# The development of Gerris in simulating the multiphase MHD flows

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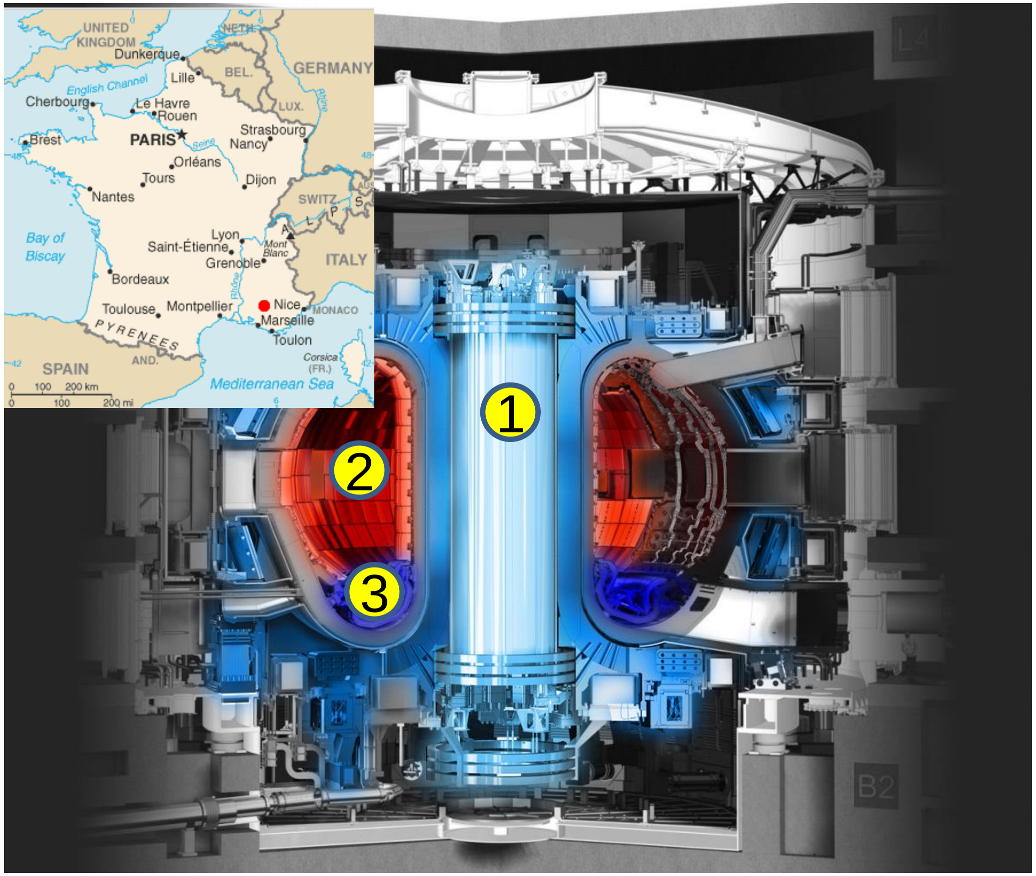
# Contents

# Tokamak(ITER)

**TOKAMAK**： **To**roidal-**ka**mera-**ma**gnet-**k**otushka

**ITER** ： **I**nternational **T**hermonuclear **E**xperimental **R**eactor

WHERE? Cadarache, Provence




① **Magnets**

to initiate, confine, shape and control the ITER plasma.

② **Blankets**

For energy conversion

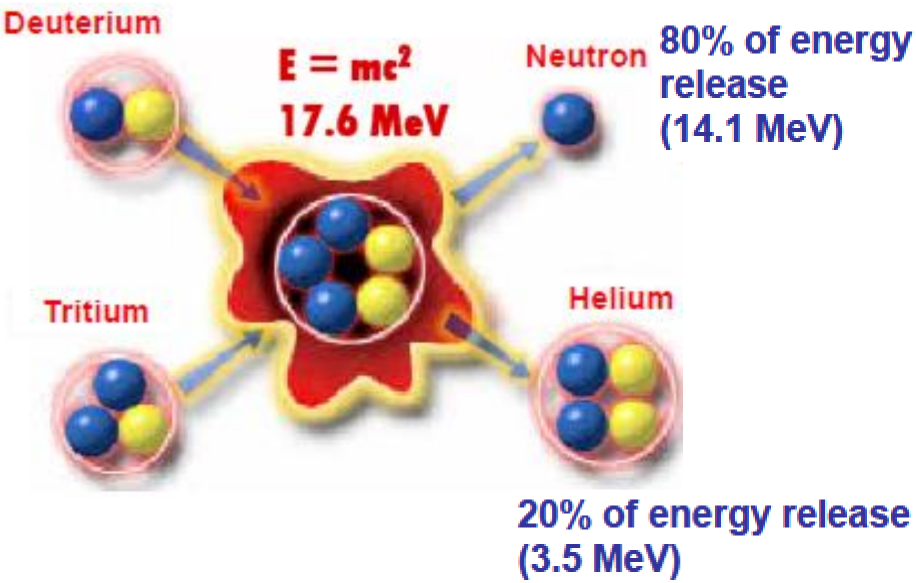
③ **Divertor**

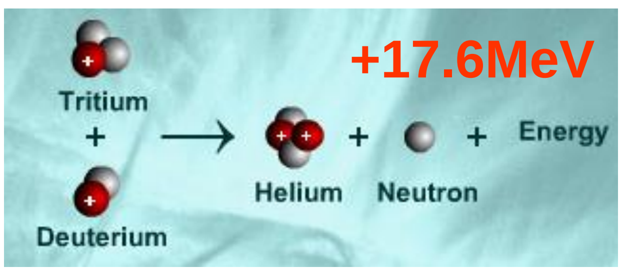
controls the exhaust of waste gas and impurities from the reactor.

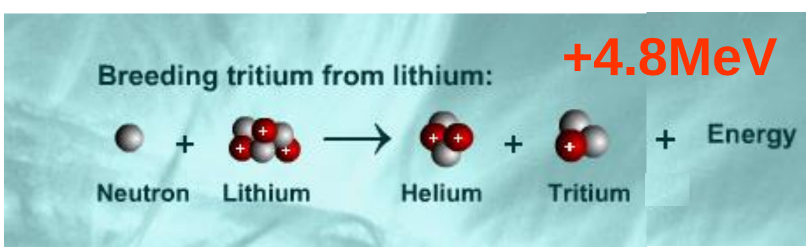
博学笃志 格物明德

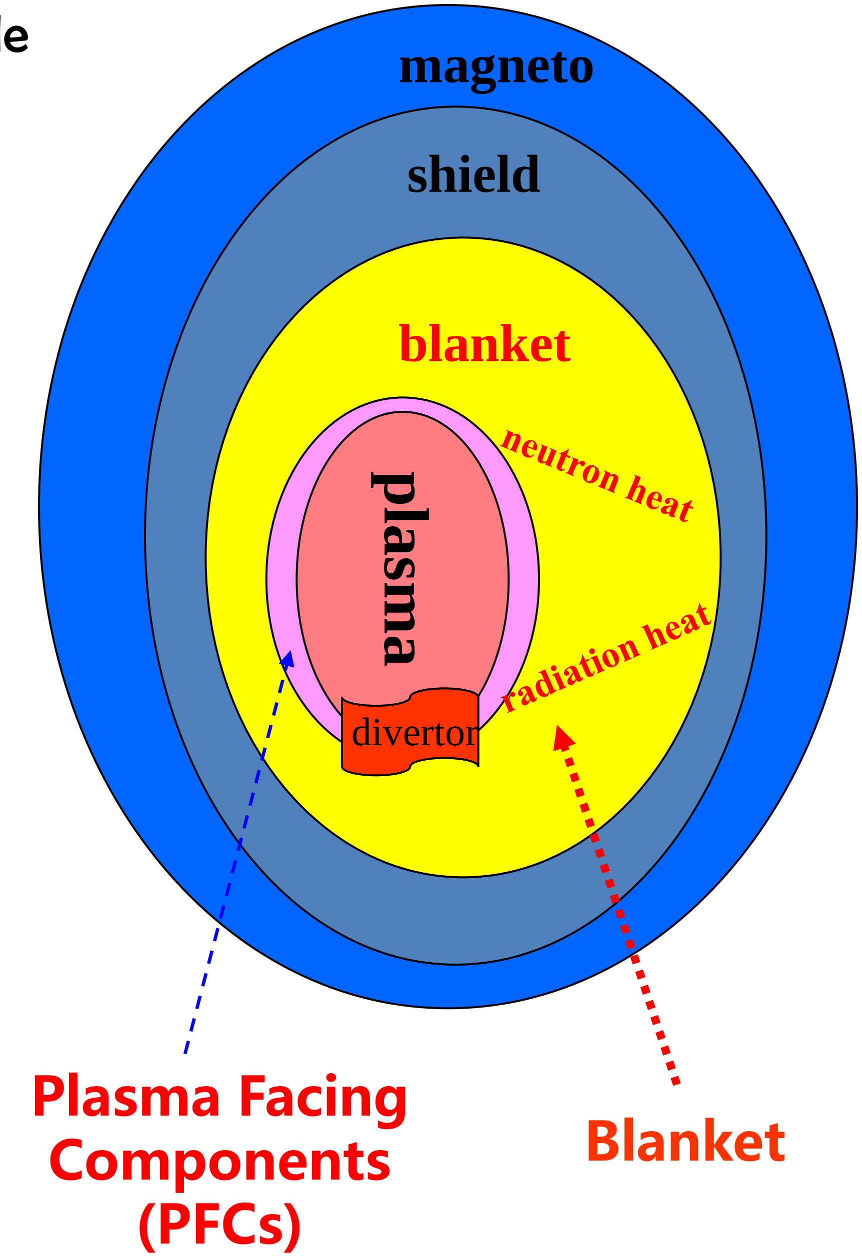
# Magnetic Confinement Fusion Reactor

The world program is focused on the D-T Cycle



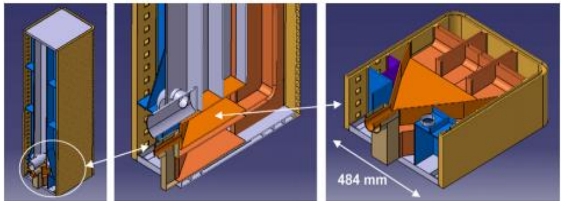



# What can we do with MHD?

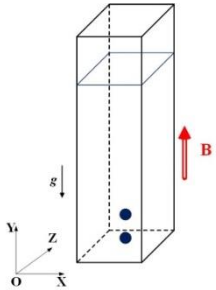
**Liquid Lead Lithium Blanket:**

heat transfer, MHD, complicated geometry, thermal-stress coupling between fluids and ducts, tritium permeation， corrosion

Complex boundary



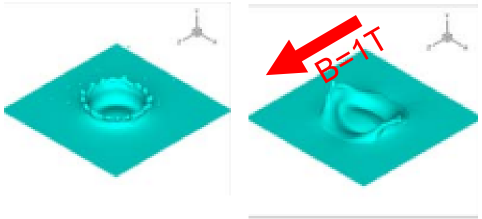

Bubbly Flows




**Liquid Divertor:**

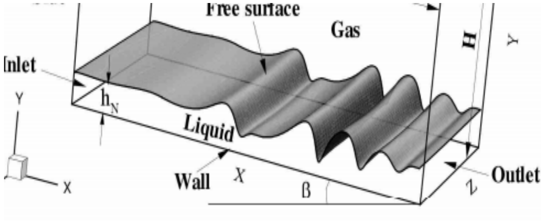
plasma effect, MHD, Marangoni effect, Seebeck effect, phase change , interfacial flows, large heat flux, corrosion

Droplet splash




Thin film MHD flow

…



# Self-Introduction

西安交通大学
XI'AN JIAOTONG UNIVERSITY
中国科学院大学
University of Chinese Academy of Sciences

## Working with Gerris (2009-2017)

**2009-2014 PhD study**

University of Chinese Academy of Sciences

2010.08 touch the first line of Gerris...

~2012.08 implement the MHD module into Gerris

~2014.08 the single bubble motion in MHD flows

**2015- Assistant professor**

Xi'an Jiaotong University

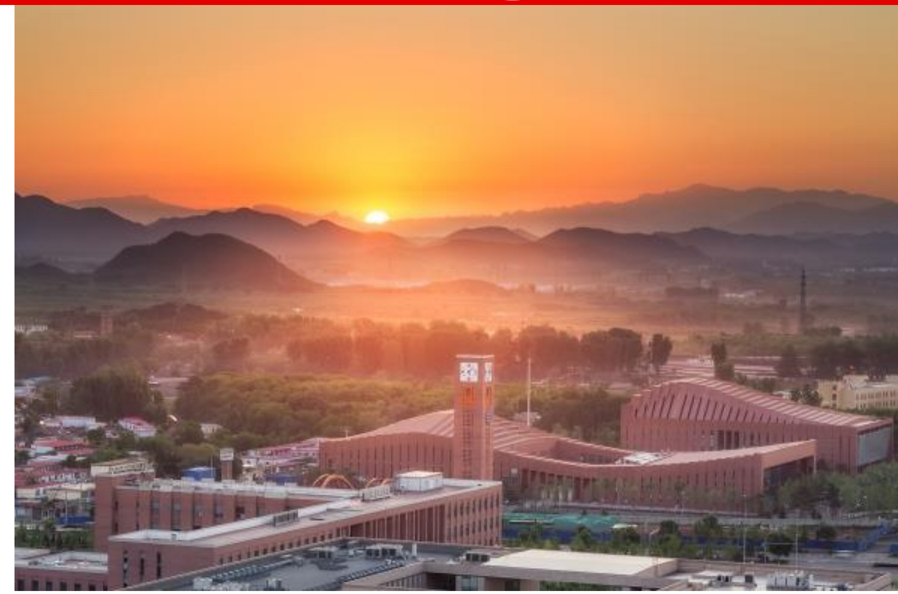
~2016.08 the droplet splashing with MHD effect

~2017.10 implement the phase change module

**2016-2017 Visiting scholar**

IMFT, Toulouse, France

working with J.Magnaudet, stratified flows



# What I did ? ——Numerics



## Incompressible Navier-Stokes Equation

EMF

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla \cdot (\eta \nabla \boldsymbol{u}) + \sigma_s k \delta \nabla c + \boldsymbol{J} \times \boldsymbol{B} + \rho g (1 - \beta \Delta T)$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\boldsymbol{u}) = 0$$

VOF for interface tracing

## Charge Conservation Law

$$\nabla \cdot \boldsymbol{J} = 0$$

## Electrical Potential Poisson Equation

$$\boldsymbol{J} = \sigma(-\nabla \varphi + \boldsymbol{u} \times \boldsymbol{B})$$

$$\nabla \cdot (\sigma \nabla \varphi) = \nabla \cdot (\sigma \boldsymbol{u} \times \boldsymbol{B})$$

$$\mathrm{We} = \frac{\rho u^2 L}{\sigma}$$

$$Ha = LB_0 \sqrt{\sigma / \eta}$$

$$Re = u_0 L / \eta$$

$$N = Ha^2 / Re$$

## Phase Change

$$\nabla \cdot \boldsymbol{u} = \left( \frac{1}{\rho_V} - \frac{1}{\rho_L} \right) \dot{m}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\boldsymbol{u}) = -\frac{\dot{m}}{\rho_L}$$

# Single phase MHD flows

## Numerical Methods (J. ZHANG, MJ. NI, J. Comp. Physics, 2014)

1. MHD flows with complex electrically insulated boundaries
2. Fluid-Solid coupling problems with electrically conducting boundaries

### 1. The interpolation for current density *j* in mixed cells

**Normally:**

$$(j_x)_c = \frac{\sum_{f=1}^{nf}(j_x|s_x|)_f}{\sum_{f=1}^{nf}|s_x|_f}$$

Based on the face area average

$$\boldsymbol{J} = \sigma(-\nabla\varphi + \boldsymbol{u}\times\boldsymbol{B})$$

$$\nabla\cdot(\sigma\nabla\varphi) = \nabla\cdot(\sigma\boldsymbol{u}\times\boldsymbol{B})$$

**Improvement:**

$$\boldsymbol{J}\times\boldsymbol{B} = \nabla\cdot(\boldsymbol{J}(\boldsymbol{r}\times\boldsymbol{B})) = \nabla\cdot(\boldsymbol{J}\boldsymbol{r})\times\boldsymbol{B}$$

→ $$\boldsymbol{J} = \nabla\cdot(\boldsymbol{J}\boldsymbol{r})$$

$$\boldsymbol{J}_c = \frac{1}{\Omega_c}\int_{\Omega_c}\boldsymbol{J}\,d\Omega = \frac{1}{\Omega_c}\int_{\Omega_c}\nabla\cdot(\boldsymbol{J}\boldsymbol{r})\,d\Omega = \frac{1}{\Omega_c}\oint_{S_c}J_n\boldsymbol{r}\,ds = \frac{1}{\Omega_c}\sum_{f=1}^{nf}(J_n)_f\boldsymbol{r}_f s_f$$

$$= \frac{1}{\Omega_c}\sum_{f=1}^{nf}(J_n)_f d_f s_f \boldsymbol{n}_f$$

$d_f$ is the distance from the cell center to the face center

# Single phase MHD flows

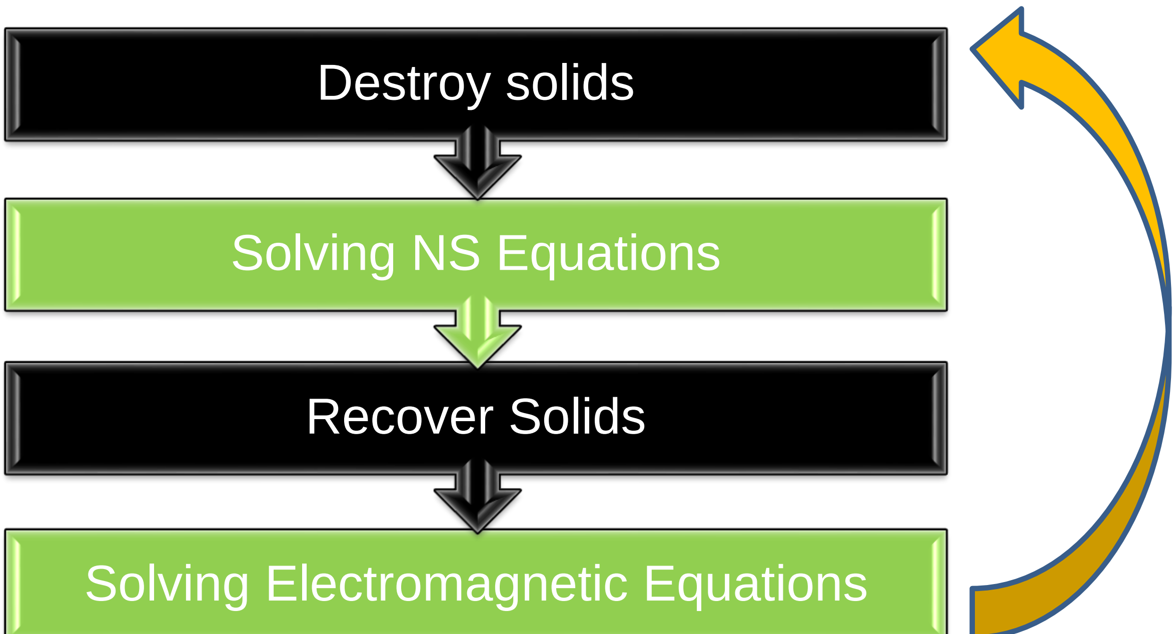
## 2. Fluid-Solid Coupling

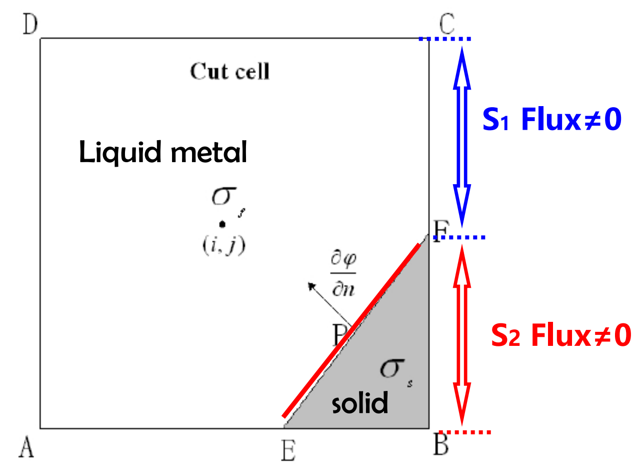
If the solids are electrically conducting, the current density also exists in the solids.

$$\sum_{f=1}^{nf}\left(\sigma_{f1}\left(\frac{\partial\varphi}{\partial n}\right)_{f1} S_{f1} + \sigma_{f2}\left(\frac{\partial\varphi}{\partial n}\right)_{f2} S_{f2}\right) = \sum_{f=1}^{nf}\sigma_{f1}(\boldsymbol{u}\times\boldsymbol{B})_f\cdot \boldsymbol{n}_f S_{f1}$$

Navier-Stokes equations : Fluids

EMHD equations: Fluids+ Solids

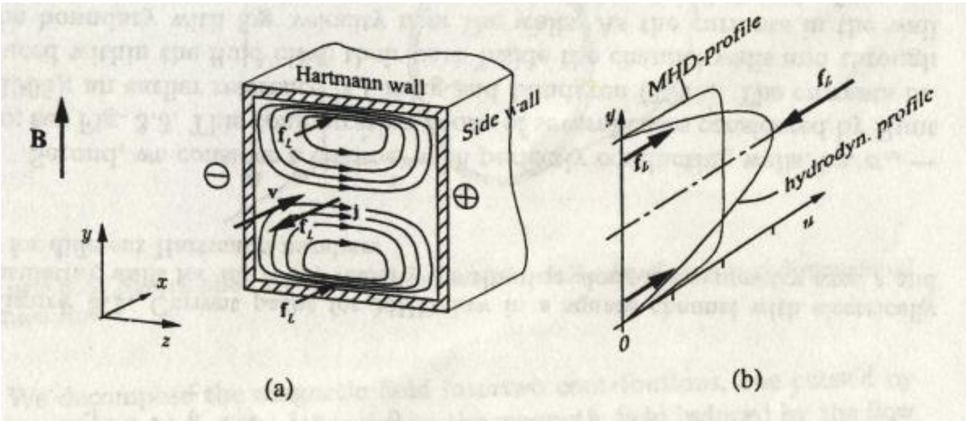


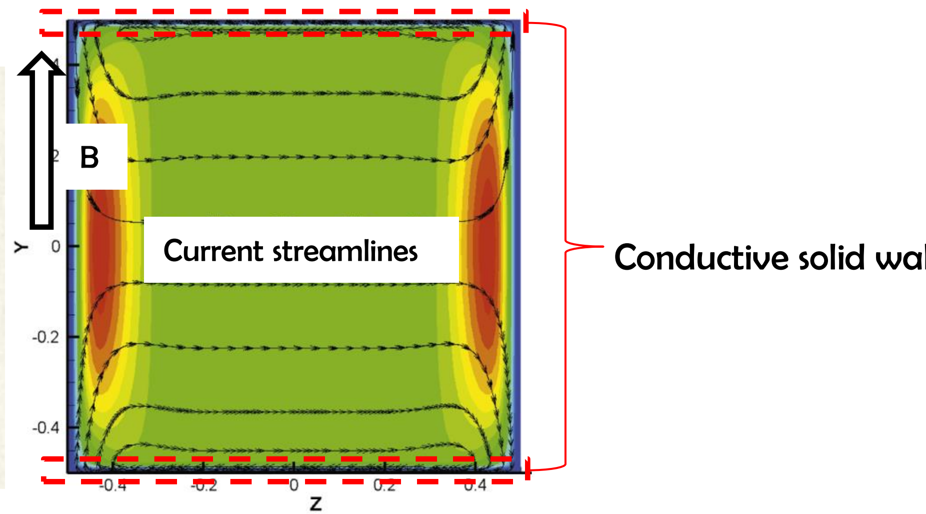



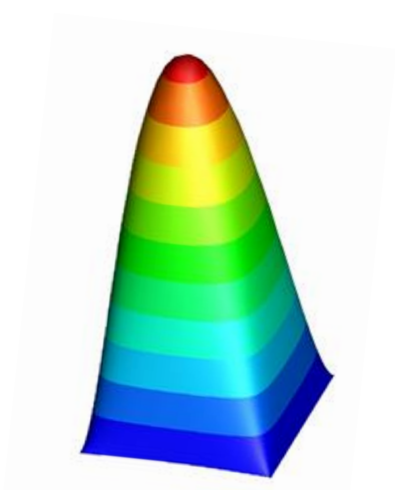

Fluid-solid coupling problems when the solid is electrically conducting

# Single phase MHD flows

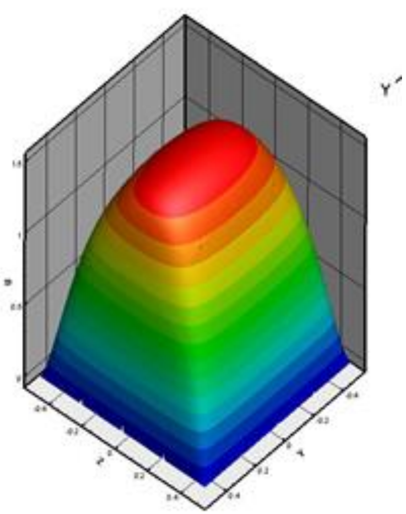
## Results for MHD channel flows:



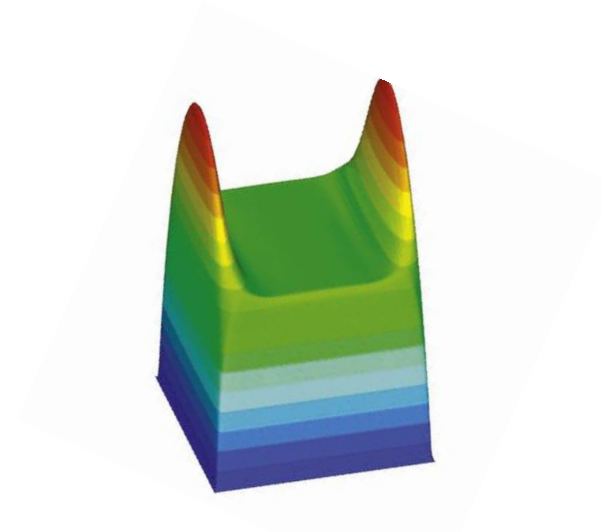



1. Hydrodynamics flow



2. MHD flows in insulated duct



3. MHD flows in conducted duct

# Multiphase phase MHD flows



## Numerical Methods ( J. ZHANG, MJ. NI, J. Comp. Physics, 2014)

1. MHD module with free surface flows
2. Marangoni effects are included

### If Marangoni effect is considered

$$F_s = \sigma\kappa\delta_s \boldsymbol{m} + \nabla_{\|}(\sigma)\delta_s \quad \text{and} \quad \sigma = \sigma_0 + \sigma_T(T - T_0)$$

$$\rightarrow F_s = \sigma\kappa\delta_s \boldsymbol{m} + \sigma_T\nabla_{\|}(T)\delta_s$$

$$\nabla_{\|}T = (\boldsymbol{I} - \boldsymbol{m}\otimes\boldsymbol{m})\nabla T = \nabla T - (\nabla T\cdot\boldsymbol{m})\boldsymbol{m}$$

$$\sigma_T\nabla_{\|}T\delta_s = \sigma_T\big(\nabla T - (\nabla T\cdot\boldsymbol{m})\boldsymbol{m}\big)\delta_s$$

$$= \sigma_T\left(\nabla T - \left(\nabla T\cdot\frac{\nabla h}{|\nabla h|}\right)\frac{\nabla h}{|\nabla h|}\right)|\nabla h|$$

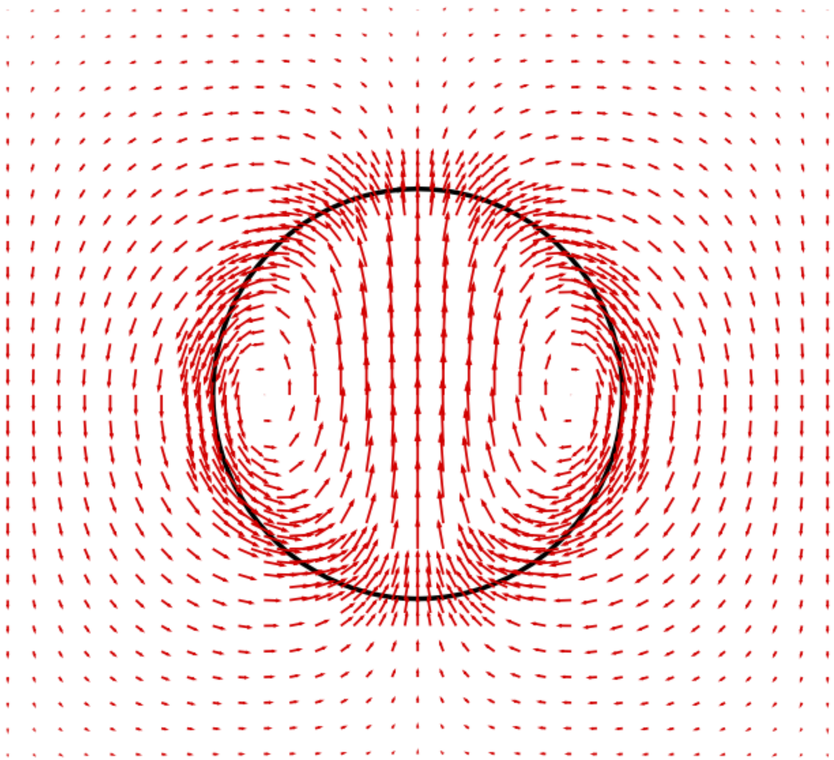


**Fig. 16.** Velocity profile for thermocapillary droplet motion.

# Further improvement?

**1.** In solving the MHD multiphase flows, the electric conductivity is calculated as

$$\sigma = \sigma_1 T + \sigma_1 (1 - T)$$

There is no problem when calculating $\rho$ and $\mu$, however, it will introduce error in calculating the electric conductivity !!

Why ??

Let us consider the jump condition across the interface

The electric current and electric potential is continuous

$$\begin{aligned} [\![\boldsymbol{J}]\!] &= 0 \\ [\![\varphi]\!] &= 0 \end{aligned}$$

$$\boldsymbol{J} = \sigma_e(-\nabla\varphi + \boldsymbol{u} \times \boldsymbol{B})$$

Therefore, the Ohm's law yields

$$\begin{aligned} [\![\sigma_e(\nabla\varphi)]\!] &= [\![\sigma_e(\boldsymbol{u} \times \boldsymbol{B})]\!] \\ &= [\![\sigma_e]\!](\boldsymbol{u} \times \boldsymbol{B})^{\Gamma} \end{aligned}$$

It is obvious that if the interface is translating or ratating, the flux of the $\nabla\varphi$ across the interface is not continuous any more

# Cut-cell approach



## How to improve the scheme?

We use the Cut-cell scheme to separate the interfacial cell into two parts: liquid_1 and liquid_2

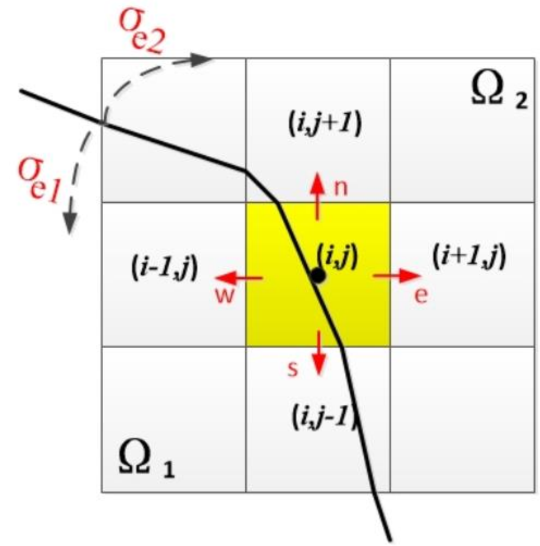
Solve this equation

$$\sum_{n=1}^{nf} \sigma_e \frac{\partial \varphi}{\partial n} ds = \sigma_e \theta \Delta V$$

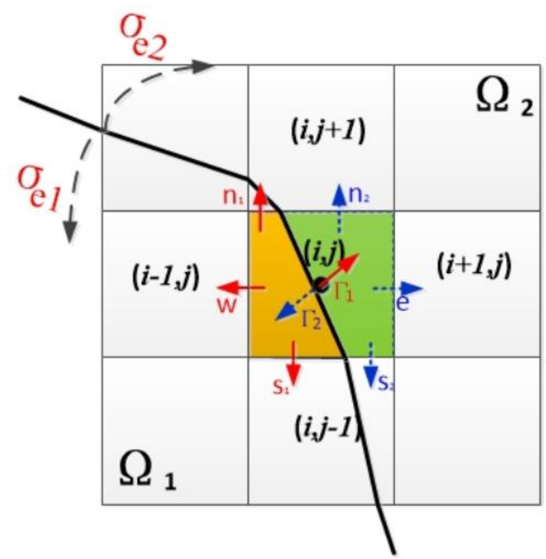
$$(\sigma_{e2})(\frac{\partial \varphi}{\partial n})^e - (\sigma_{e1})(\frac{\partial \varphi}{\partial n})^w + (\sigma_{e1} f^n + \sigma_{e2}(1-f^n))(\frac{\partial \varphi}{\partial n})^n - (\sigma_{e1} f^s + \sigma_{e2}(1-f^s))(\frac{\partial \varphi}{\partial n})^s + \underbrace{[(\sigma_{e1}\frac{\partial \varphi}{\partial n})_1 - (\sigma_{e2}\frac{\partial \varphi}{\partial n})_2]^\Gamma S^\Gamma}= (\sigma_{e1} f + \sigma_{e2}(1-f))\theta \qquad (34)$$

The flux through the interface

$$(\frac{\partial \varphi}{\partial n})^\Gamma = (\boldsymbol{u} \times \boldsymbol{B})^\Gamma \cdot \boldsymbol{n}$$




(a) volume averaged scheme




(b) cut-cell scheme

Struct GfsVOFState

Struct GfsVOFSurfaceBc

# Cut-cell approach



## Validations

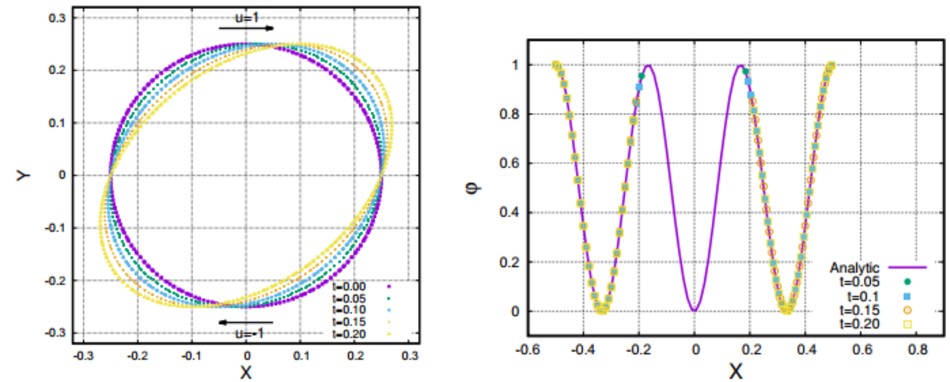
We solve the Poisson equation with a widely used test case

$$\nabla^2 \varphi = \theta$$

$$\theta = -\pi^2(k^2 + l^2) sin(\pi k x) sin(\pi l y)$$

$$\rightarrow \quad \varphi(x, y) = sin(\pi k x) sin(\pi l y)$$

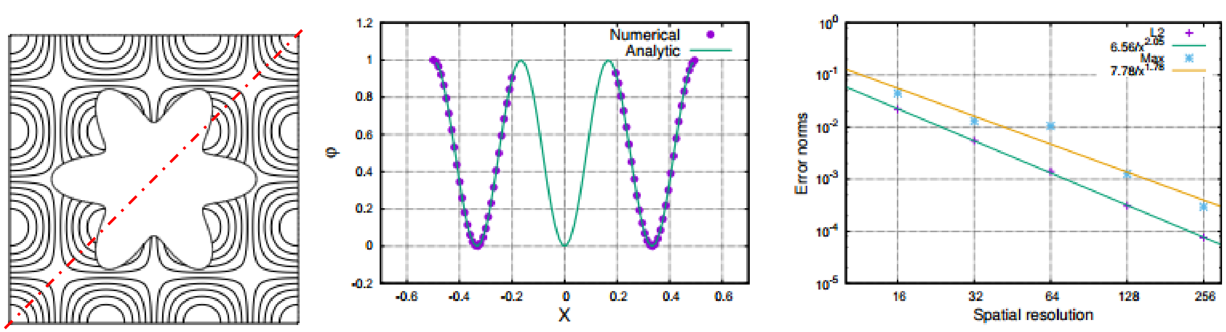
## 2. The interface is rotating in shear flow




(a) (b)

(a)The evolution of the interface shape over time;

## 1. Star shaped interface ($\sigma_2 = 0$)

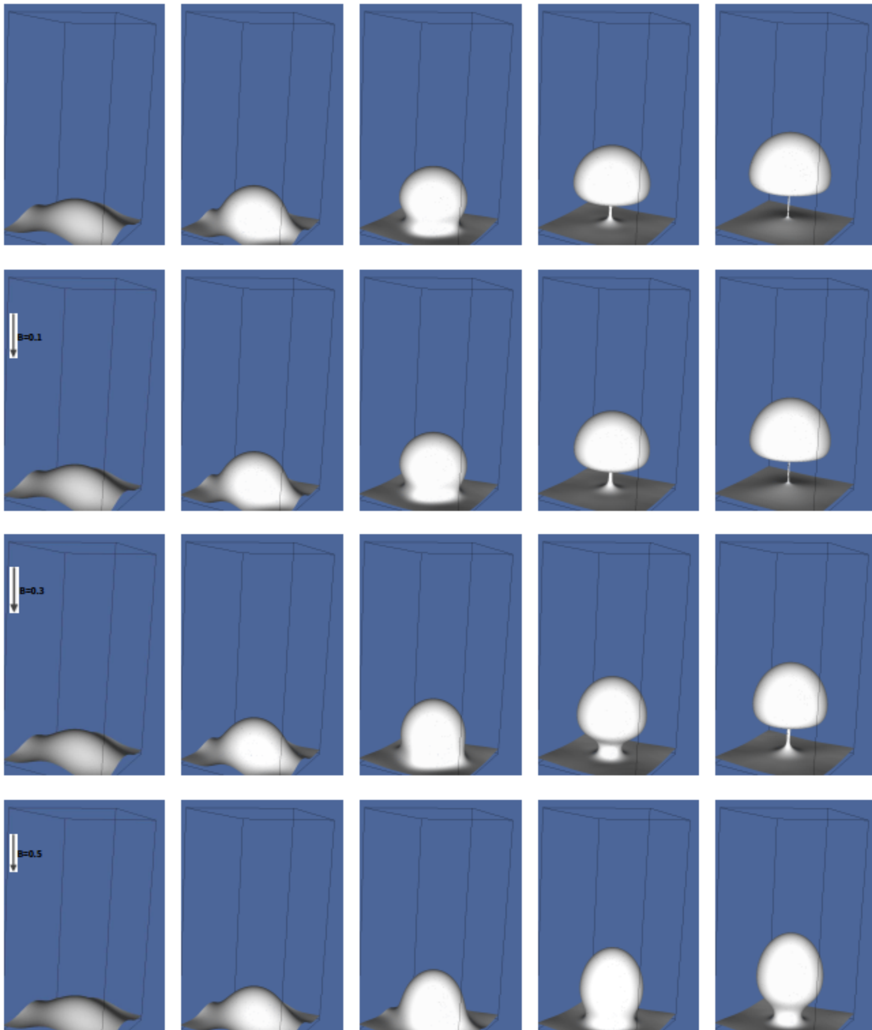



(a) The contour map of $\varphi$ in the domain of $\Omega_1$;
(b) the distribution of $\varphi$ along the diagonal line
(c) the evolution of the error and associated convergence order

# MHD flows with phase change

3D film boiling flows under the influence of MHD effect

$$\nabla \cdot u = \left(\frac{1}{\rho_V} - \frac{1}{\rho_L}\right)\dot{m} \qquad \frac{\partial c}{\partial t} + \nabla \cdot (cu) = -\frac{\dot{m}}{\rho_L}$$

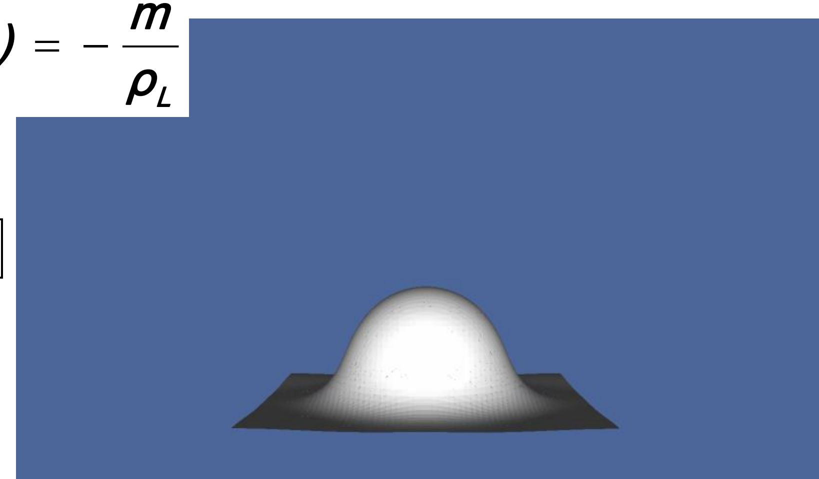



B=0T

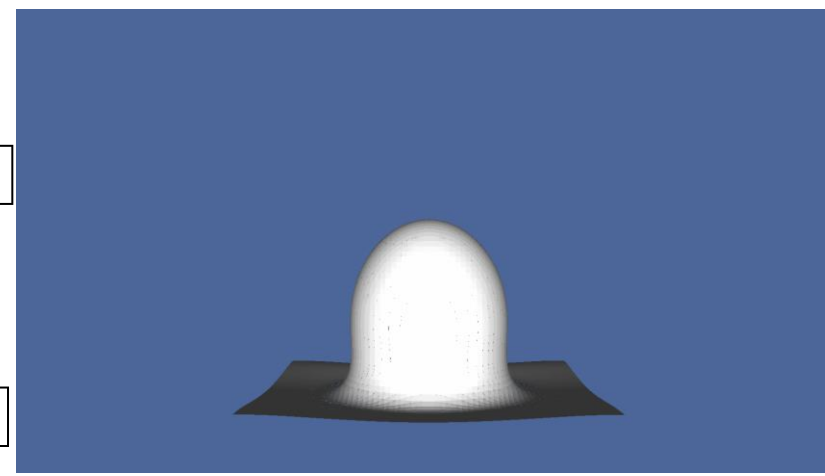
B=0.1T

B=0.3T

B=0.5T



Without magnetic field



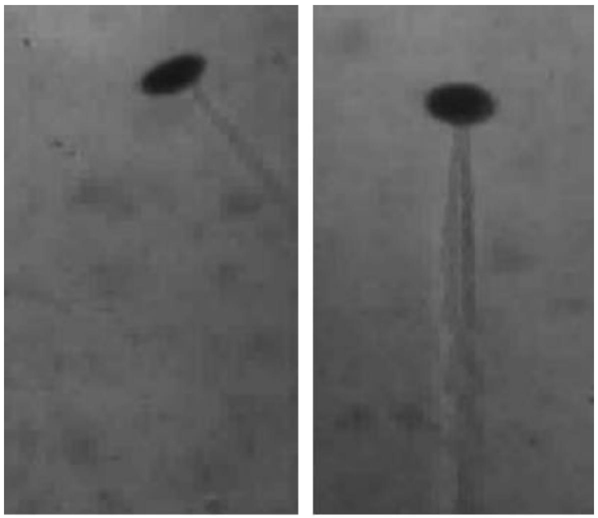
Vertical magnetic field

# Single bubble with MHD

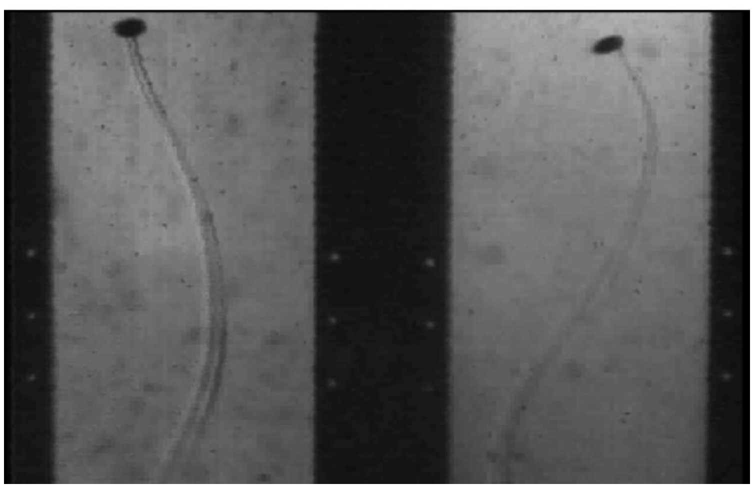
## Background:

Vortex-structures —Relations?→ Rising path

zigzag



spiral



## Purpose: the influence of MF on:

1. rising path
2. vortex structures
3. shape deformations
4. rising velocities

## Parameter spaces

- Model: Re=2000~4000, We=2~5, Eo=1.2~4.9.
- Various vertical magnetic fields are imposed.

A.W.G. de Vries, 2003, PhD thesis

# Without MFs

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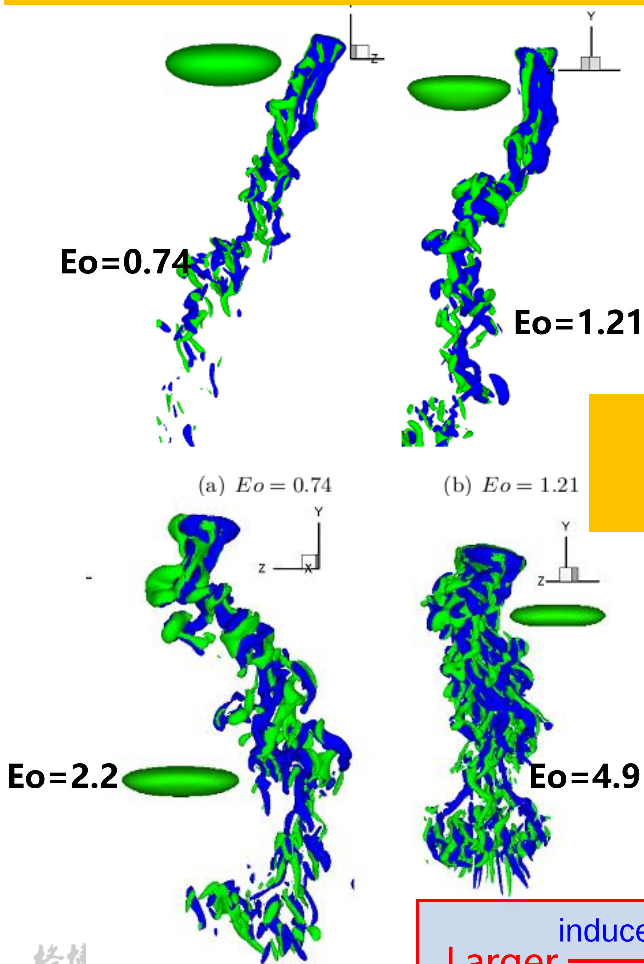
中国科学院大学 University of Chinese Academy of Sciences

## Without MFs: Vortex structures of different sized bubble

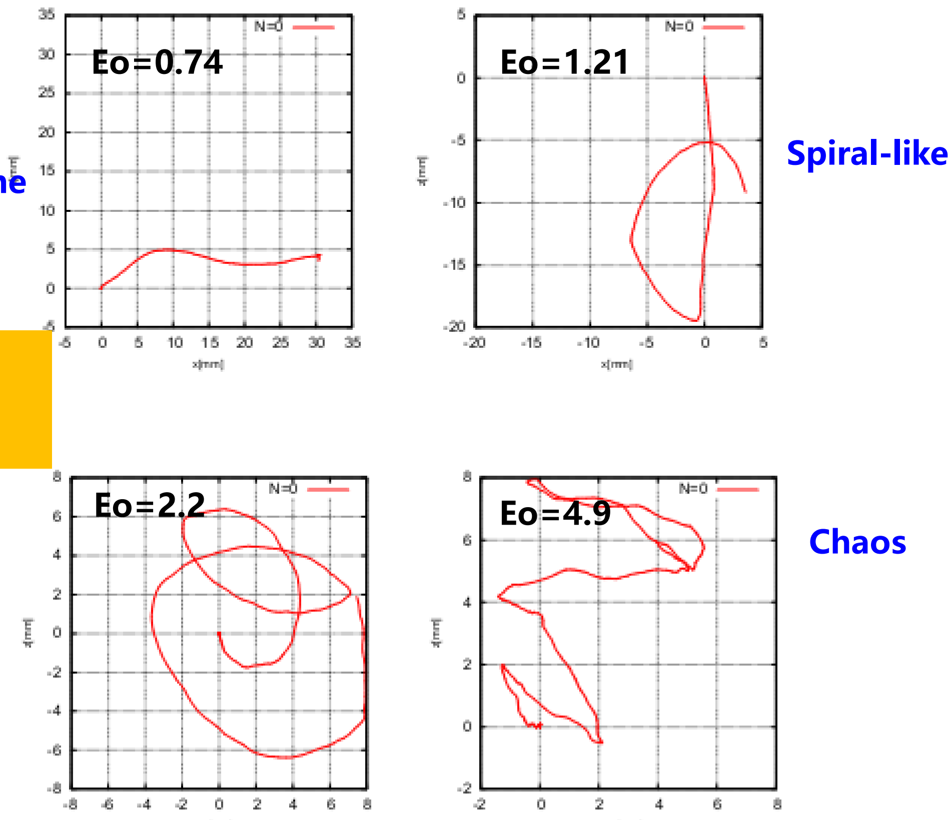


(a) $Eo = 0.74$ (b) $Eo = 1.21$

(c) $Eo = 2.2$ (d) $Eo = 4.9$

$$Eo = \frac{\rho\sqrt{gD}D}{\mu}$$

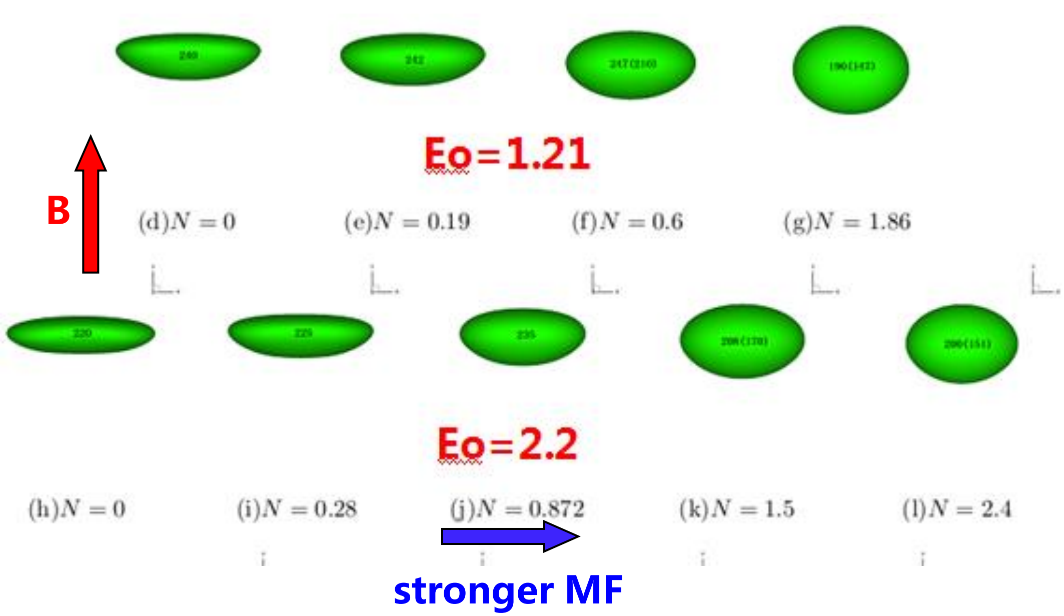
## Without MFs: top view of the bubble path



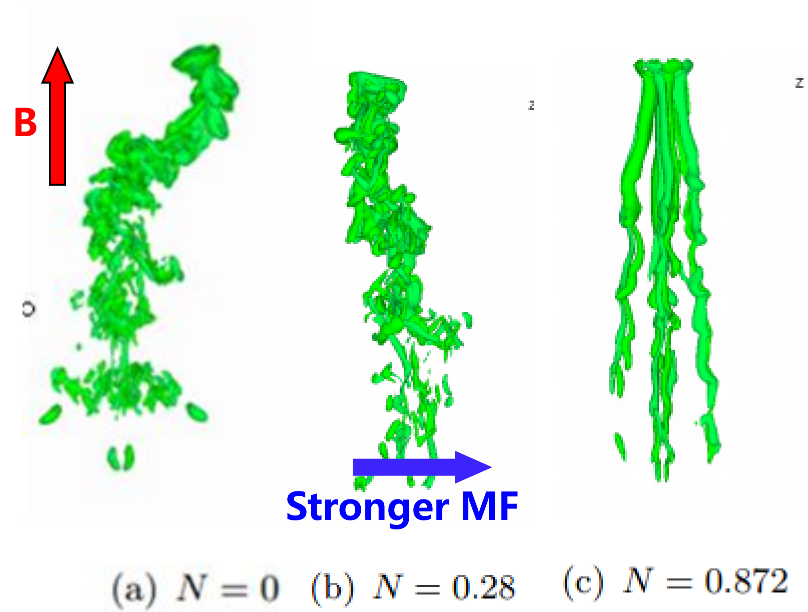

Larger **bubble** —induce→ More flatten shape —induce→ More unstable vortex —induce→ More unstable path

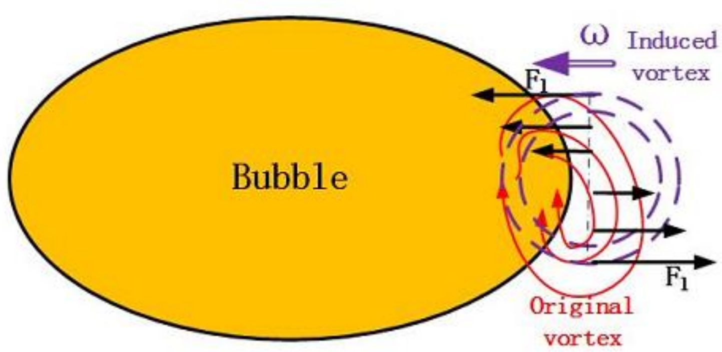
# vertical MFs

Variations of bubble shape




Variations of the vortex structures

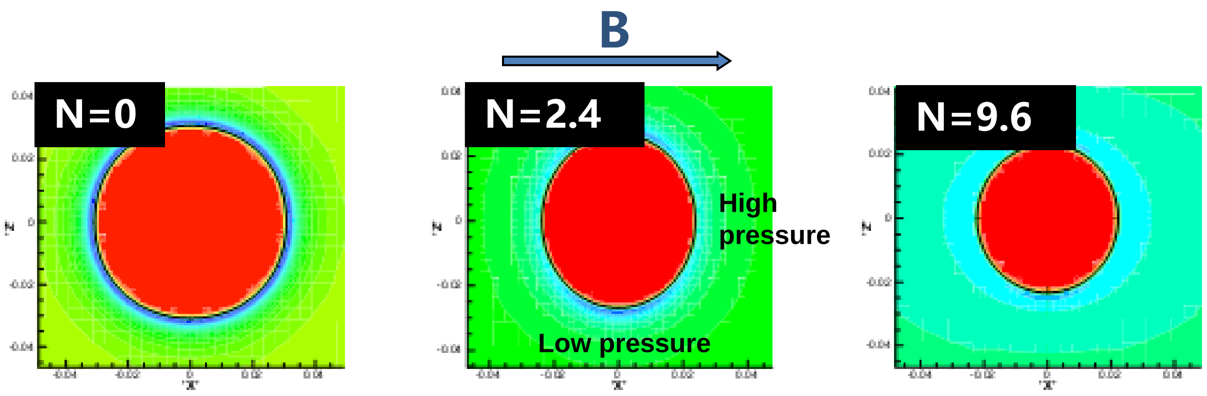



**Why bubble is more round?**

# horizontal MFs

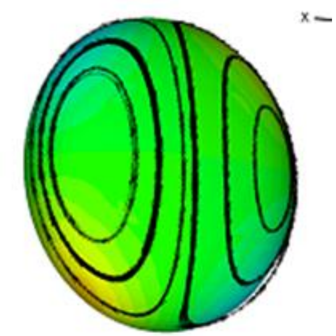
## Top view of bubble




(a) $N = 0$ (b) $N = 2.4$ (c) $N = 9.6$

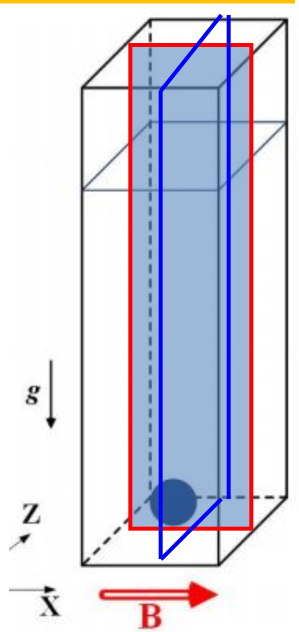
most obvious feature :

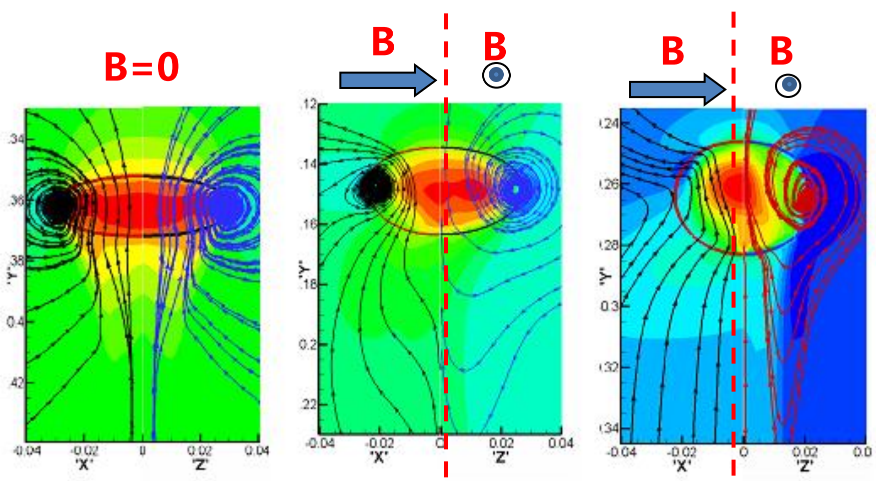
**Anisotropic!!!**



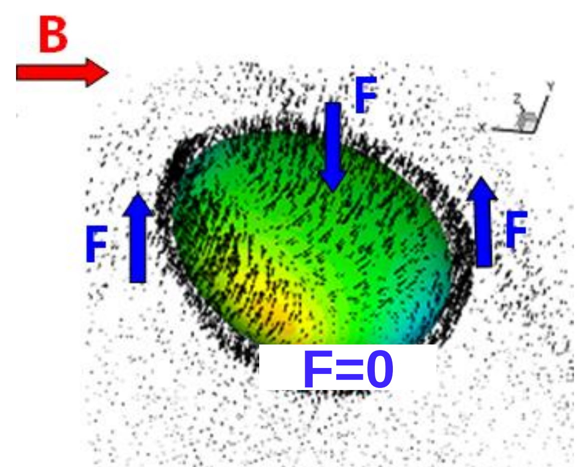
current density

## Flow field




(a) $N = 0$ (b) $N = 0.872$ (c) $N = 9.6$




Lorentz force

# horizontal MFs

## Pressure force is balanced by Lorentz force

The Lorentz torque is transferred to viscous dissipation an
d Joule dissipation

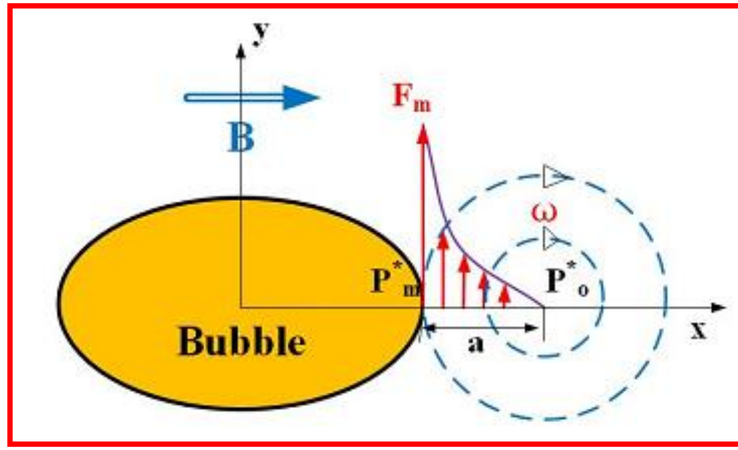
$$j_z B \cdot a \approx \mu \frac{\omega a}{a^2} + \sigma_e u B \cdot Ba$$
$$\approx \sigma_e \omega a \cdot B \cdot Ba$$
$$\omega \approx \frac{j_z}{\sigma_e B a}$$

The elevated pressure induced by the Lorentz torque, Centrifugal effect in the interface vicinity

$$p_2^* \approx \rho u^2 = \rho \omega^2 a^2$$
$$= \rho a^2 \frac{j_z^2}{\sigma_e^2 B^2 a^2} = \frac{\rho j_z^2}{\sigma_e^2 B^2}$$

The ratio of induced pressure jump versus original pressure drop

$$\frac{p_2^*}{p_1^*} \approx \frac{\rho j_z}{\sigma_e^2 B^3 a}.$$



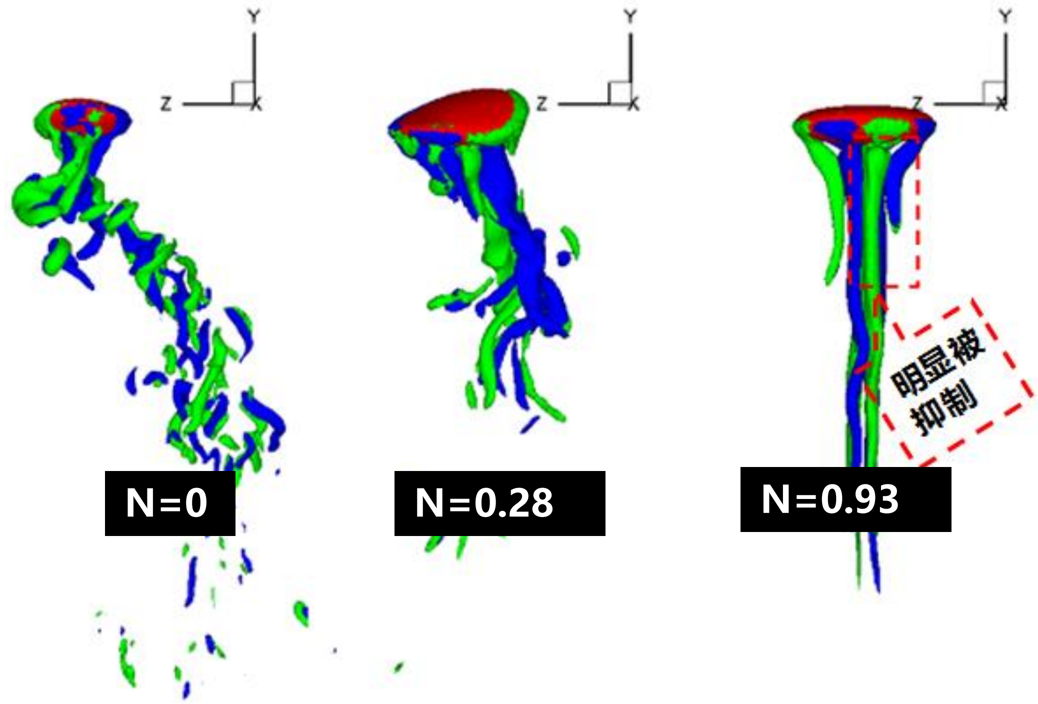

Moderate N, the ratio scale with $N^{-1}$, and the centrifugal effect is dominant. It squeezes the bubble.

High N, the ratio scale with $N^{-2}$, the centrifugal effect disappear and the bubble recover its circular shape. $j_z \approx \rho u^2 / Ba$

# horizontal MFs



B ⊙




(a) $N = 0$ (b) $N = 0.28$ (c) $N = 0.93$

The wake behind the bubble is also anisotropic

J. ZHANG, MJ.NI, 2014, Phy. Fluids

J. ZHANG, MJ.NI, 2016, Phy. Fluids

# Single bubble motion without MFs

## Why revisit the single bubble motion without MFs ?

Some problems are still not clear

Why does bubble transits from zigzag to spiral ? What happens to the bubble wake during the transition?

Some references:

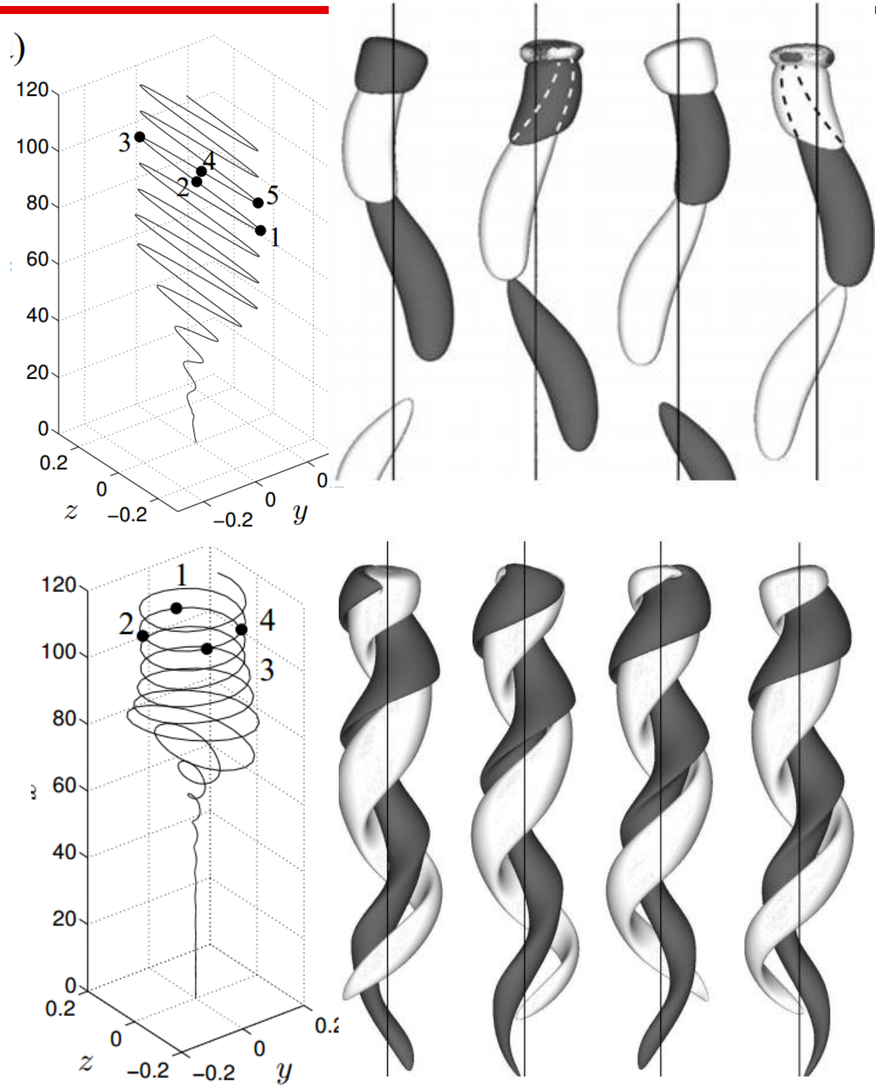
Shew(2002): "to unravelling the causes of the transition from zigzag to spiral. Is one wake vortex stronger or, as we suggest, are the wake vortices simply unstable to rotation?"

Patricia Ern(2012) "The way the transition to such paths occurs, especially the nature of the corresponding bifurcation(s), and the reorganization of the wake dynamics it implies are mostly unknown presently and need to be clarified"

# horizontal MFs

Zigzag

Spiral

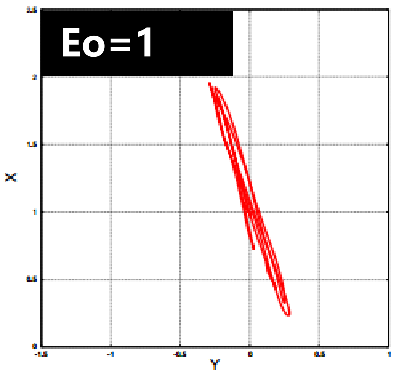


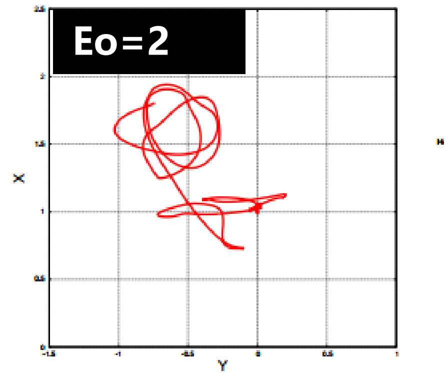

J. Cano-Lozano and J.Magnaudet, 2016, *Physical Rivew : Fluids*

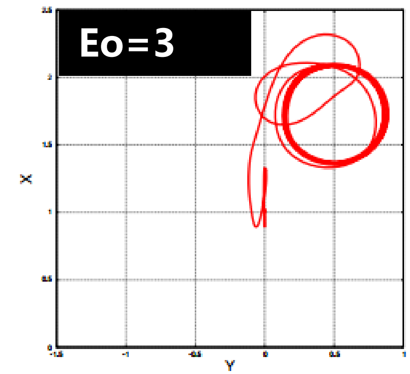
S.Popinet, 2017, Basilisk, http://basilisk.fr/src/examples/bubble.c

# Zigzag and Spiral motions

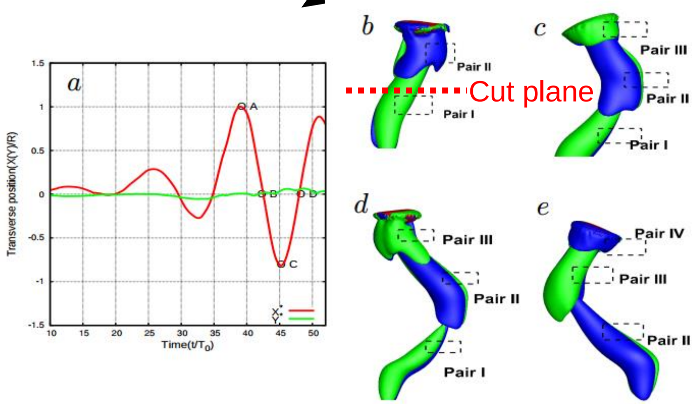
Three cases: zigzag, zigzag-spiral, spiral, Re~ 150



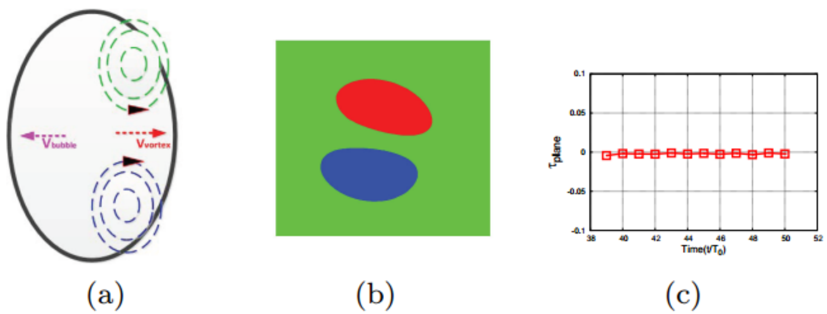




(1). Zigzag




Evolution of the vortex structures during a zigzag period



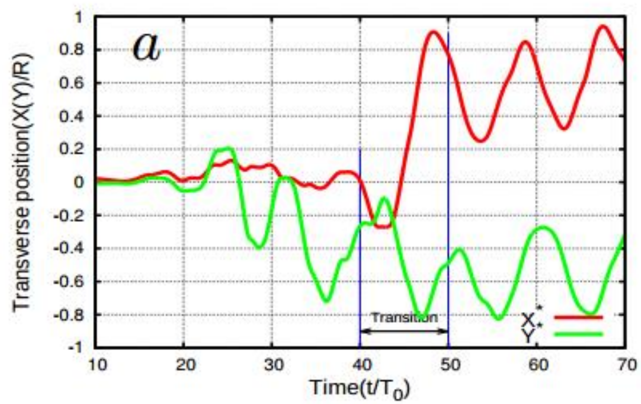

The bottom view of the vortex structures. (c) is the time series of the arithmetic integration of the streamwise vorticities $\tau_{plane} = \int \omega_z dS$, proving that the counter-rotating vortices are perfectly symmetric

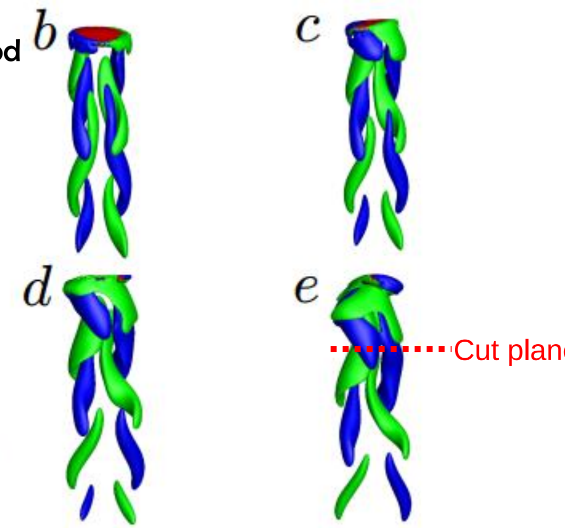
# Zigzag and Spiral motions



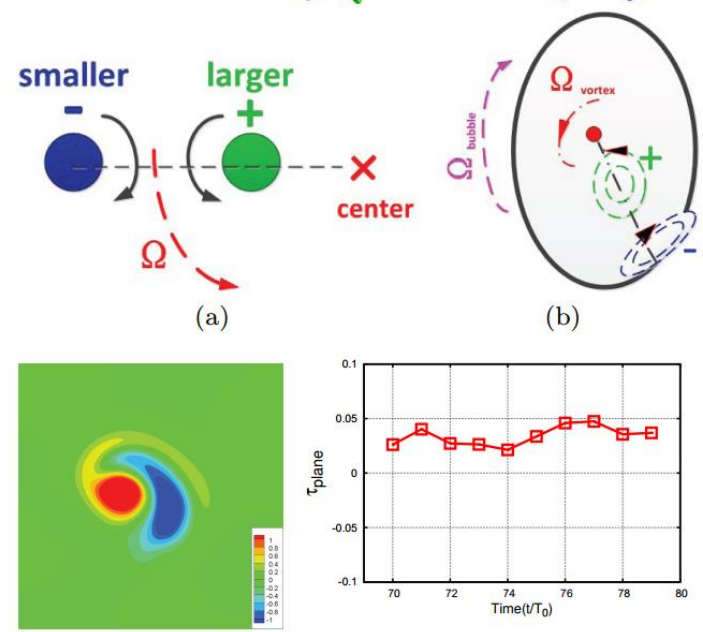
**(2). Zigzag-spiral**

Evolutions of vortex structures in the transition period




The inner positive thread is stronger than the outside negative one gradually!!




(a) (b) (c) (d)

(a) Sketch of the motion of the counter-rotating vortex pairs when they are unequal in strength.
(b) The top view of the vortex structures behind spiraling bubble.
(c) The contour of the streamwise vorticities in the cut plane 4*R* downstream the bubble.
(d) The time evolution of the arithmetic sum on the cross plane $\tau_{plane} = \int \omega_z dS$

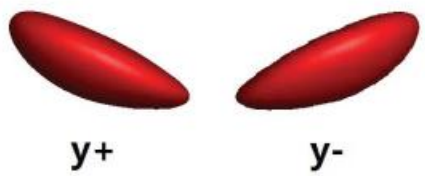
# Zigzag and Spiral motions

## Why the vortex strengths are different between the two threads?

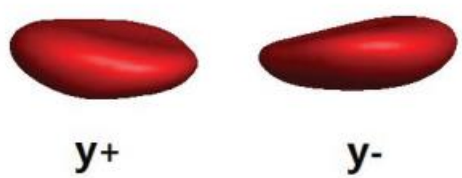
In fact, Brucker (1999, Phys. Fluids) observed this in experiments, but he did not give more discussions

Because in case of large We, the bubble deformation is more significant, therefore, asymmetric bubble shape is appeared

Asymmetric shapes
↓
asymmetric vortex structures
↓
Angular velocity
↓
Spiral motion




(b)zigzag motion




(a)spiral motion

So we think:

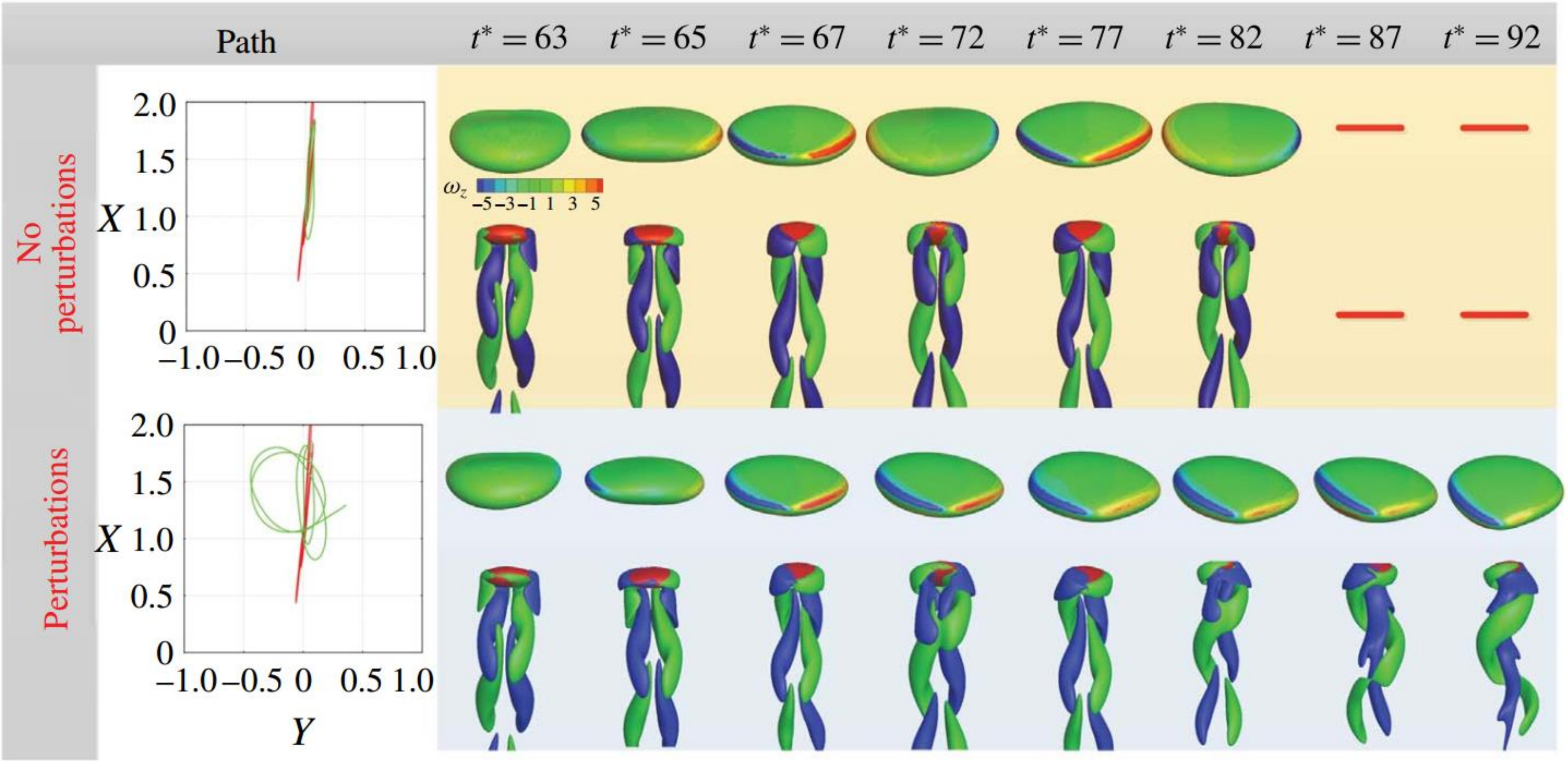
In case of low Re
No flow instability, asymmetric shape induce asymmetry between the double vortex threads

In case of high Re
Many factors will take effect to destroy the balance between the double vortex threads

# Zigzag and Spiral motions



For a zigzag-spiral transitional bubble, what happens if we impose a perturbation on the surface tension when it rises in the zigzag stage?




By imposing the disturbance at the surface tension, the balance between the double-threaded vortex structures are destroyed, therefore, the vortex threads twine with one another gradually.

# Zigzag and Spiral motions



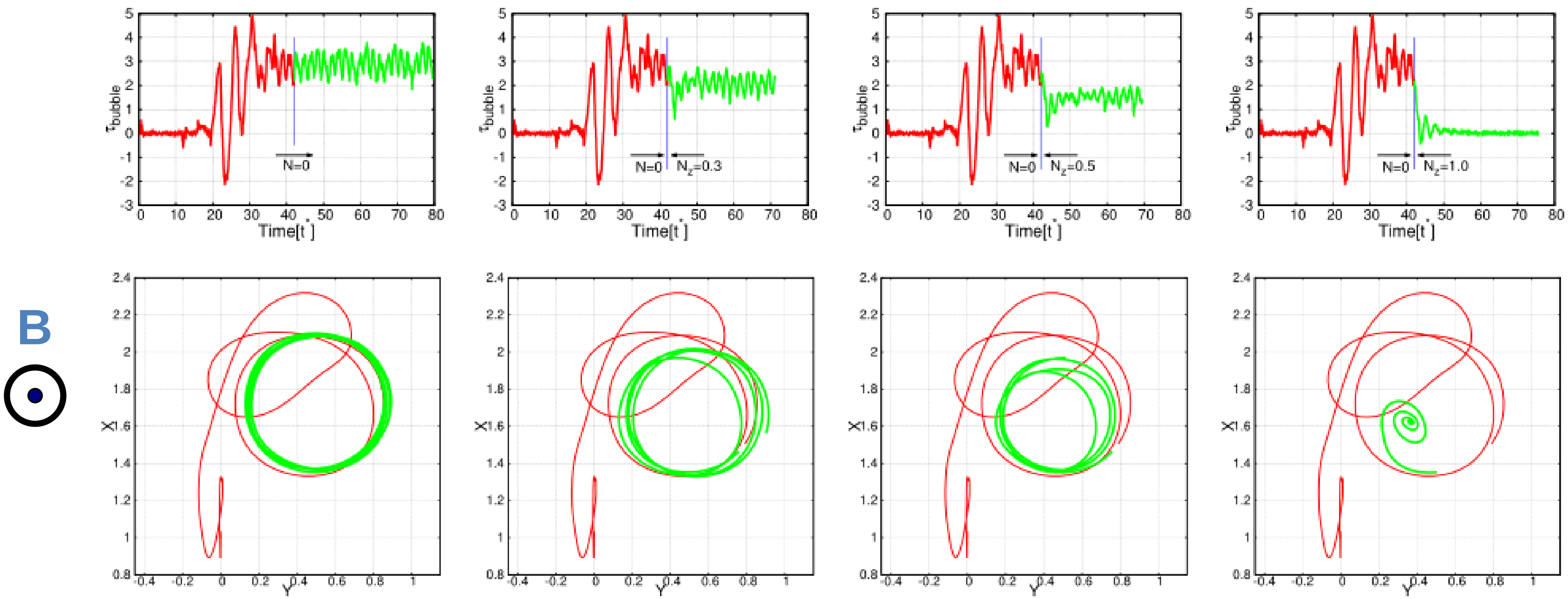
## How to verify this with MF?

If we narrow the imbalance between the double vortex threads, can the spirally rising bubble transit to zigzag or even rectilinear motion? Imposing MF onto a spiral motion bubble

## 1. Vertical MF

Red: before imposing MF;
Green: after imposing MF.

B




**UP**: The gap between the positive and negative vortex strength

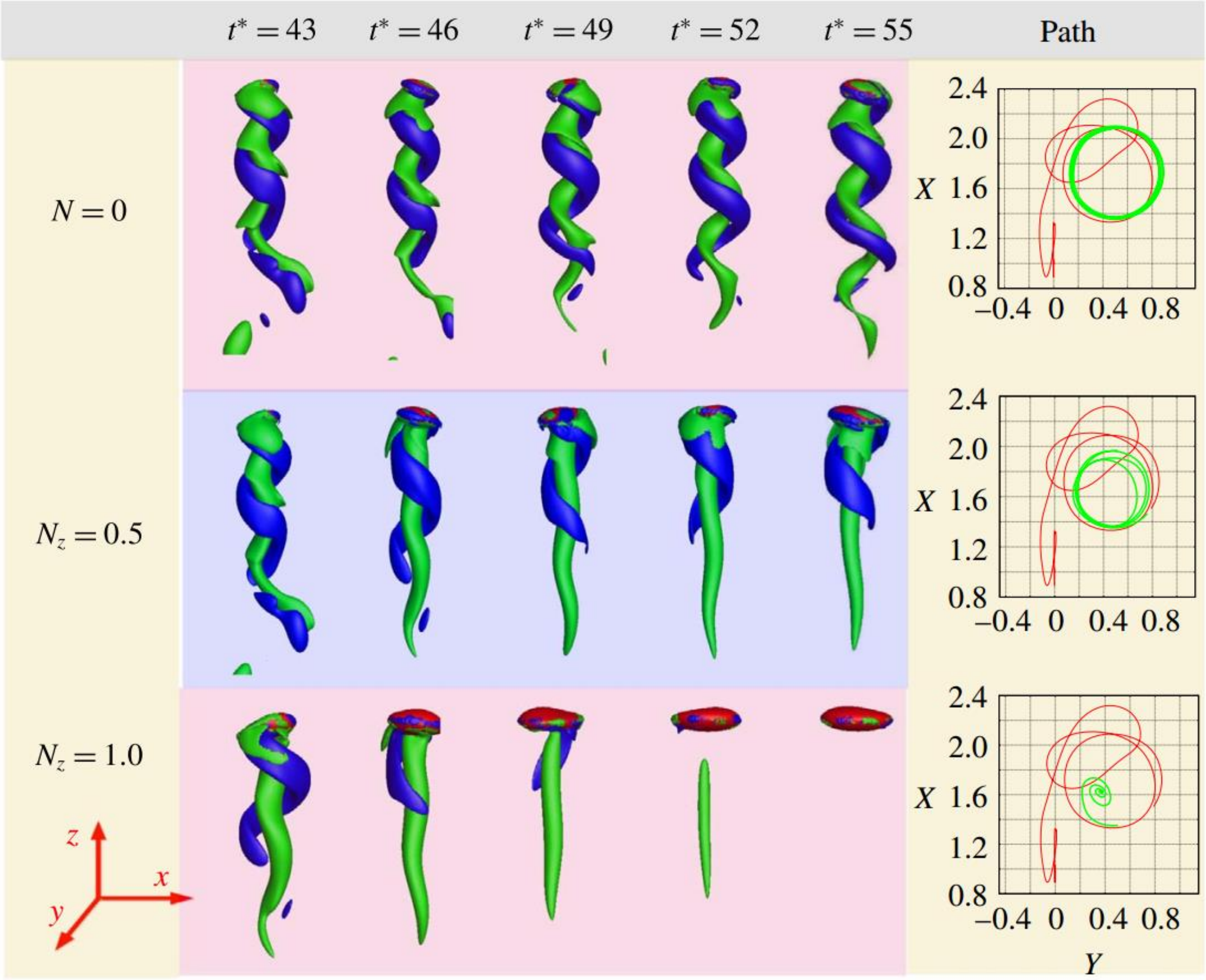
arithmetic sum of $\omega_z$ $\tau_{bubble} = \int_S \omega_z dS.$

**BOTTOM**: The radius of spiral motion.

More symmetric vortex threads
⬇
Smaller spiral radius

# Zigzag and Spiral motions

## The influence of the vertical MFs




J. ZHANG, MJ.NI, 2017, J. Fluid Mech.

# drop impact on liquid film

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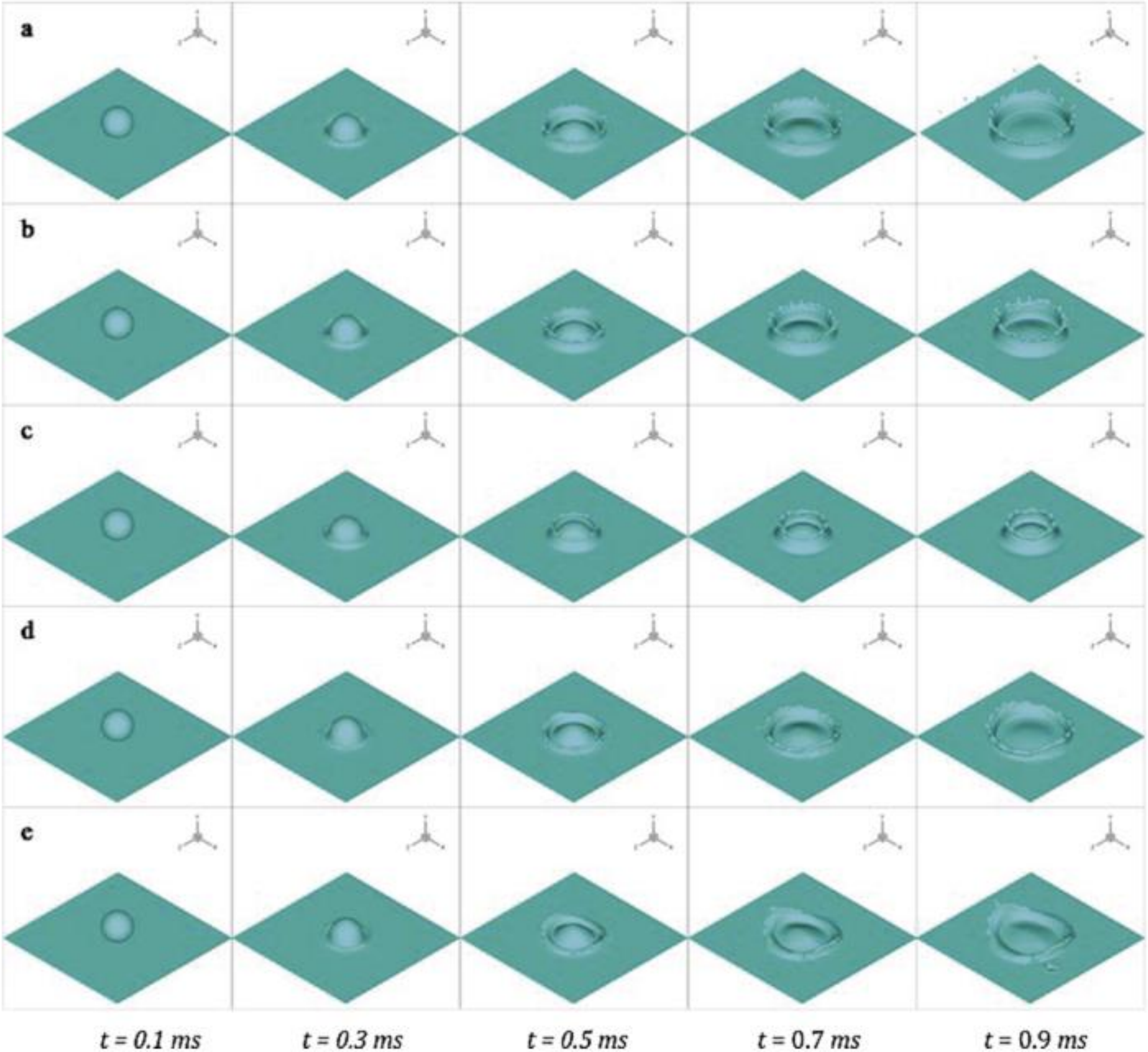
(a) Without MFs, Re=21622, We=190

(b)Vertical MFs B=0.5

(c)Vertical MFs B=1.0

(d)Horizontal MFs B=0.5

(e)Horizontal MFs B=1.0

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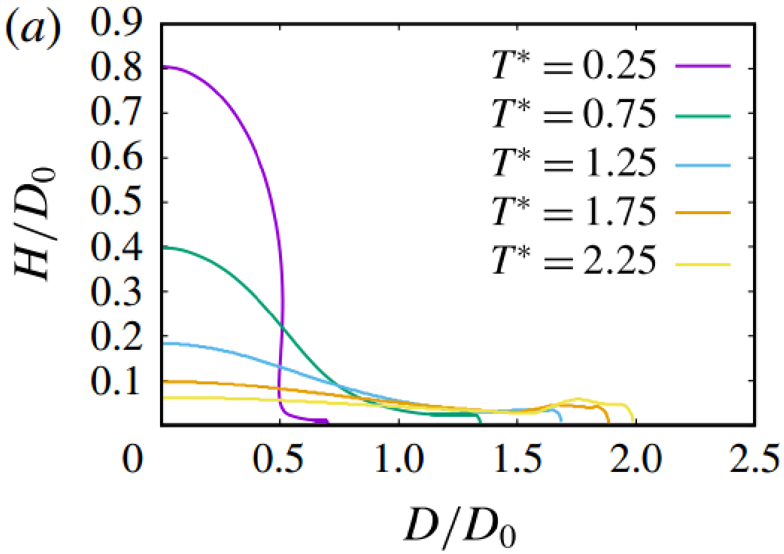
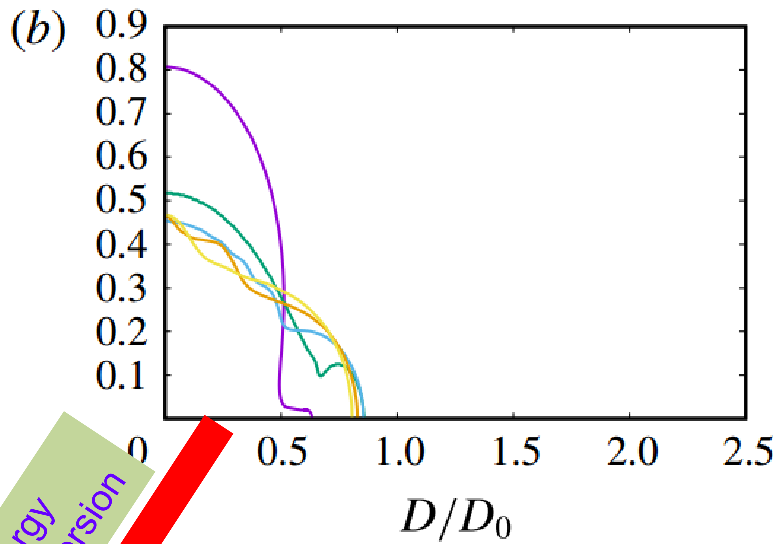
# drop impact on solid surface

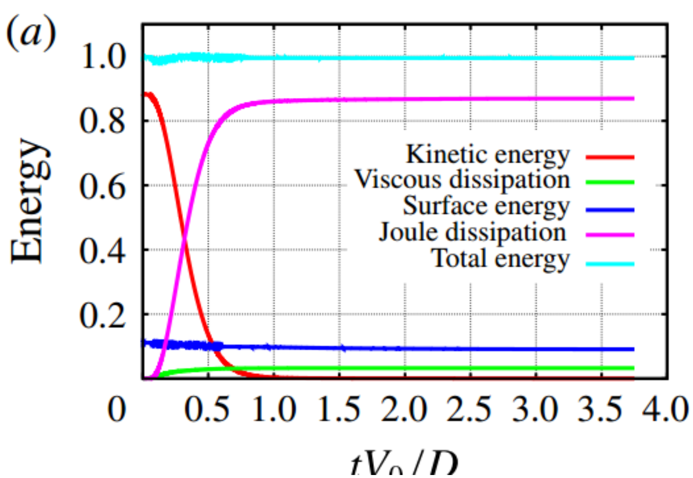


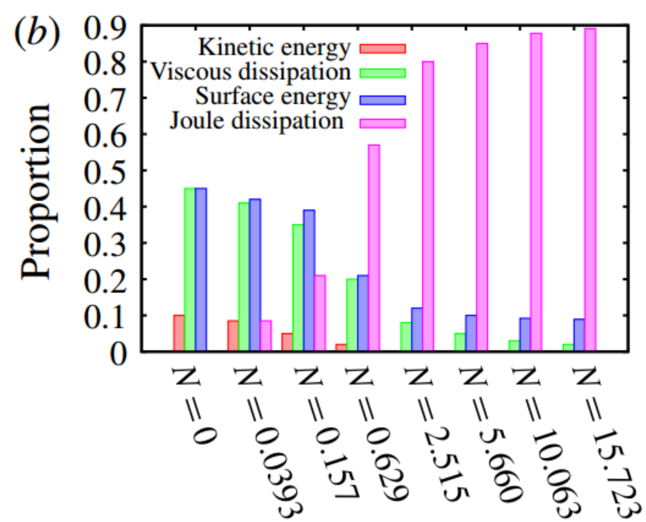

Without MFs




Under Vertical MFs

Energy conversion



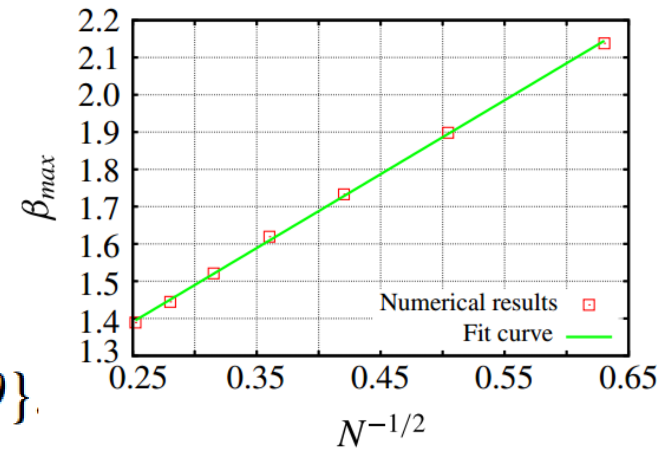



# drop impact on solid surface

We know that without MFs

$$\beta_{max} \propto Re^{1/5} \qquad \beta_{max} \propto We^{1/2}$$

With MFs, we prove that $\beta_{max} \sim N^{-1/2}$

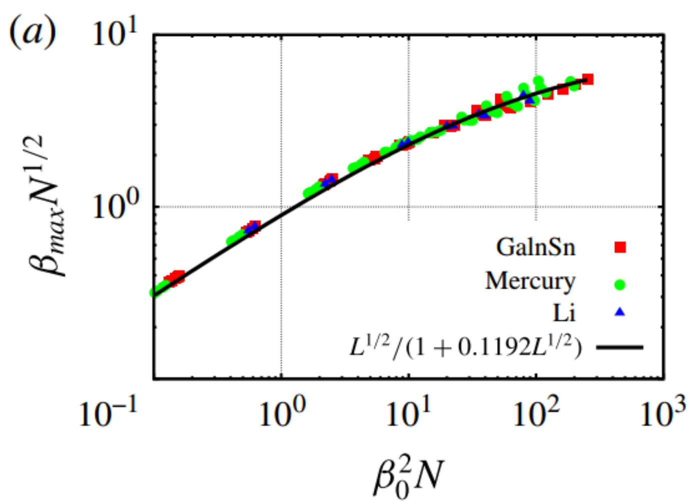



Finally:

$$\beta_{max} \propto \{Re^{1/5},\ We^{1/2},\ N^{-1/2},\ \theta\}$$

We establish a correlation to predict the maximum spreading factor

$$\beta_{max}N^{1/2} = L^{1/2}/(1+BL^{1/2}), \qquad L = \beta_0^2 N,$$




J. ZHANG, MJ.NI, 2016, J. Fluid Mech

# Summary

## Bubbles

1. Single bubble motion under vertical/horizontal MFs
2. Single/double bubble motion without MFs

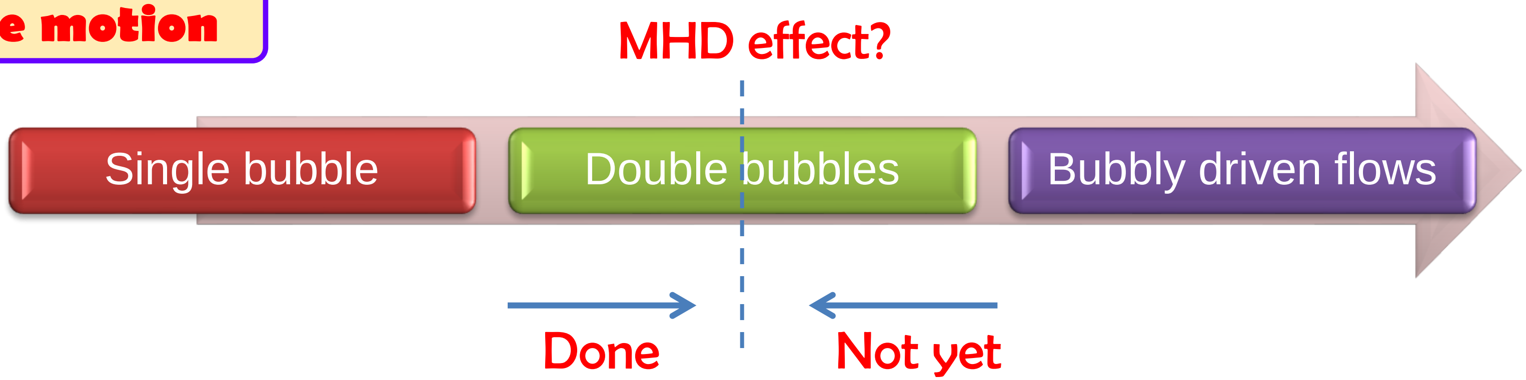
J. ZHANG, MJ.NI, 2014, Phy. Fluids
J. ZHANG, MJ.NI, 2016, Phy. Fluids
J. ZHANG, MJ.NI, 2017, J. Fluid Mech.

## Droplets

1. Droplet impact onto liquid thin film under MFs
2. Droplet impact onto solid surface under MFs
3. Droplet falling across a non-uniform MFs

J.J. WANG, J. ZHANG, MJ.NI, 2014, Phy. Fluids
J. ZHANG, MJ.NI, 2016, J. Fluid Mech
S. WU, J. ZHANG, MJ.NI, 2017, submitted

**Bubble motion**



# Prospective





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