Basilisk (Gerris) Users' Meeting 2023

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Mixing induced by (large amplitude) Faraday waves

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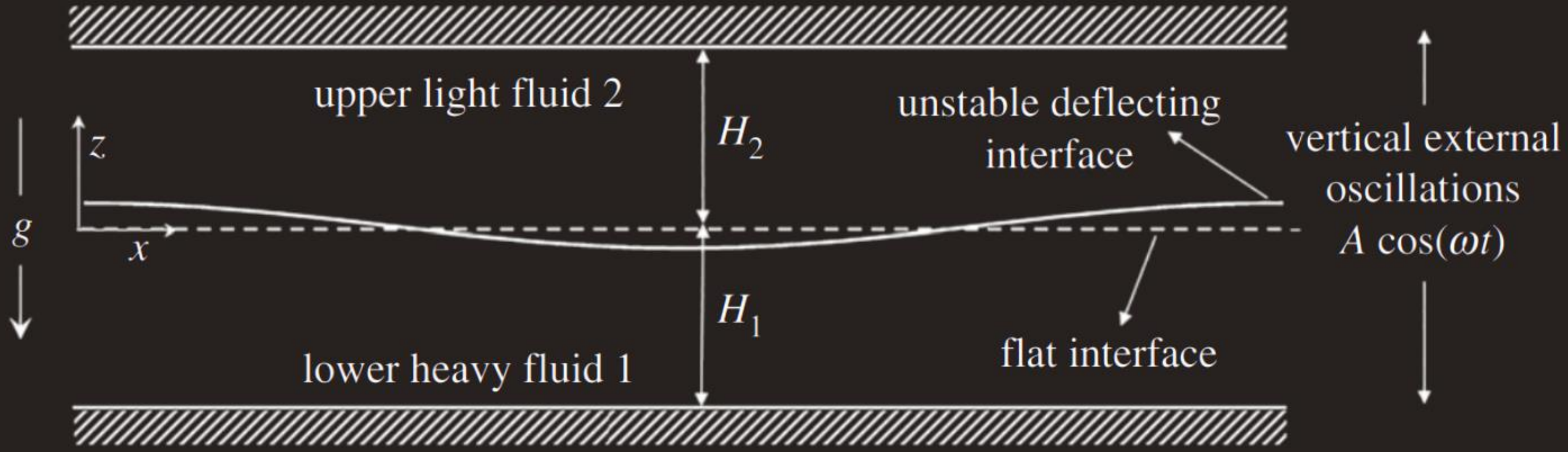
In 1831, Faraday observed the instability of vertically oscillating liquids



Schematic depicting two-layers as done in a Faraday experiment, image taken from Dinesh et al. (2022)

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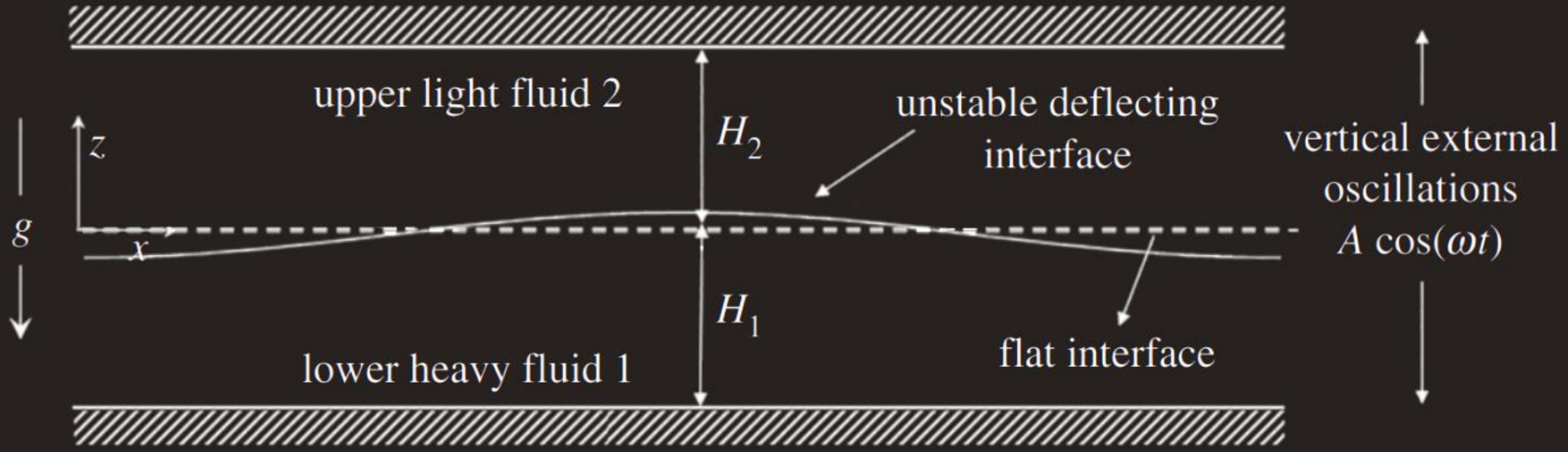

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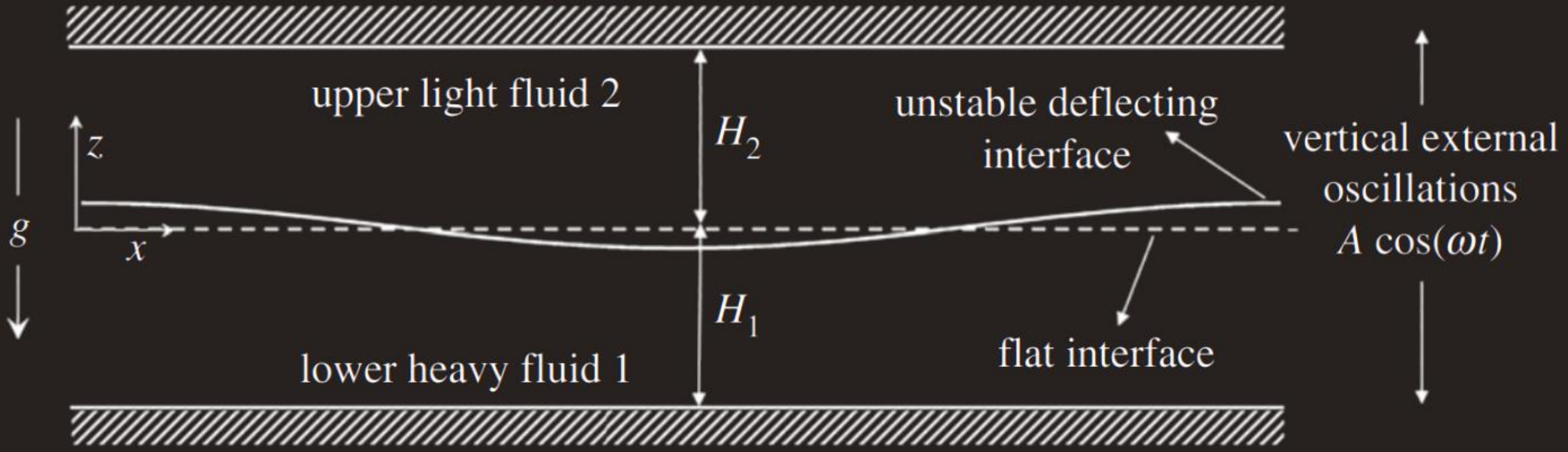
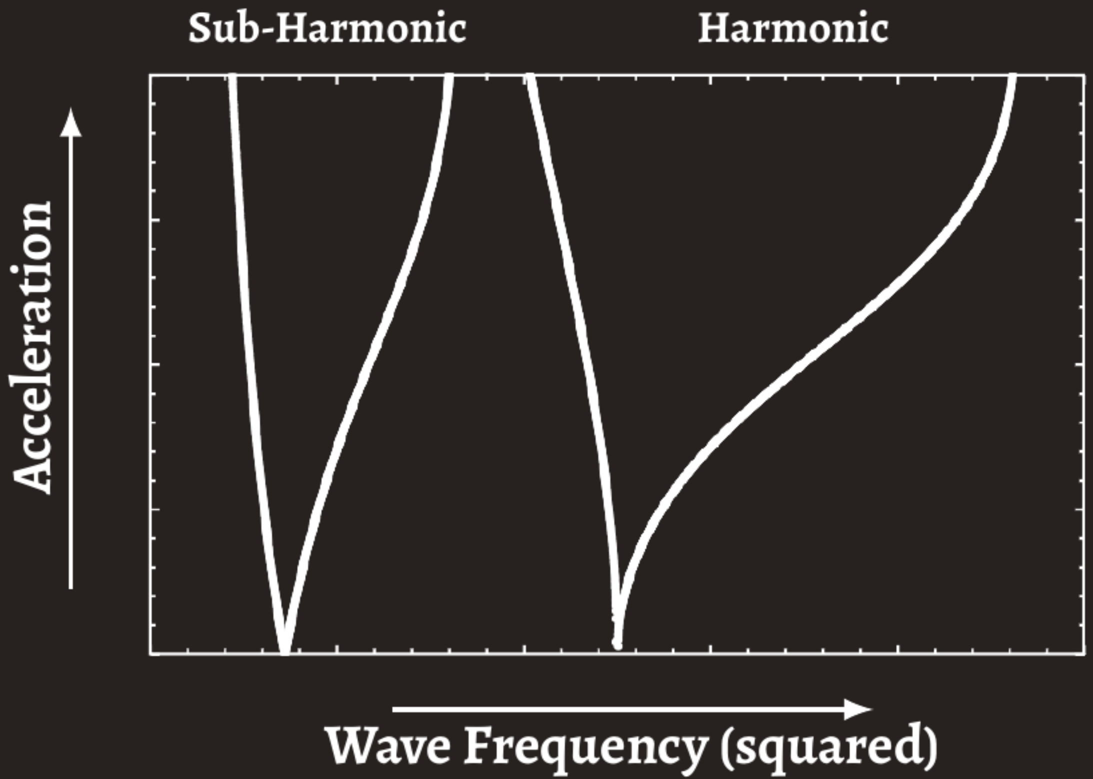


Schematic depicting two-layers as done in a Faraday experiment, image taken from Dinesh et al. (2022)

This phenomenon is well-described by a Matthieu equation

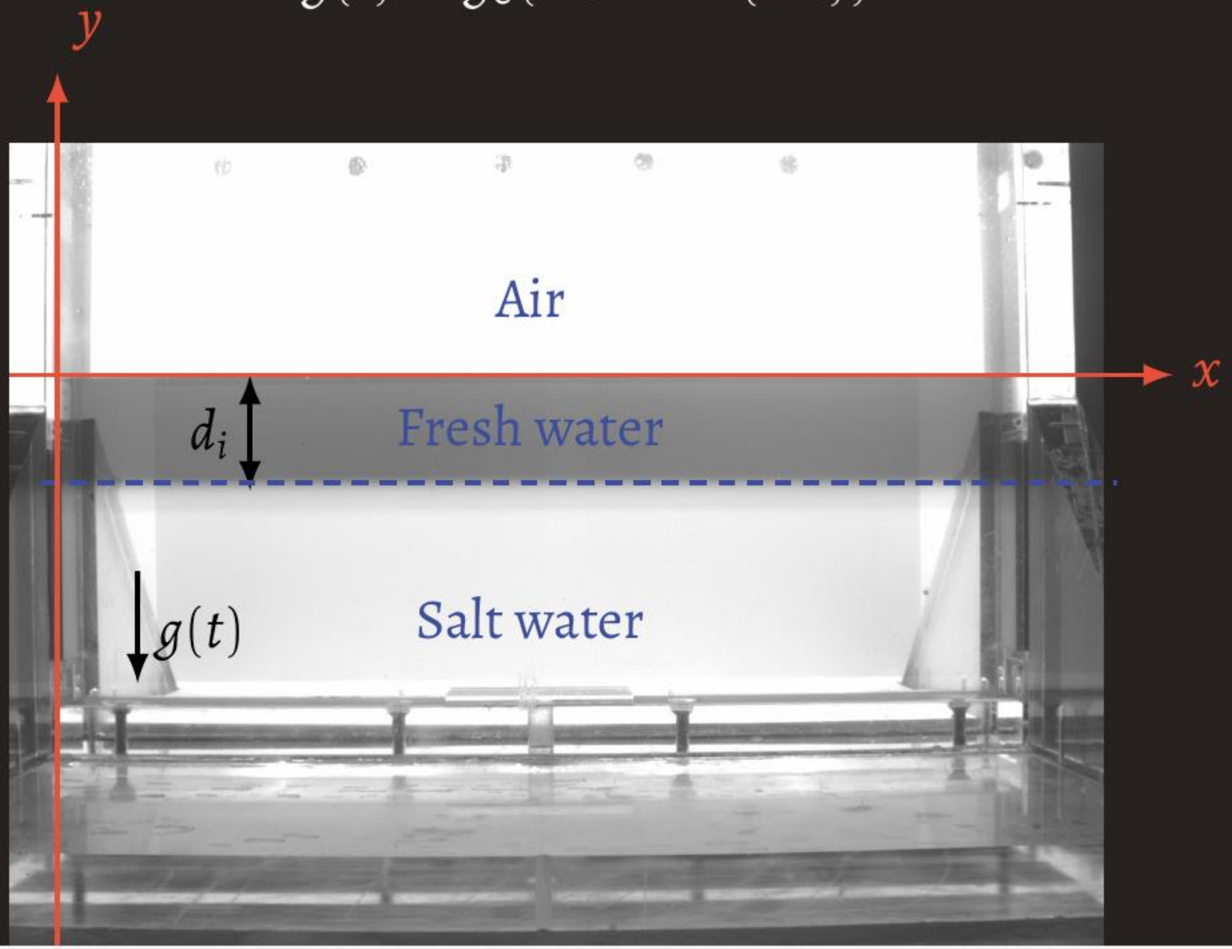
$$\ddot{\eta} + \delta\dot{\eta} + \Omega^2(1 + F\cos(\omega t))\eta = 0$$

where $F = A\omega^2$ and $\Omega^2 \sim \mathcal{A}_t G_0 k$ is the dispersion relation of the interface



We study the influence of a free-surface close to a miscible interface when subject to a periodic acceleration in the vertical direction

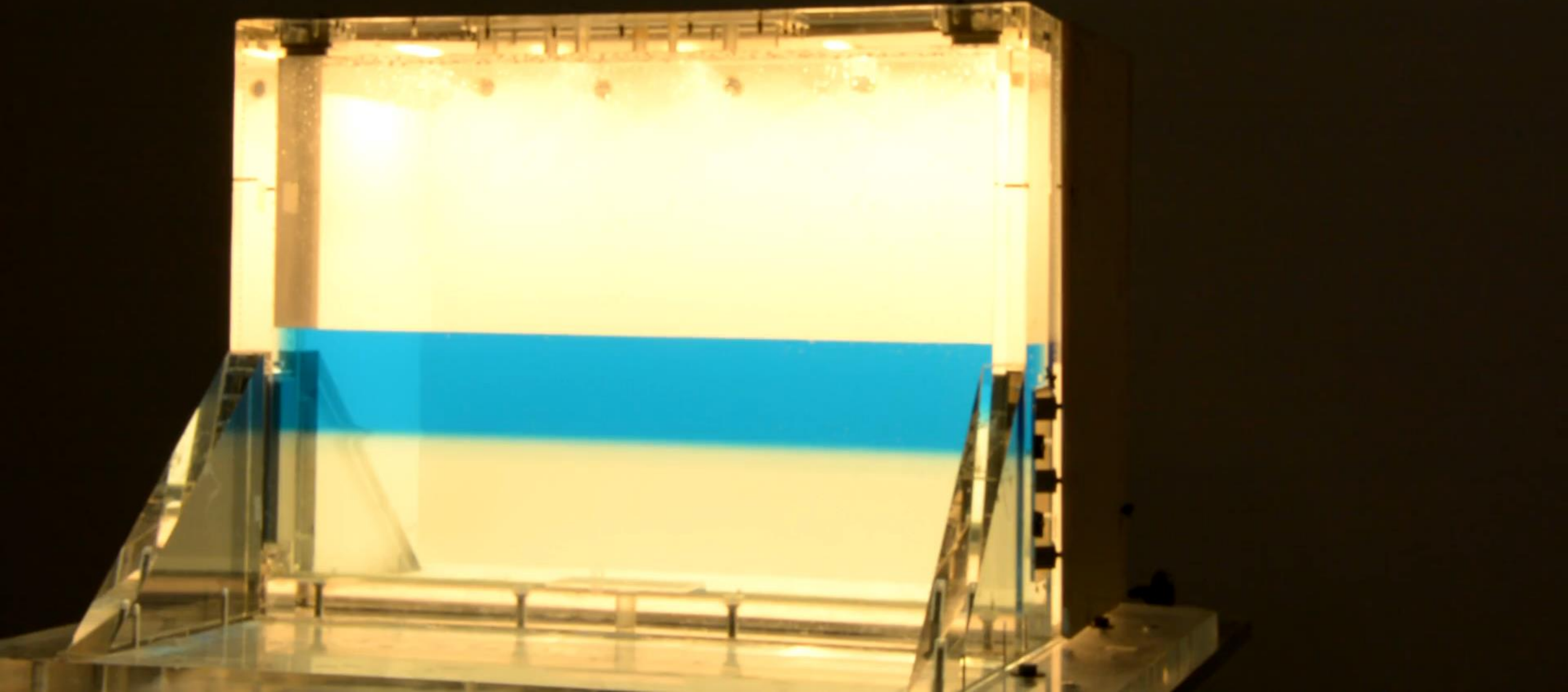
$$g(t) = g_0(1 + F\cos(\omega t))$$



Depending on the forcing frequency ω, a Faraday instability may create standing waves on the **free-surface**, on the **miscible interface**, or **both**

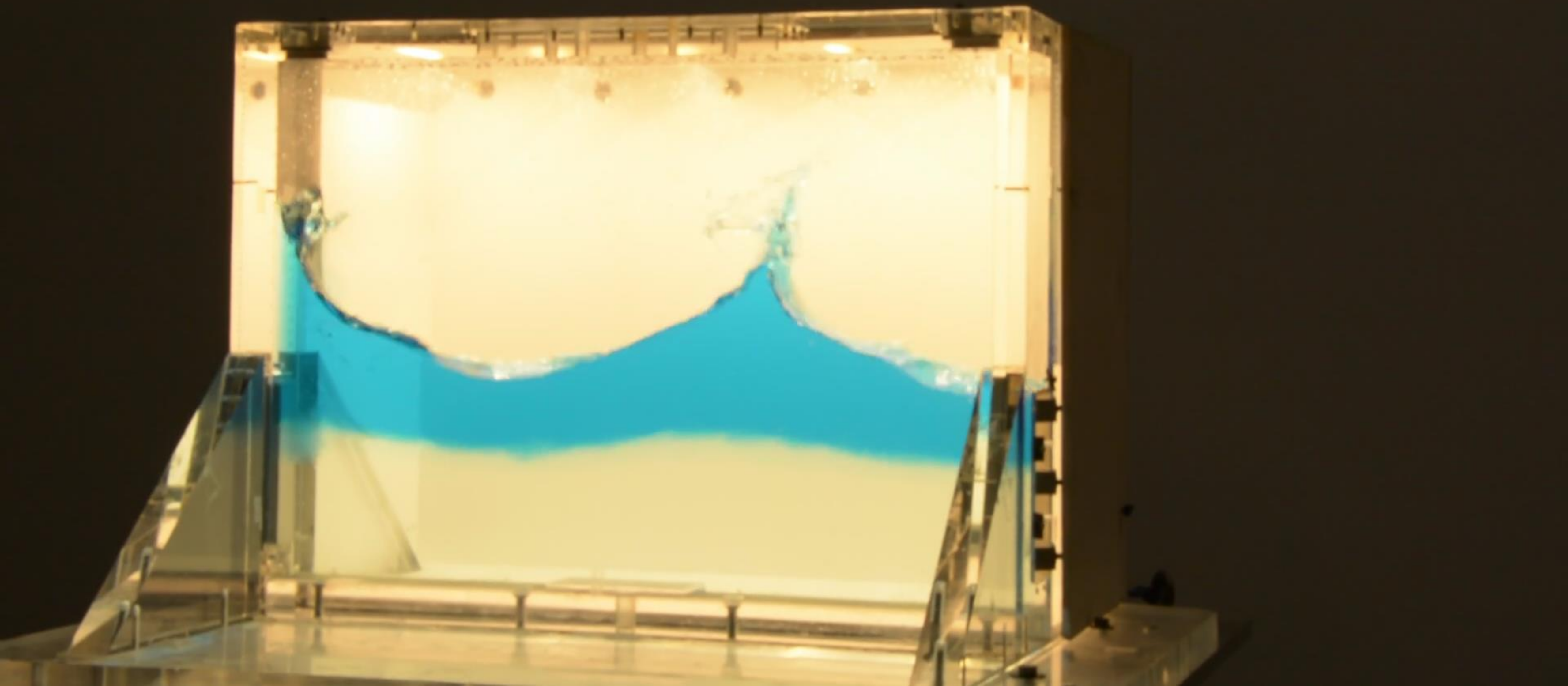
Let’s focus on the first scenario

For surface waves, a Faraday instability may create
a large amplitude standing wave with frequency $\omega/2$
a **sub-harmonic** instability

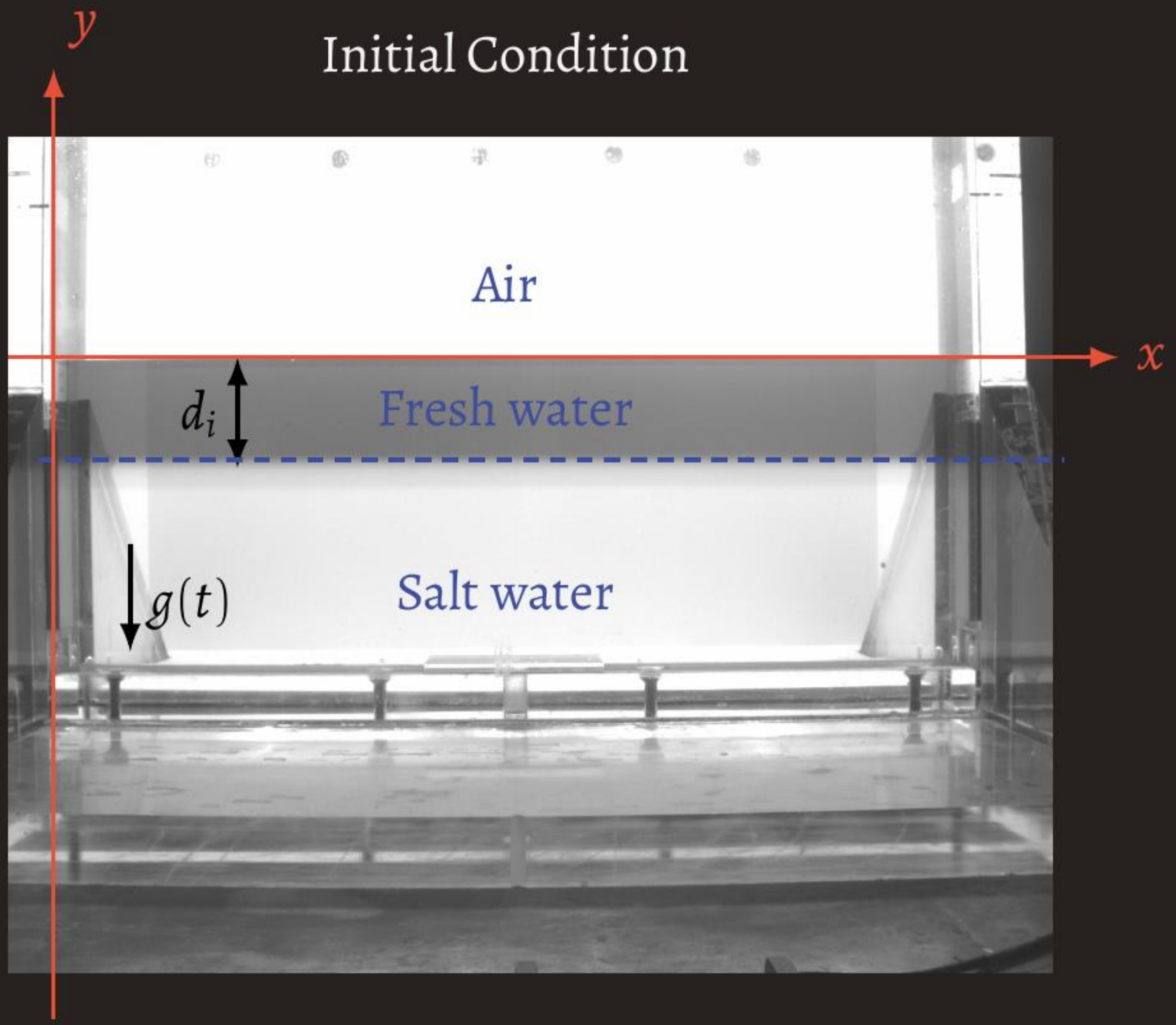


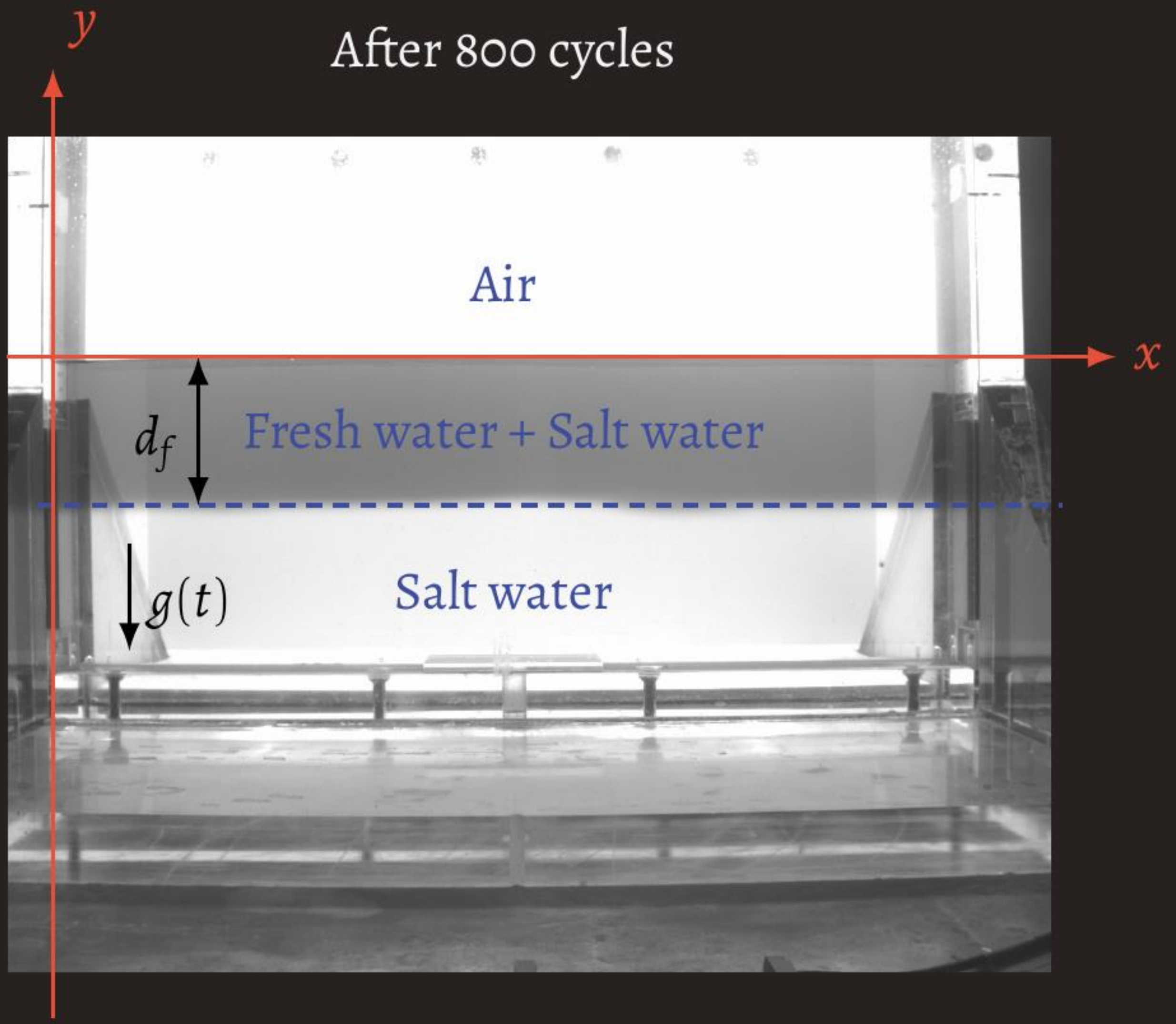
Sub-harmonic instabilities modify the miscible interface in two forms
Through the **displacement** induced by the standing wave

But also through the ejection of droplets and collapsing air pockets which injects bubbles deep into the stratified layer



This entrainment of the stratified fluid pushes the interface downwards until the bubbles injected no longer reach the interface and saturates





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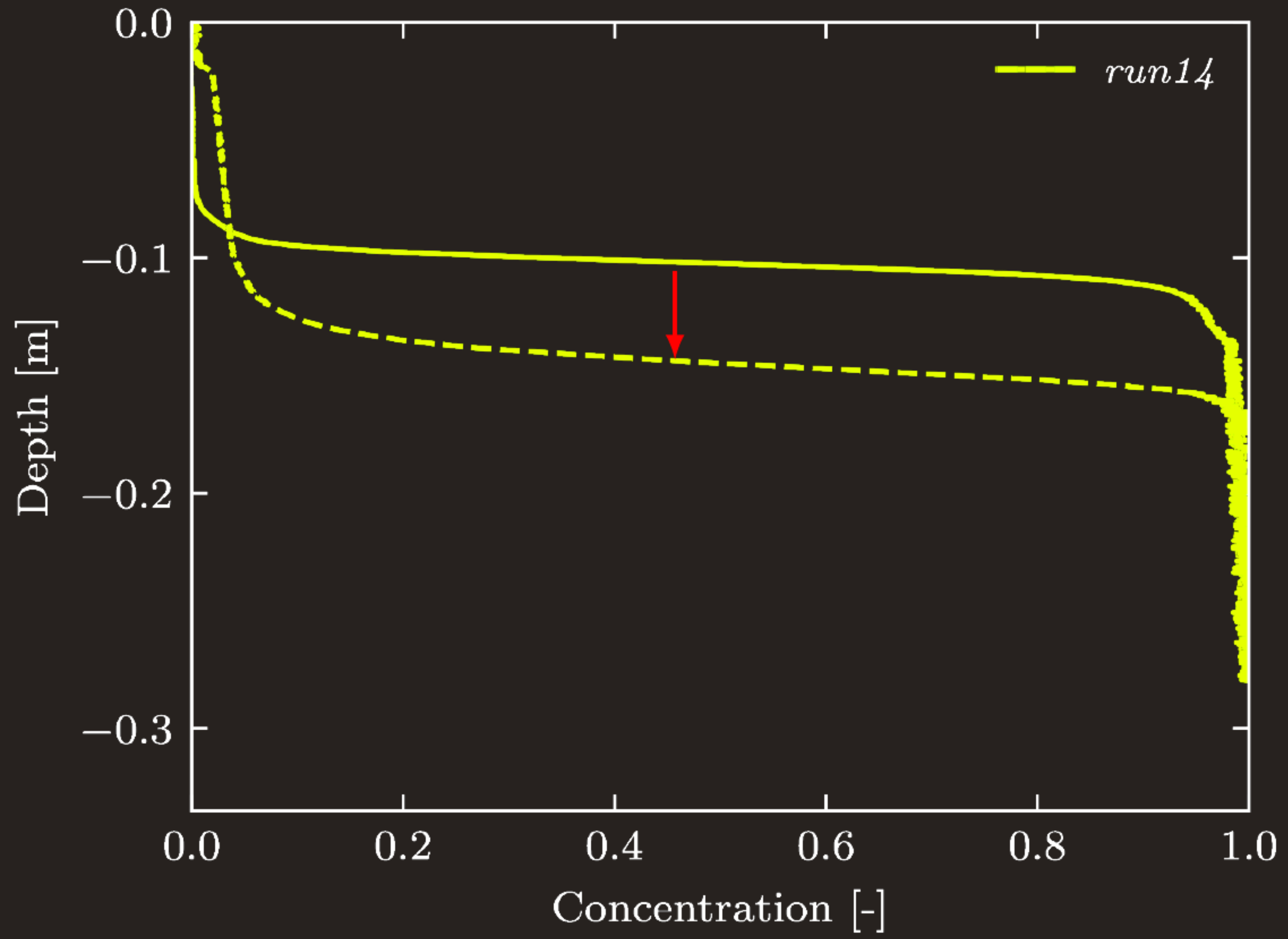
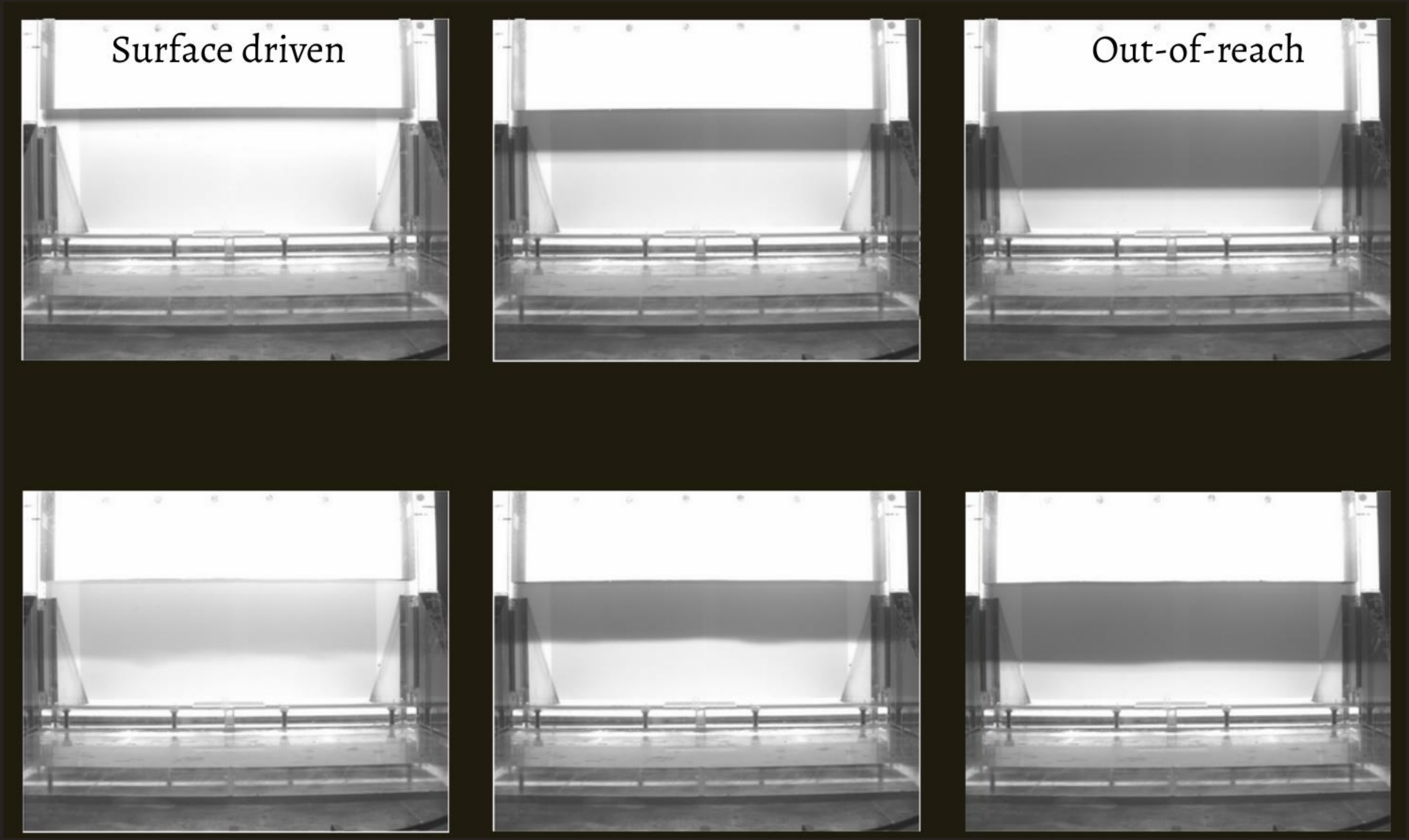


Mixing seems to be driven by the interaction free-surface and the miscible interface

Initial conditions

After 800 cycles



Direct Numerical Simulations using Basilisk

- Volume-of-fluid approach with volume fraction $\mathcal{T}(\boldsymbol{x}, t) \in [0, 1]$

$$\begin{aligned} \partial_t \mathcal{T} + \nabla \cdot (\mathcal{T}\boldsymbol{u}) &= 0 \\ \rho_0 \left[\partial_t \boldsymbol{u} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u})\right] &= -\nabla p + \nabla \cdot \left[Re^{-1} \frac{\mu_0}{\mu_{ref}} (\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u}) \right] + We^{-1} \boldsymbol{f}_\sigma + Fr^{-1} \rho(c) \boldsymbol{g}(t) \end{aligned}$$

where $\rho_0 = \rho_{ref,water}\mathcal{T} + \rho_{air}(1 - \mathcal{T})$, $\mu_0 = \mu_{ref,water}\mathcal{T} + \mu_{air}(1 - \mathcal{T})$

- Advection/diffusion of solutant c with Henry's law for solubility/volatility $(\boldsymbol{x}, t) \in [-0.5, 0.5]$

$$\partial_t c + \nabla \cdot (c\boldsymbol{u}) = \nabla \cdot \left(Sc^{-1} Re^{-1} \frac{D}{D_{ref}} \nabla c + \beta c \nabla \mathcal{T} \right)$$

- Dimensionless parameters

$$Re \equiv \frac{\rho_{ref}[U][L]}{\mu_{ref}}, \quad Fr \equiv \frac{[U]^2}{[L]g_0}, \quad We \equiv \frac{\rho_{ref}[U]^2[L]}{\sigma}, \quad Sc \equiv \frac{\mu_{ref}}{\rho_{ref} D_{ref}}, \quad At \equiv \frac{\rho_{salt\ water} - \rho_{fresh\ water}}{\rho_{salt\ water} + \rho_{fresh\ water}}, \quad \beta, \quad F, \quad \omega$$

Implementation in Basilisk C with minimal changes

We treat fluid 1 as a binary mixture characterised by $c \in [0, 1]$,

$$\rho_1 = \rho_{salt\ water} c + \rho_{fresh\ water}(1 - c), \quad \rho_2 = \rho_{air}$$

`faramix.c`

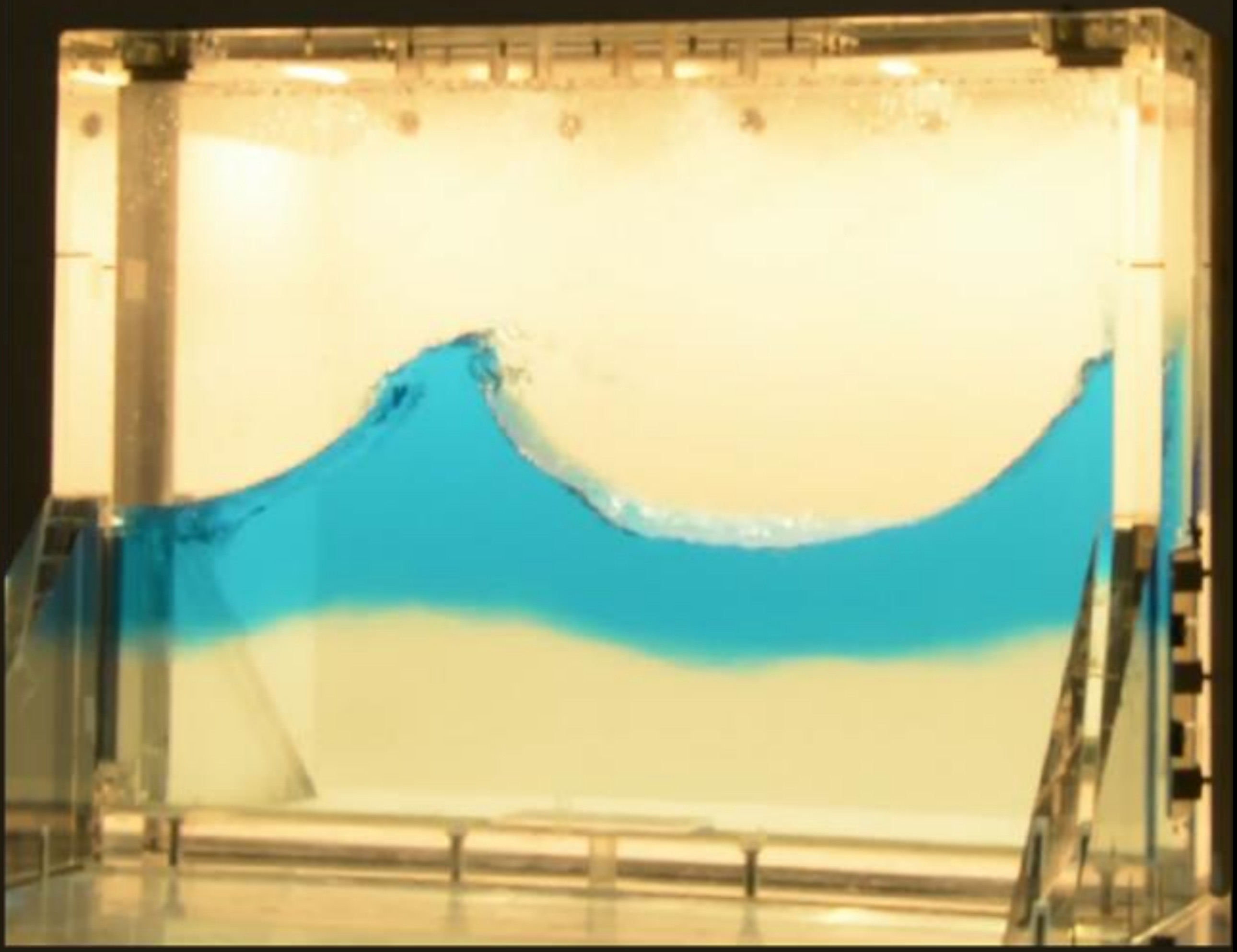
```c
#include "grid/multigrid3D.h"
#include "navier-stokes/centered.h"

vector h[];
scalar c[], * stracers = {c};
#include "./my-two-phase.h"

#define rho1 (rho1t + clamp(c[],0.,1.)*(rho1b-rho1t))
#undef  rho
#define rho(f) (clamp(f,0.,1.)*(rho1 - rho2) + rho2)
#include "tension.h"
#include "navier-stokes/conserving.h"
#undef rho1
#include "./henry.h"
```

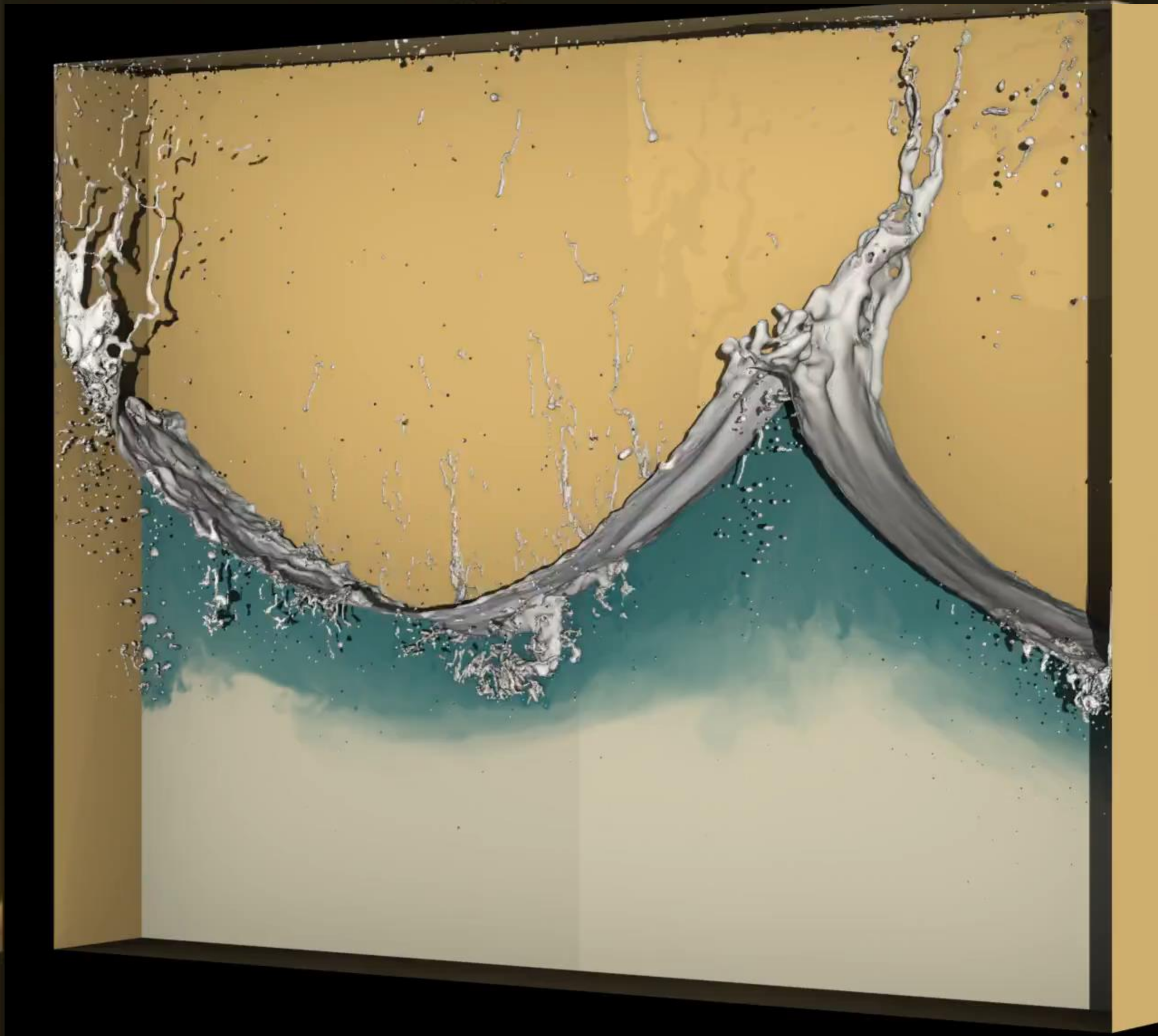
`my-two-phase.h`

```c
extern scalar c;
double rho1t = 1., rho1b = 1.;
#define rho1(c) ( rho1t + clamp(c,0.,1.)*(rho1b - rho1t) )
```

AMR Implementation also possible using embedded boundaries using `"contact-embed.h"`

Direct Numerical Simulations using Basilisk and carried at **CCRT**
aim to reproduce the surface dynamics and the mixing process





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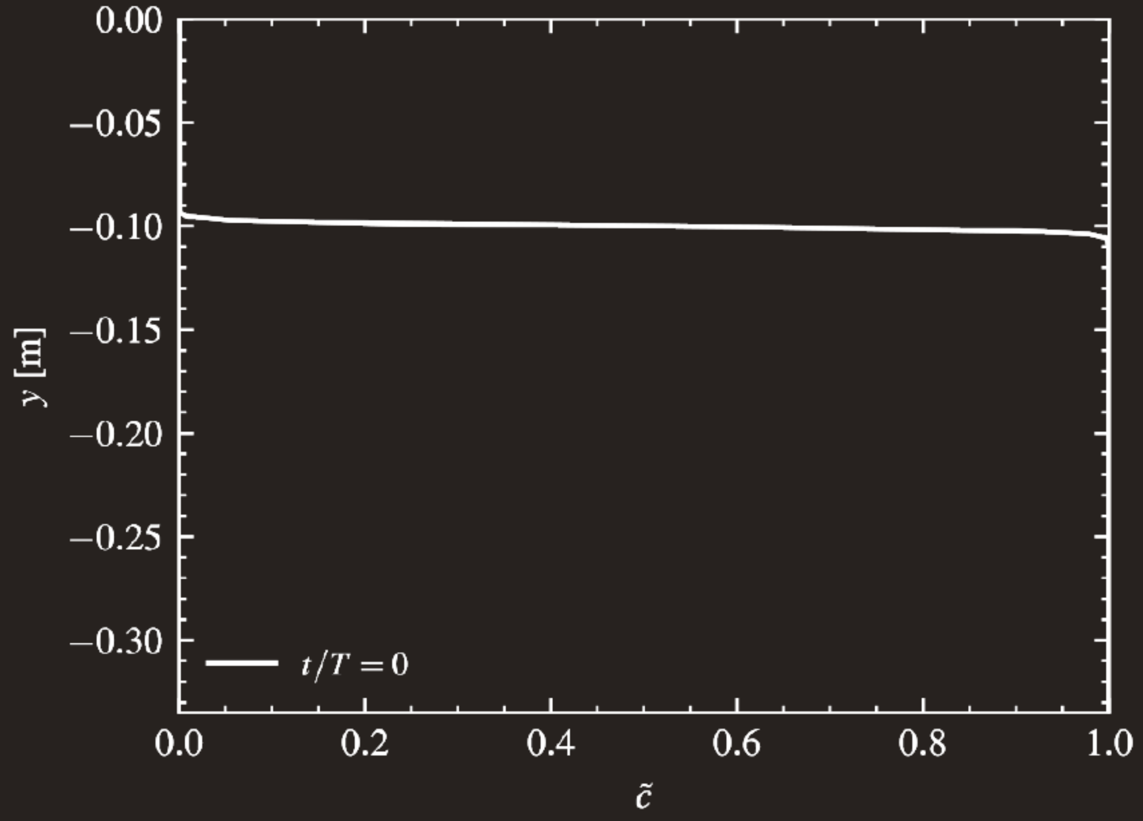
Density profiles



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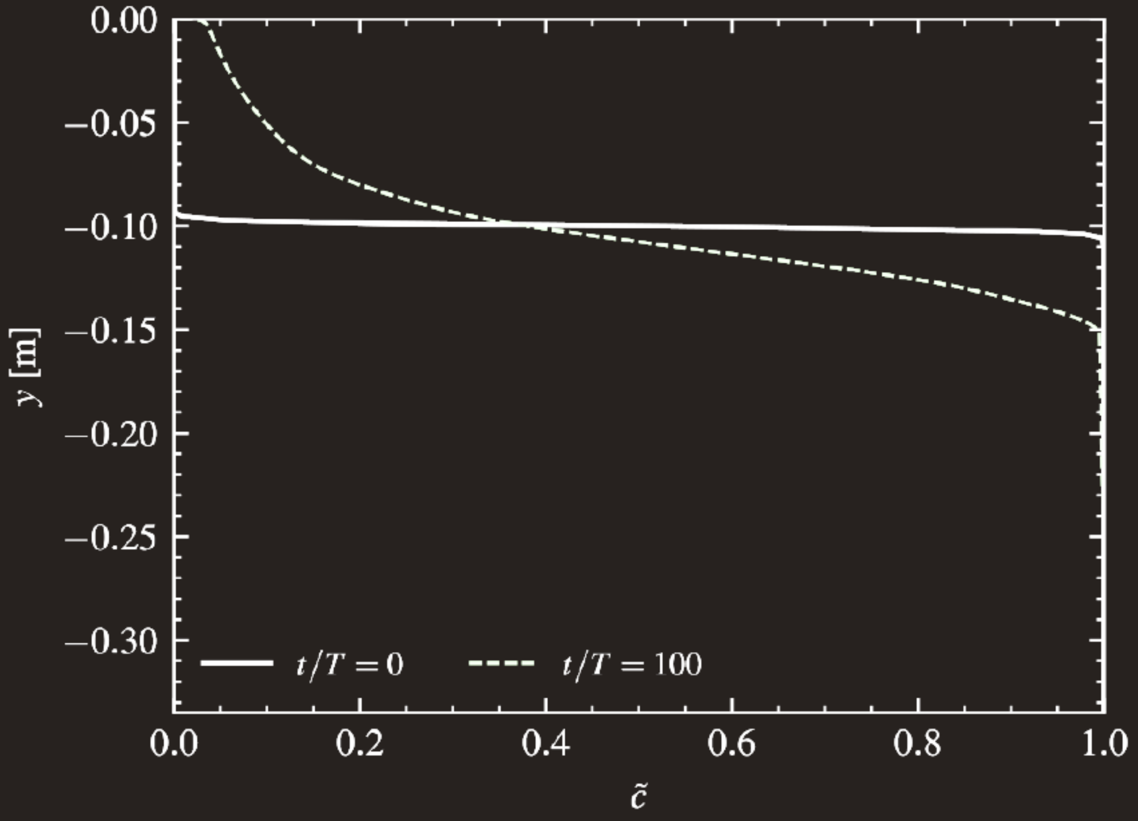
Density profiles



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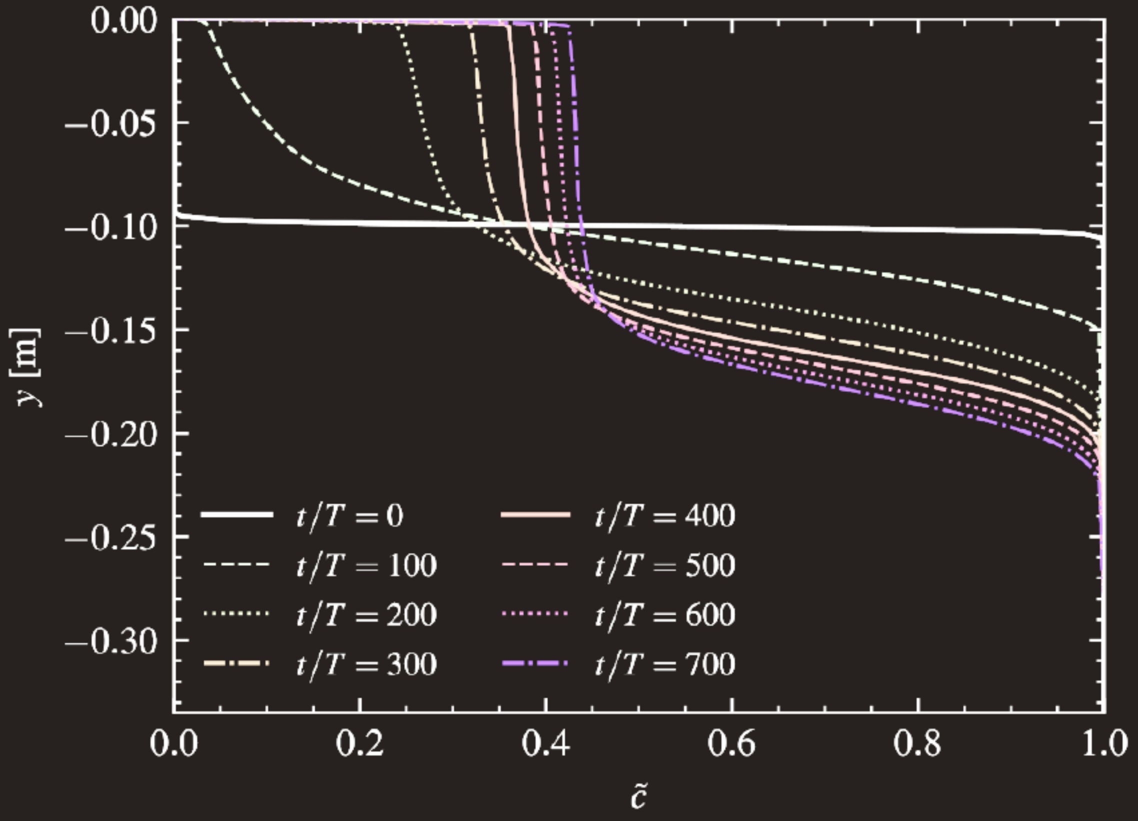
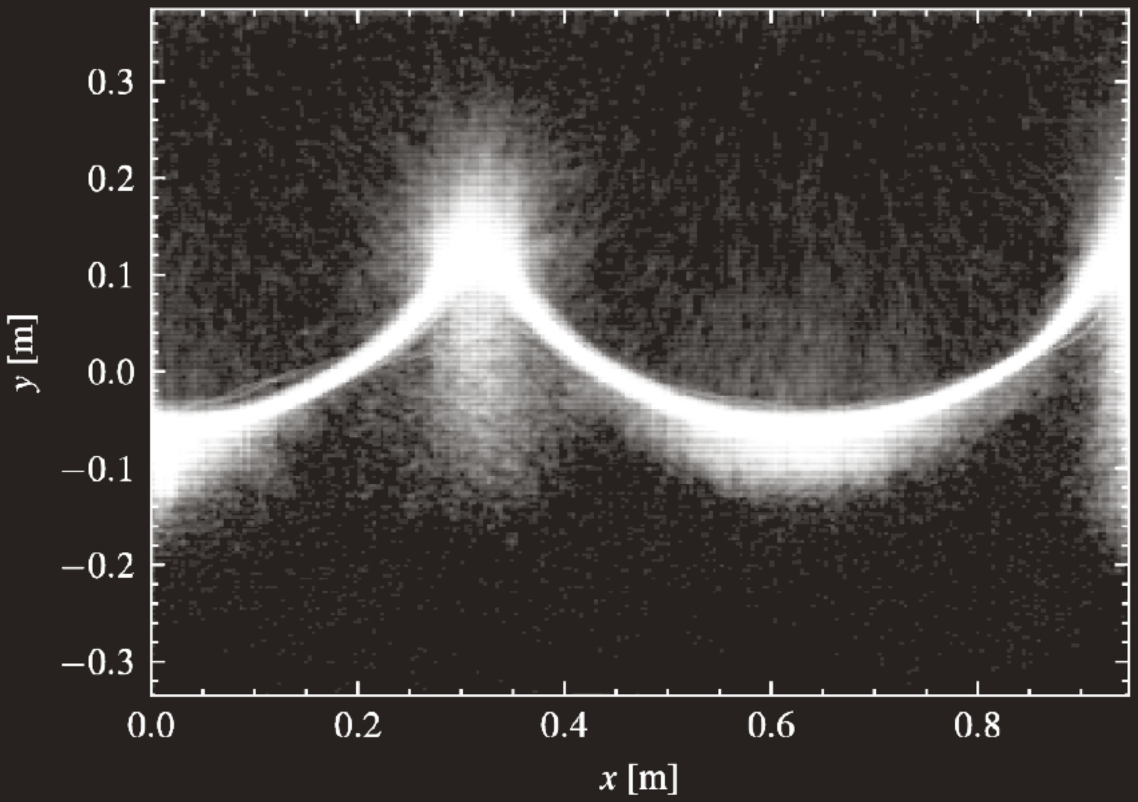


Typical scales of surface dynamics



Conclusions and Perspectives

- Experimental campaign revealed complex interactions between the free surface and the miscible interface
- Use of DNS to complements existing observations
- For large forcing amplitudes, the wave breaking is observed with droplet-ejection and collapsing cavities
- Surface dynamics accelerate the entrainment process, creating a homogeneous layer of fluid at the top
- This seems to be correlated to the penetration depth of the injected bubbles. Deeper layers are unfazed