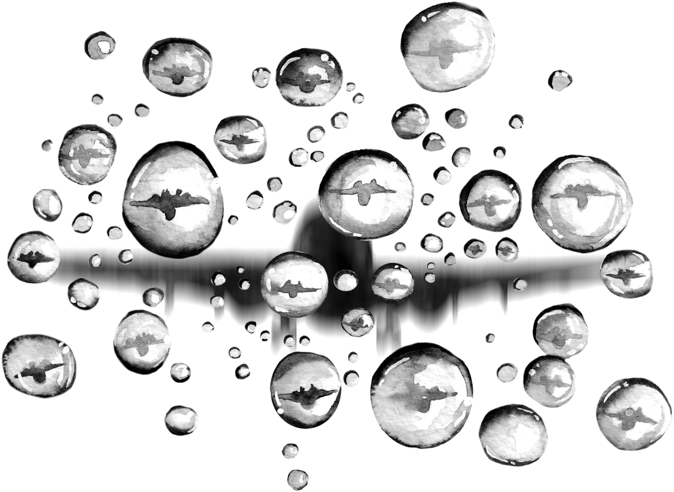
WARWICK
THE UNIVERSITY OF WARWICK

UK FLUIDS NETWORK

@rcimpeanu

GitHub rcsc-group

Throwback Thursday



Motivation

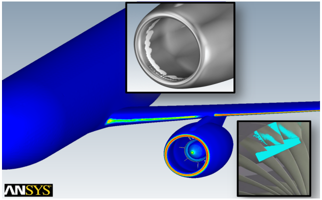
Background

Studying the dynamics of droplets in high speed flow conditions has ramifications particularly in the aeronautics industry, where the following flight conditions:

- heavy rainfall
- high liquid water content (LWC) regions - clouds

are commonplace.

Ice formation severely affects aerodynamic performance. Of particular interest are the adverse effects on the nacelle system.



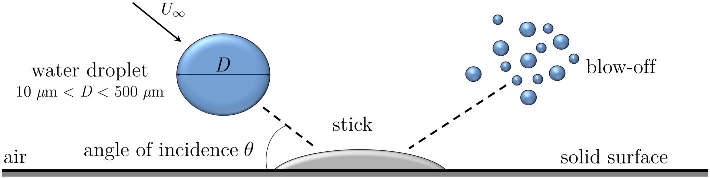
Introduction

Fundamental Research Question

Proposed new rulemaking by both EASA and the FAA is requiring water catch analysis to consider freezing rain and drizzle, which are characterized by water droplets of larger diameter than what are general considered for certification.

Task description:

- Blow-off factors for water catch are currently not well understood;
- required in the analysis of Nacelle ice protection system performance;
- would result in reduced bleed air off-take demand on the engine compressor, hence reduced fuel burn.



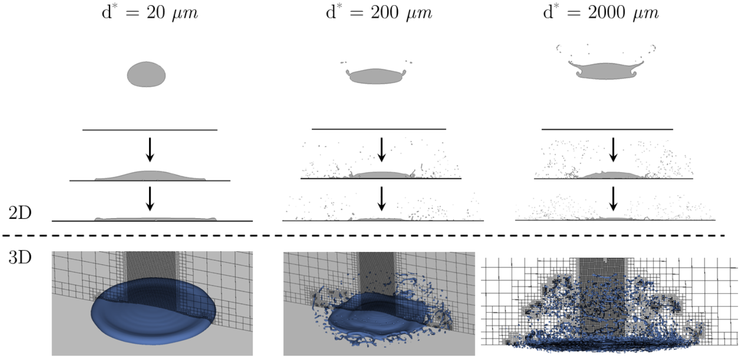
Primary objective: Using modelling, analysis and computational fluid dynamics (CFD) tools in order to study the rich dynamics of drop impact onto solid surfaces (including drop formation and pinch-off) within parameter ranges suitable from the physical standpoint.

Single drop impact

Droplet behaviour - summary - RC & Papageorgiou, IJMF 107, 192-2017 (2018)

Detailed analysis of splashing dynamics has been performed in **both two and three dimensions**, revealing **similar qualitative and quantitative features.**

- small drops: pancaking behaviour, **no splashing**;
- medium drops: deformation before impact, **moderate splashing**;
- large drops: stronger deformation, **violent splashing**;



Pre-impact dynamics

Experiments (NASA & INTA)

During 2012 – 2015, Vargas, Sor and Magarino (NASA - INTA Madrid collaboration) have conducted a set of experiments on droplet breakup near the leading edge of an airfoil.

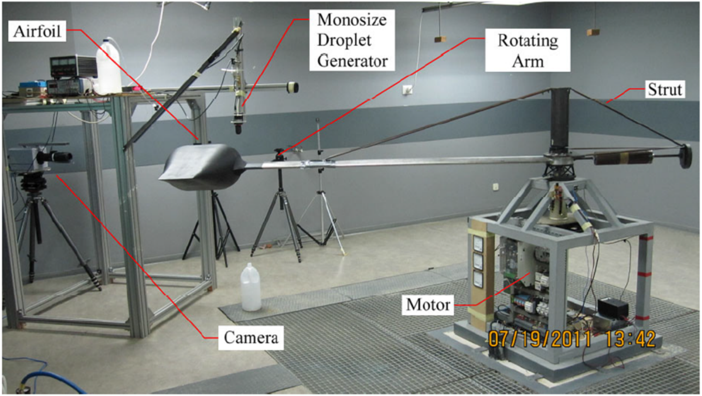


Figure 2.—Experiment set-up in the INTA test cell.

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Key observations:

- small drops tend to retain their shape;
- medium sized drops flatten into squashed ellipsoidal shapes;
- large drops eventually break under violent rupture.

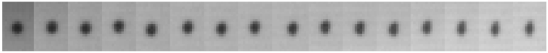


Fig. A1 Case 1249 droplet deformation evolution: airfoil velocity = 90 m/s, chord size = 690 mm, and droplet radius = 181 μm.

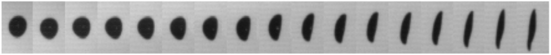
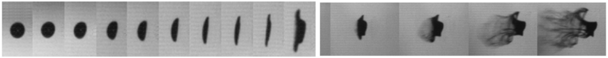


Fig. A5 Case 1041 droplet deformation evolution: airfoil velocity = 90 m/s, chord size = 690 mm, and droplet radius = 565 μm.

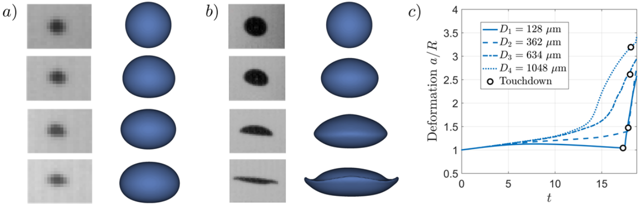


Drop deformation and breakup as $D = 362\ \mu\text{m}$ (above), $D = 1130\ \mu\text{m}$ (middle) and $D = 2064\ \mu\text{m}$ (below)

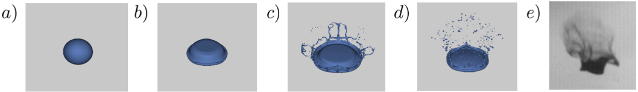
Pre-impact dynamics

Comparison - RC & Papageorgiou, IJMF 107, 192-207 (2018)

Excellent qualitative agreement is found between the experiments and computations in the same parameter range, with good quantitative agreement also found in the proposed deformation rate metric.

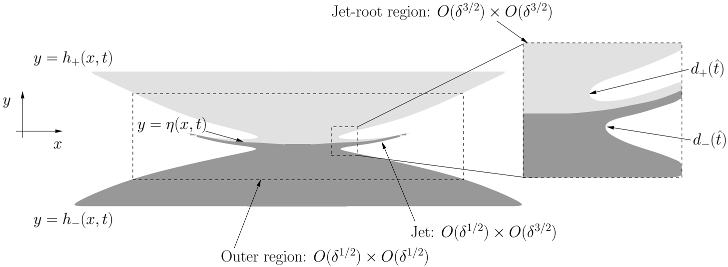


With the aid of the simulations dynamics for much smaller drops is analysed, while also gathering break-up data in the timesteps just before impact.



Impact

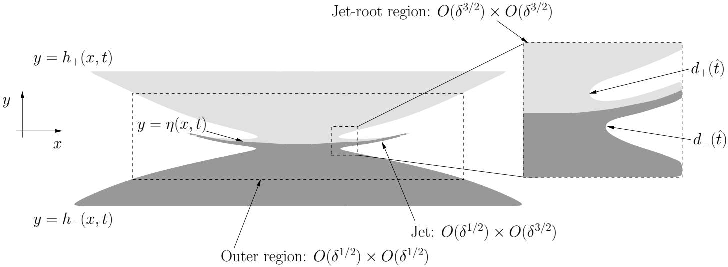
Asymptotic structure - RC & M.R. Moore, JFM 856, 764-796 (2018)



Early-time asymptotic structure according to Wagner theory.

Impact

Asymptotic structure - RC & M.R. Moore, JFM 856, 764-796 (2018)



Early-time asymptotic structure according to Wagner theory.

- Outer: boundary conditions linearise onto $y = 0$, solved using Riemann-Hilbert techniques.
- Inner ('jet-root'): quasi-steady Helmholtz flow, solved using Schwarz-Christoffel mappings.
- Jet: thin, high-speed jet governed by zero-gravity shallow-water equations, solved using characteristics.

Results

Summary - RC & M.R. Moore, JFM 856, 764-796 (2018)

Given time and patience, useful quantitative information about the impact process and properties of the resulting jet is retrieved:

Location of jet root:

$$\hat{x}_j = 2\sqrt{\frac{(1+V)R}{1+R}\hat{t}},$$

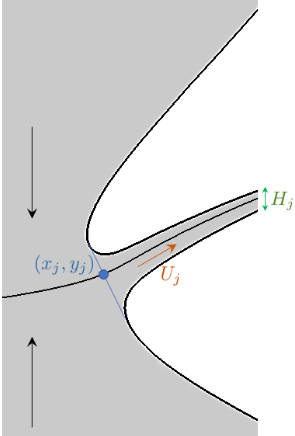
$$\hat{y}_j = \left(\frac{(3RV+R-V-3)}{2(1+R)}\hat{t}\right).$$

Jet thickness:

$$H_j(\hat{t}) = \frac{\pi(1+V)^{3/2}}{8}\sqrt{\frac{1+R}{R}}\hat{t}^{3/2},$$

Jet velocity:

$$U_j = 2\dot{\hat{d}}_0 = 2\sqrt{\frac{(1+V)R}{1+R}}\hat{t}^{-1/2},$$

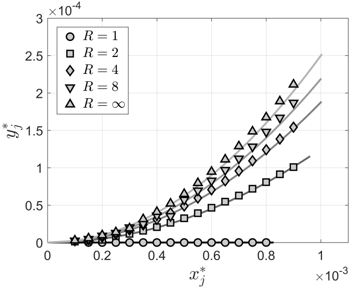


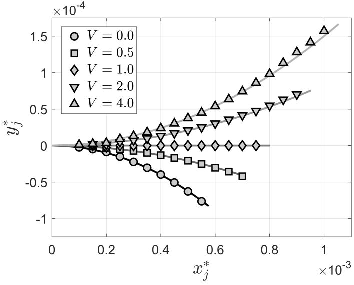
Results

Jet root location - RC & M.R. Moore, JFM 856, 764-796 (2018)

At early times we find excellent agreement between the two approaches. This begins to deteriorate (in an anticipated manner):

- at the tip of the jet;
- when not correcting for the presence of entrapped air bubbles;
- once we force the underlying assumptions (e.g. lower impact velocities);
- at sufficiently large times;





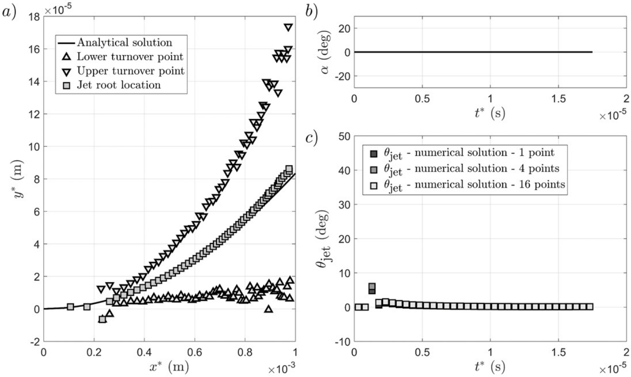
Results

Jet angle - RC & M.R. Moore, JFM 856, 764-796 (2018)

From the inner region we can also derive the jet angle, found to be

$$\alpha = \delta^{1/2}\left((R-1)\sqrt{\frac{1+V}{R(1+R)}}\right)\sqrt{\hat{t}} + o(\delta^{1/2}),$$

where α is small. Abstractly it represents the slope of the vortex sheet in the outer region as it approaches the turnover point. $R = 1$, $V = 2$:



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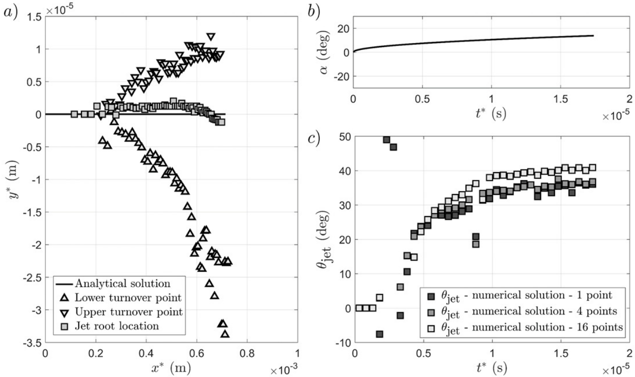
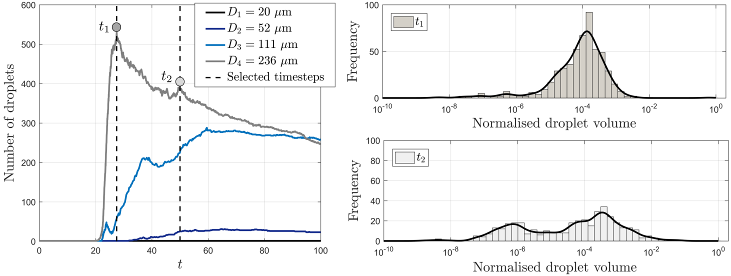


Post-impact dynamics

Droplet distribution - RC & Papageorgiou, IJMF 107, 192-2017 (2018)

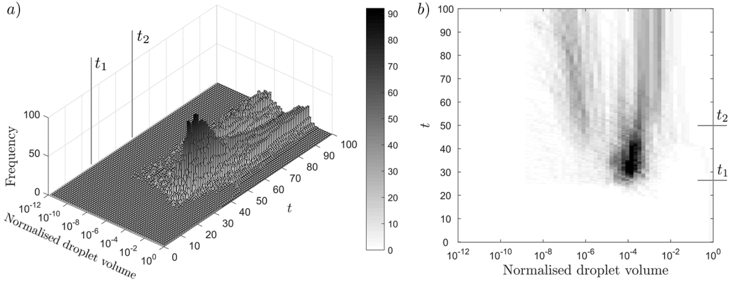
The volumes of the satellite droplets in the system follow a log-normal distribution during most of the simulation, with an average area equal to roughly 1/10000 relative to the initial droplet. This behaviour is consistent with experimental observations (Mundo, *IJMF*, 95 and Yarin, *JFM*, 1995) for lower velocity impacts onto solid surfaces.



Droplet distribution for an $\theta = 60^\circ$ angle of incidence impact in the case of a range of droplet sizes.

Post-impact dynamics

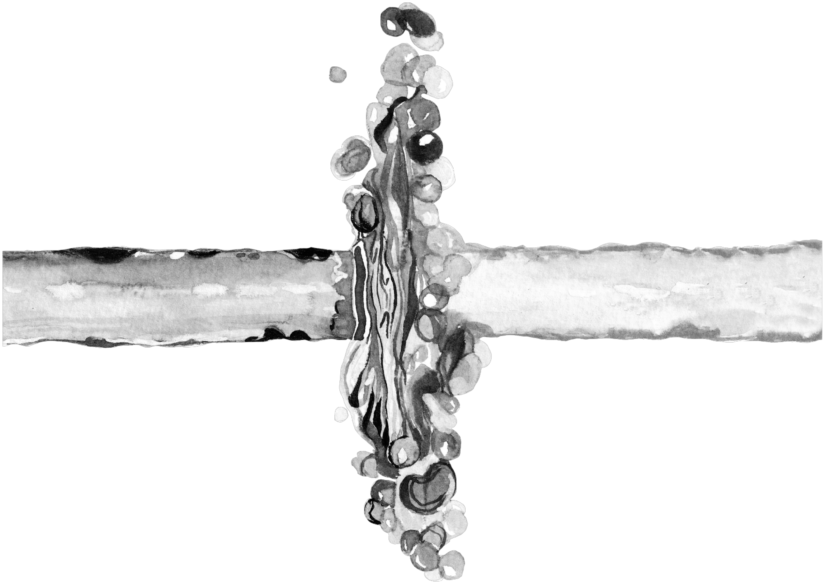
Twin peaks - RC & Papageorgiou, IJMF 107, 192-2017 (2018)



The evolution of the drop size distribution hints at a separation in drop behaviour depending on size:

- the larger fluid volumes are found as spherical caps in contact with the solid surface.
- are more likely to grow in time as coalescence events take place.
- the smaller drops are airborne and subjected to the strong background flow.
- tend to break up as long as capillary forces allow it.

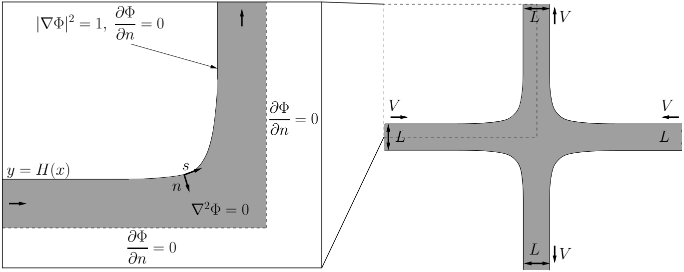
So . . . what have we been up to since then?



Results

Symmetric jets - M.R. Moore, RC, Ock^2, J.M. Oliver, JFM 882, A19 (2020)

The simplest problem where we can study the effect of viscous corrections near the free surface is a pair of jets colliding symmetrically at the origin.



We find that the perturbation to the free surface satisfies the singular integro-differential equation

$$0 = \frac{\mathrm{d}\tilde{h}_0}{\mathrm{d}s} + 2\kappa(s) + \frac{1}{2}\int_{-\infty}^{\infty} \operatorname{cosech}\left(\frac{\pi(s-t)}{2}\right)\left[\tilde{h}_0(t)\kappa(t) - 2\kappa(t)\tilde{w}^*(t)\right]\,\mathrm{d}t,$$

where $\tilde{w}^* = -2\arctan\left(e^{\pi s/2}\right)$.

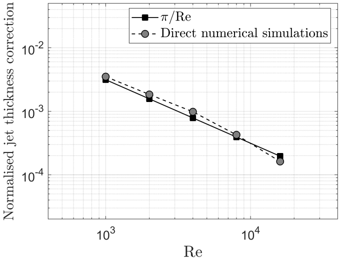
Results

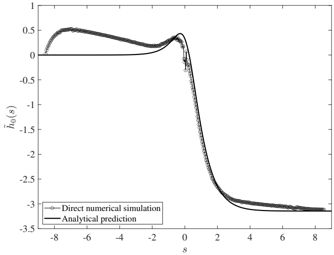
Symetric jets - M.R. Moore, RC, Ock[2], J.M. Oliver, JFM 882, A19 (2020)

The effect of viscosity is to thicken the jet at the outlet by twice the angle the tangent to the free surface turns as s increases.

- the ratio of the jet thickness at the outlet ($s = \infty$) to that at the inlet ($s = -\infty$) is given by

$$\frac{(H + \varepsilon^2 \tilde{h}_0)(\infty)}{(H + \varepsilon^2 \tilde{h}_0)(-\infty)} = 1 + \frac{\pi}{Re} + o\left(\frac{1}{Re}\right).$$





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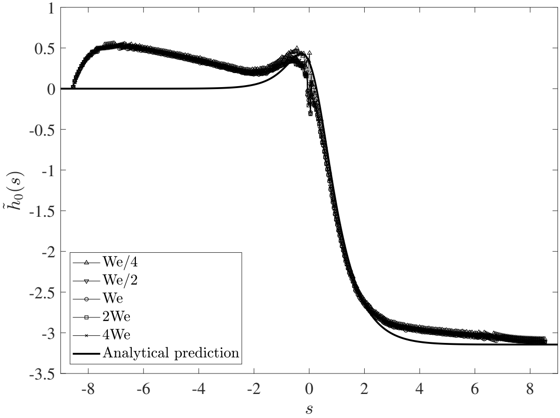
Symmetric jets - M.R. Moore, RC, Ock^2, J.M. Oliver, JFM 882, A19 (2020)

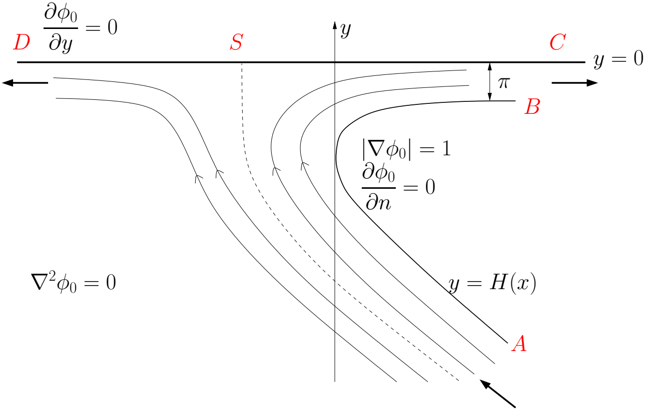
- changes in surface tension coefficient (while *Re* is large but finite with $1/We = \mathcal{O}(1/Re)$)induce no qualitative differences;
- quantitatively, the free surface suffers a normal translation given by $\phi_1 = -\kappa x(y - 1/We)$.



Results

Inner region - M.R. Moore, RC, Ock^2, J.M. Oliver, JFM 882, A19 (2020)

The key example is the (suitably scaled) leading-order-inner problem according to Wagner theory for a droplet-droplet impact at small times.

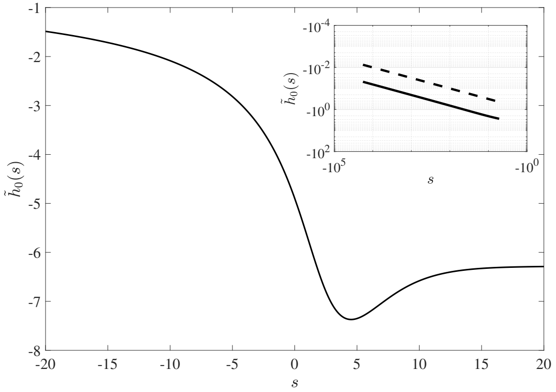


The dashed line indicates a dividing streamline that terminates at a relative stagnation point on the body at S. In a frame moving with the turnover points, the problem is a Helmholtz flow.

Results

Inner region - M.R. Moore, RC, Ock[2], J.M. Oliver, JFM 882, A19 (2020)

- Downstream as $s \to \infty$, one can see the perturbation approaching an analytically predicted value of -2π;
- Upstream we see inverse square-root decay back into the bulk (the outer Wagner region), displayed in further detail in the inset.



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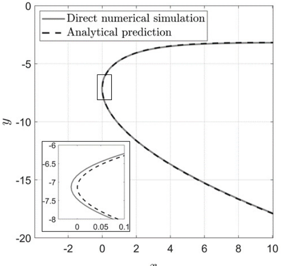
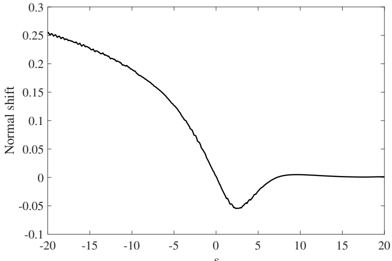


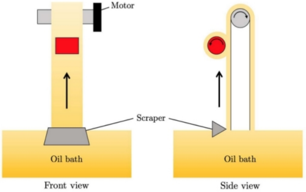


A truly symbiotic relationship?

In the same spirit ...

Levitating a cylinder - Dalwadi, RC et al., JFM 917, A28 (2021)

Direct numerical simulations can be deployed alongside beautiful matched asymptotic expansion methods for a wide class of problems as means to resolve complex physical systems with no fitting parameters, with experiments providing essential confirmation.



J. Fluid Mech. (2021), *vol.* 917, A28, doi:10.1017/jfm.2021.284

JFM PAPERS

Levitation of a cylinder by a thin viscous film

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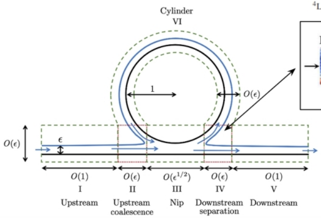

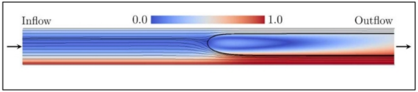

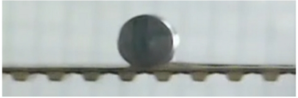

Bringing it all together - a hybrid multi-scale framework

Hybrid framework

Multi-scale methodology

For us to be able to get closer to an (accurate and efficiently obtained) answer, any solution would need to resolve:

- sub-micron scales on impact and for the fragmentation process.
- $\mathcal{O}(1)$ m or larger lengthscales for device-specific geometries.

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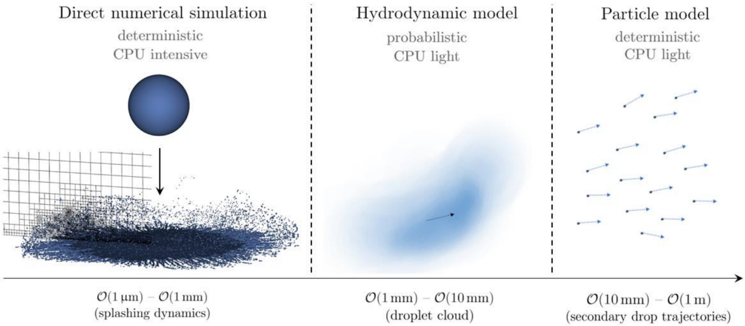

Acknowledgements

Conclusions

Acknowledgements

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- the Oxford Centre for Industrial and Applied Mathematics,
- the Warwick Mathematics Institute,

without which this work would not have been possible.

Thank you for dropping by!

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