Some aspects of Rayleigh-Bénard dynamics studied using Basilisk

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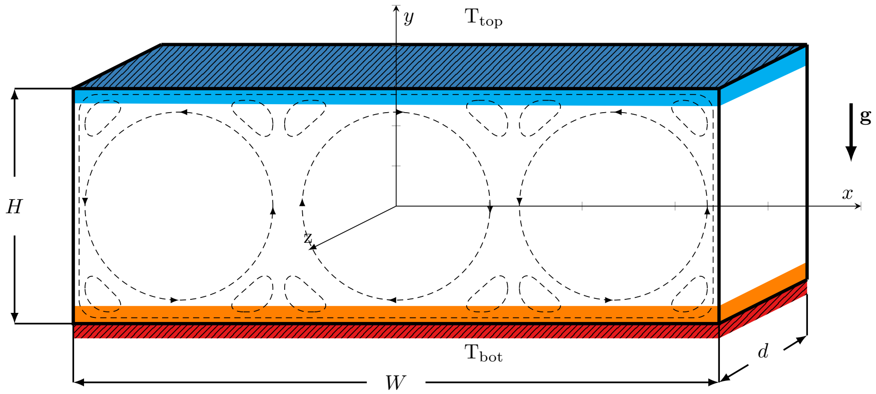
The Rayleigh-Bénard problem



$$\Delta T = T_{bot} - T_{top}$$

A horizontal layer of fluid heated from below, cooled from above

Dimensionless numbers

$$Ra \equiv \frac{\beta \Delta T g H^3}{\kappa \nu} \qquad Pr \equiv \frac{\nu}{\kappa} \qquad \Gamma_x \equiv \frac{W}{H} \qquad \Gamma_z \equiv \frac{d}{H}$$

	Ra	Pr
Indoor ventilation	10^{10}	0.7
Deep oceanic convection	10^{24}	7
Mantle convection	10^{9}	10^{23}

Experiments	$Ra \sim 10^{16}$
Direct numerical simulation	$Ra \sim 10^{13}$

Experiments and simulations: main system responses

Global Nusselt number

$$Nu \equiv \frac{q_{Tot}}{q_{cond}} = \frac{\langle \overline{vT} \rangle - \kappa \langle \overline{\partial_y T} \rangle}{\kappa \Delta T / H}$$

Global Reynolds number

$$Re \equiv \frac{UH}{\nu}$$

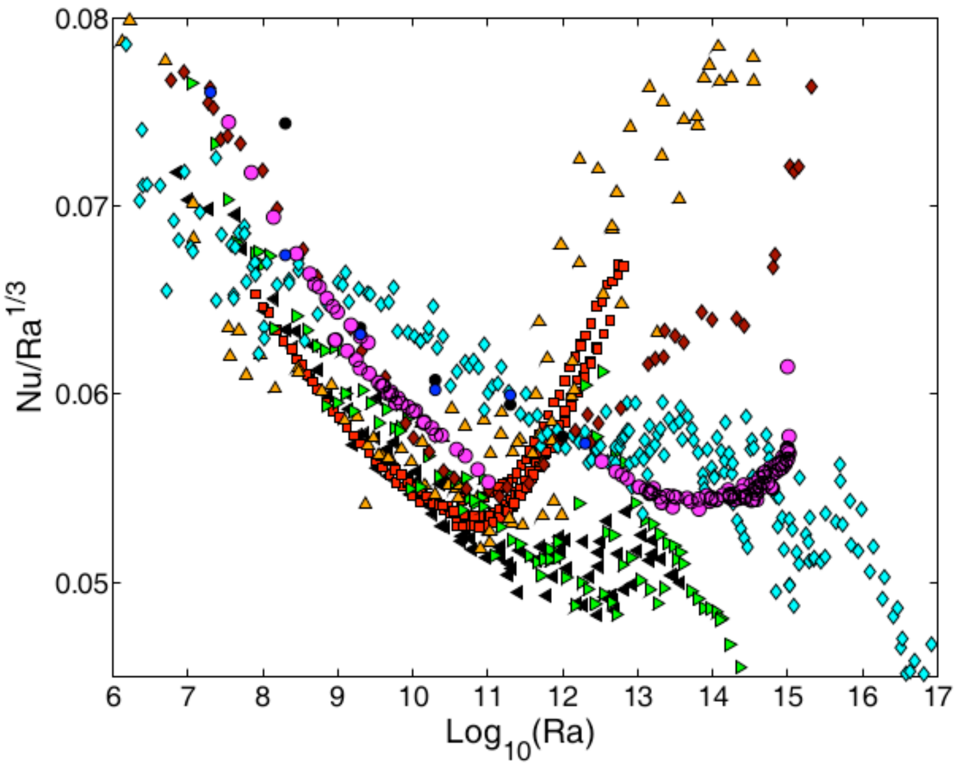
U being a characteristic velocity
(large-scale flow or RMS velocity)

Prediction of Nu and Re

$$Nu \sim Ra^{\beta_{Nu}}$$

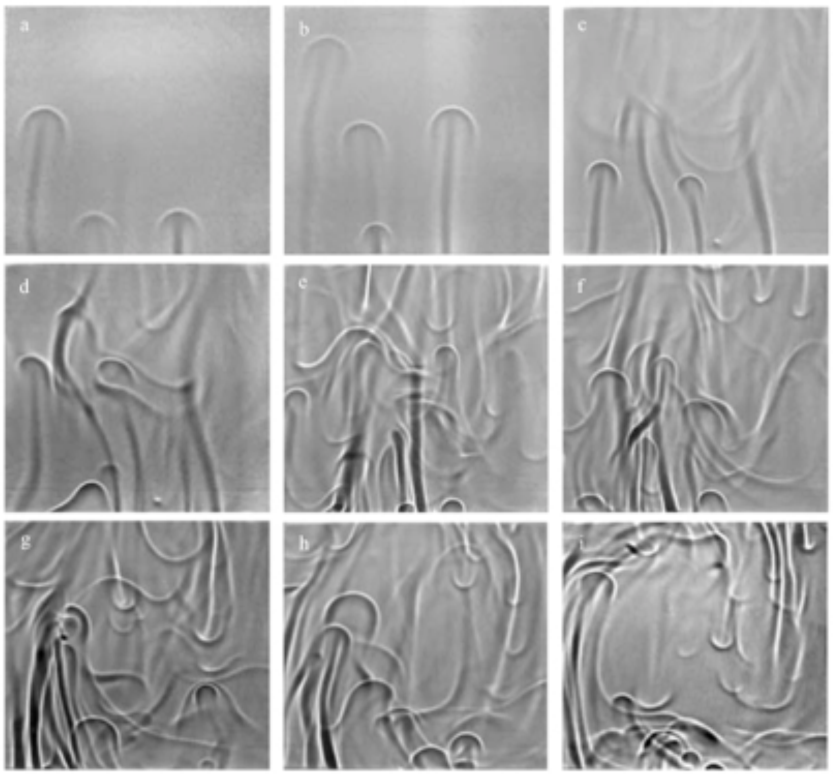
$$Re \sim Ra^{\beta_{Re}}$$

Experiments and simulations



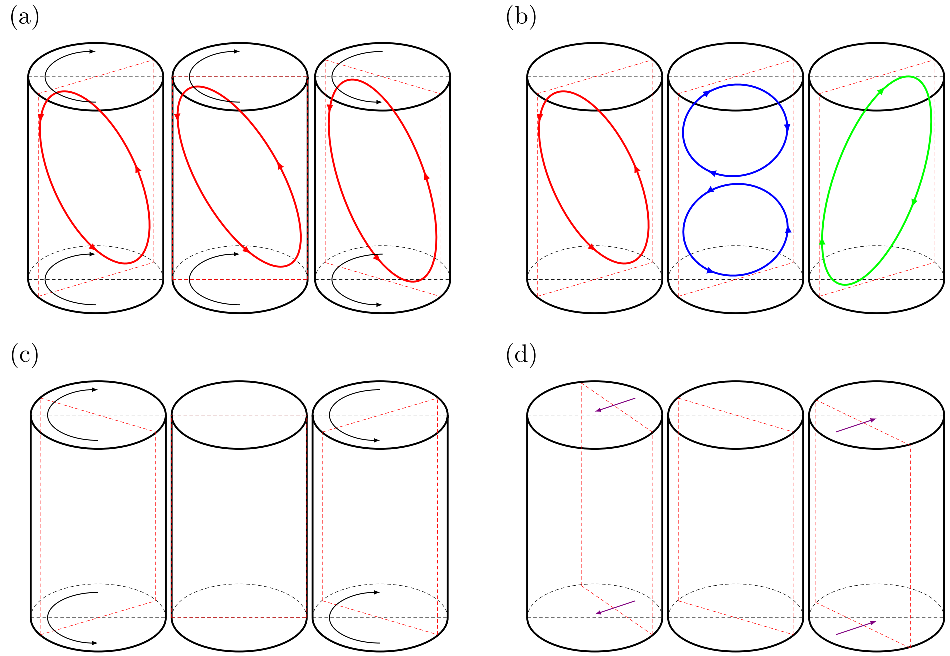
F. Chillà and J.Schumacher, Eur. Phys. J. E (2012)

Self-organization of small scales in a large-scale circulation (LSC)



Shadowgraph displays the time evolution of plumes

H.-D. Xi, S. Lam and K.-Q. Xia, (2004)



(a) Azimuthal meandering Funfschilling and Ahlers, (2004)
(b) Flow Reversal} Xi \& Xia, (2008)]
(c) Torsional Funfschilling et al., (2008)
(d) off-center oscillations or sloshing of the LSC plane (Zhou et al., (2009))

These events are often combined, producing rich and complex dynamics

To fix the plane of the large-scale circulation

Pure 2D geometry: Chandra and Verma (2011,2013)

Boussinesq equations + standard BC

$$\nabla \cdot \vec{u} = 0$$

$$\partial_t \vec{u} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + Pr Ra^{-0.5} \nabla^2 \vec{u} + Pr \theta \vec{e_2}$$

$$\partial_t \theta + \vec{u} \cdot \nabla \theta = Ra^{-0.5} \nabla^2 \theta$$

Temporal discretization scheme

$$\frac{\vec{u}_* - \vec{u}_n}{\Delta_t} + \vec{u}_{n+1/2} \cdot \nabla \vec{u}_{n+1/2} = Pr Ra^{-0.5} \nabla^2 \vec{u}_* + Pr \theta_{n+1/2} \vec{e_2}$$

$$\frac{\theta_{n+1/2} - \theta_{n-1/2}}{\Delta_t} + \vec{u}_n \cdot \nabla \theta_n = Ra^{-0.5} \nabla^2 \theta_{n+1/2}$$

$$\frac{\vec{u}_{n+1} - \vec{u}_*}{\Delta_t} = -\nabla p_{n+1/2}$$

Staggered in time discretization
Implicit viscous/BCG advection
Pressure correction scheme

Spatial discretization scheme

Cartesian grid, multi-grid approach

Linear interpolation and central differentiation scheme

Implementation in Basilisk

Simple implementation by combining existing blocks of code

Examples of the code used found on http://basilisk.fr/sandbox/acastillo

Typical values for 2D DNS

Ra	from 10^6 to $5 \cdot 10^8$
Pr	3.0 and 4.3
$Time$	from 5.000 to 40.000 t.u.
$Gridsize$	from (512^2) to (1024^2)

Typical values for 3D DNS

Ra	from 10^6 to 10^9
Pr	4.38
Γ_z	from 1/8 to 1/64
$Time$	from 500 to 5.000 t.u.
$Gridsize$	from $(512^2 \times 64)$ to $(1024^2 \times 16)$

Validation criteria: Numerical convergence

exact relations between $\overline{Nu} = Ra^{0.5}\langle\overline{v\theta}\rangle - \langle\overline{\partial_y\theta}\rangle$

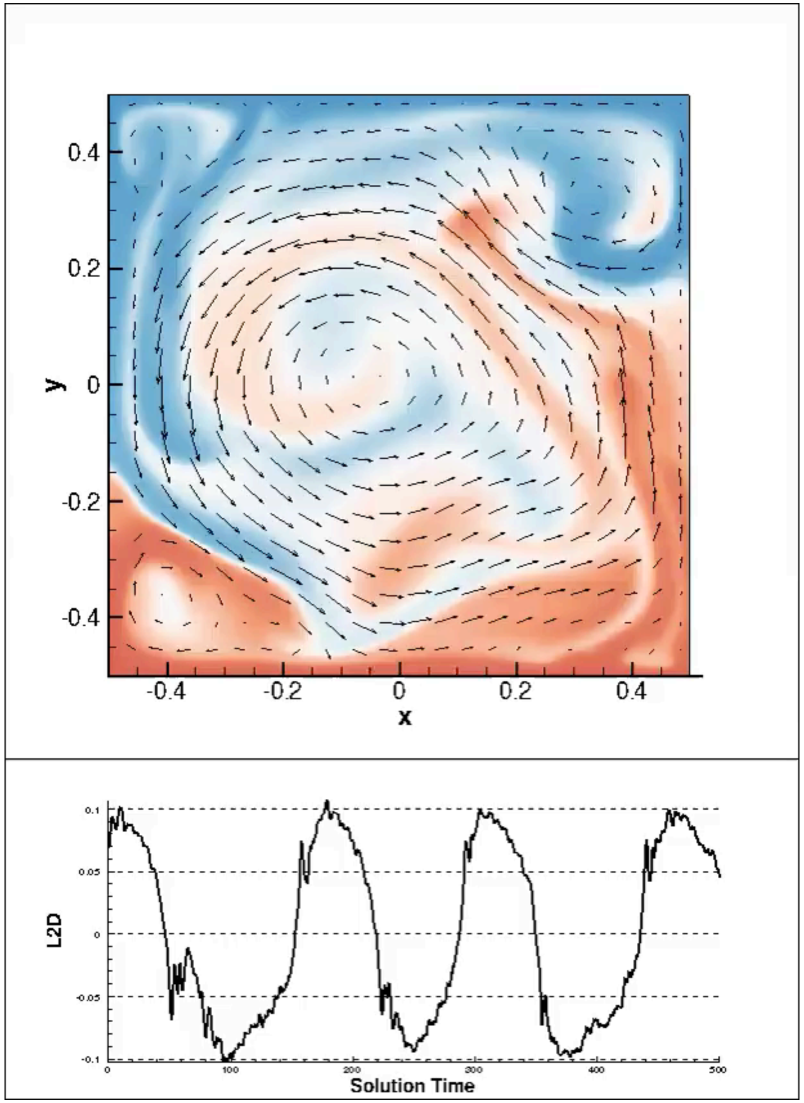
and viscous dissipation rate $\overline{Nu} = \langle\overline{\nabla\vec{u} : \nabla\vec{u}}\rangle + 1$

or thermal dissipation rate $\overline{Nu} = \langle\overline{\nabla\theta \cdot \nabla\theta}\rangle$

B. Shraiman and E. Siggia, (1990)

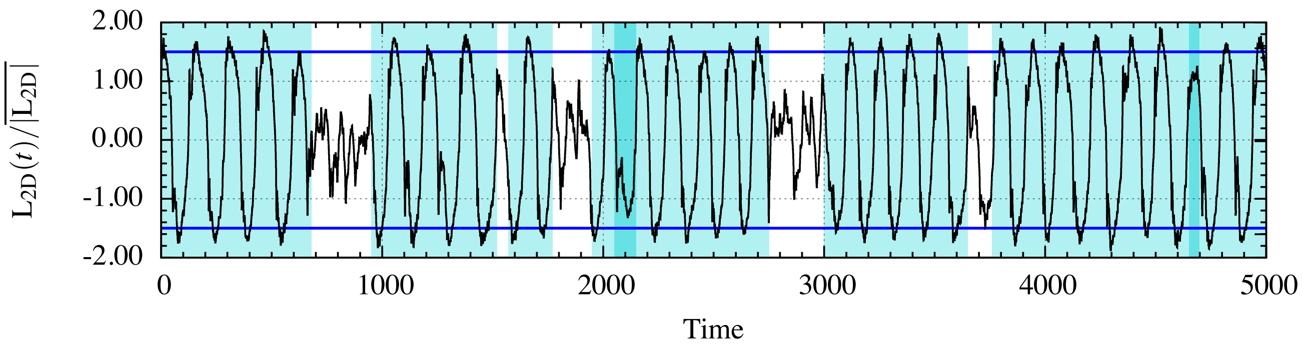
Maximum difference between these quantities is around 1%

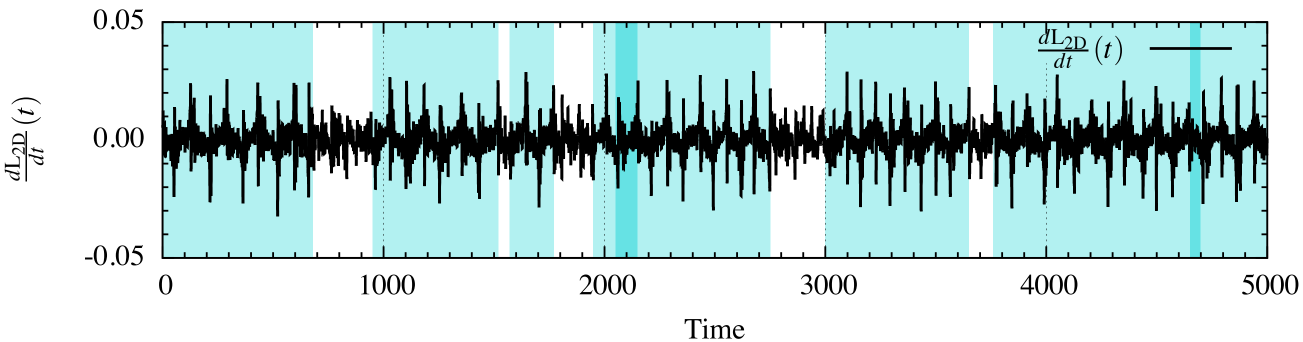
Turbulent convection inside square pure 2D RB cell



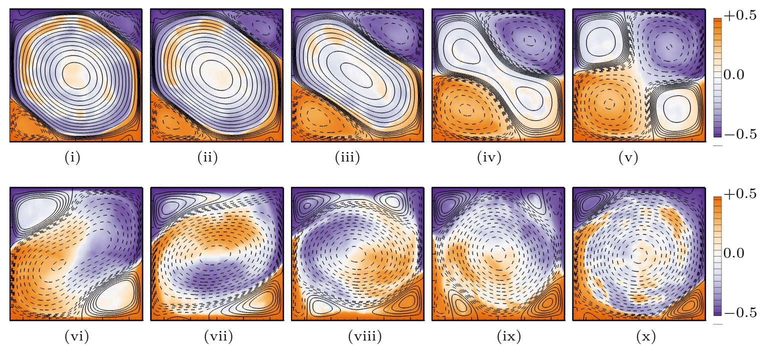
Global angular impulse $$L_{2D} \equiv -\frac{1}{2}\int \vec{x}^2 \omega dV$$

Two regimes

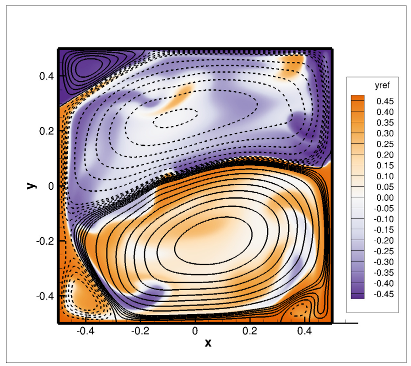




A regime composed of
consecutive reversals CR

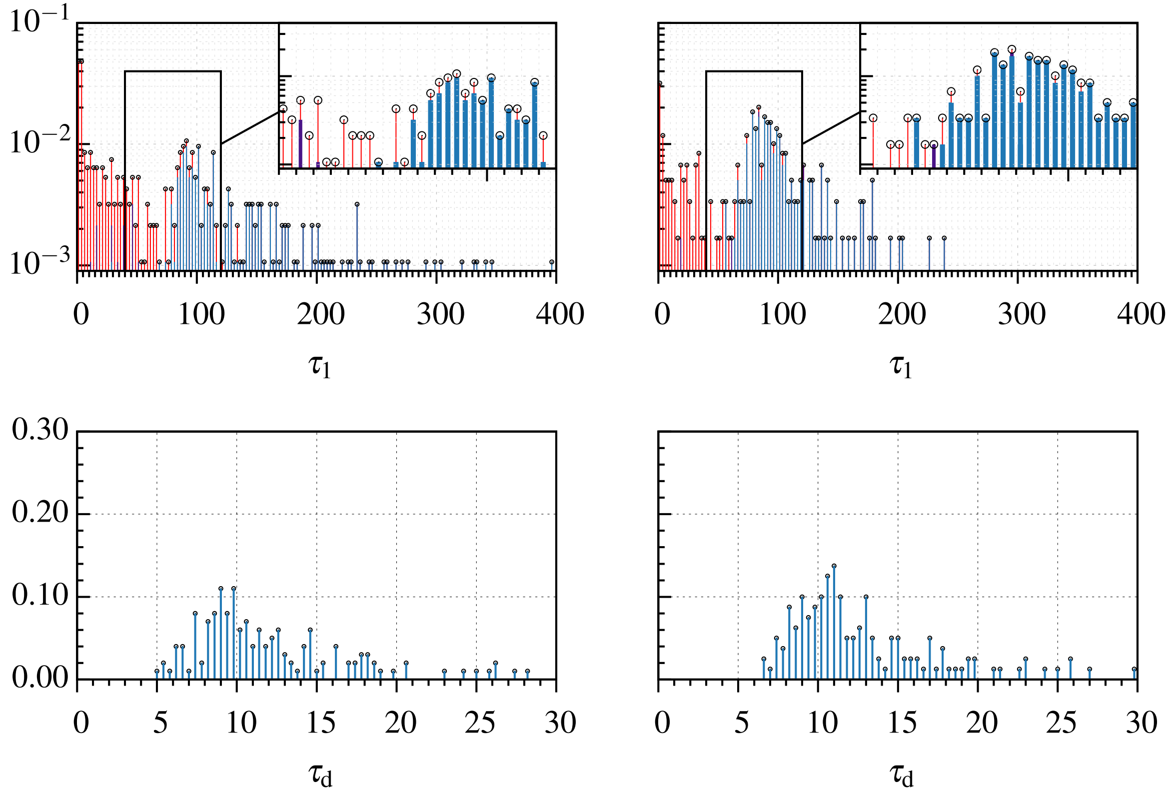


A regime composed of
extended cessations EC



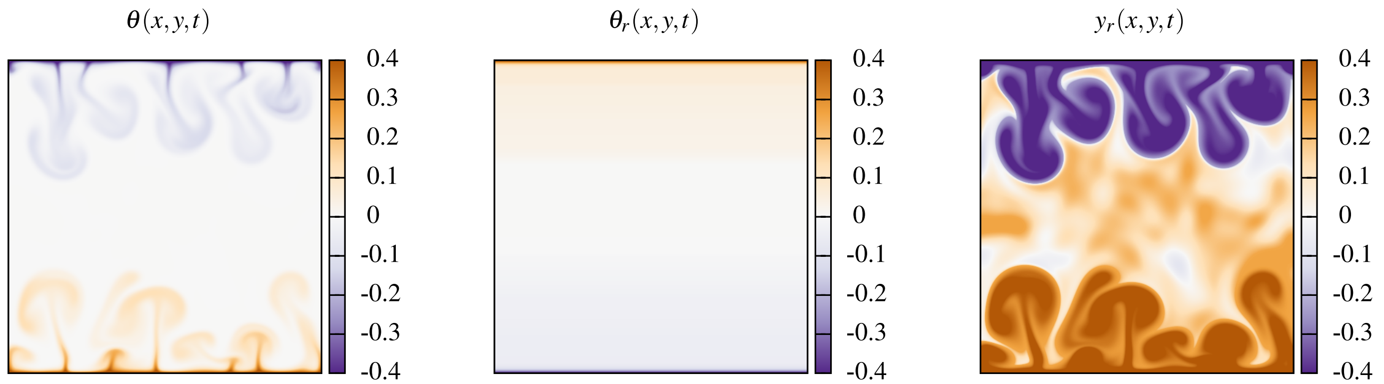
A criteria to separate both regimes

Identify time with $L_{2D} = 0$ and inter-reversal interval τ_1



Podvin and Sergent, (2015)

Global approach:
kinetic and available potential energies



Background or reference state

Imagine we adiabatically rearrange our fluid parcels into a thermally stable configuration

Background state is the density distribution with lowest potential energy, while preserving volume [Winters & Young (1995)]

Available potential energy corresponds to the part of potential energy that can be transformed into motion

$$E_{apot} \equiv -Pr \int (y - y_r(\vec{x},t))\theta(\vec{x},t) d\vec{x}$$

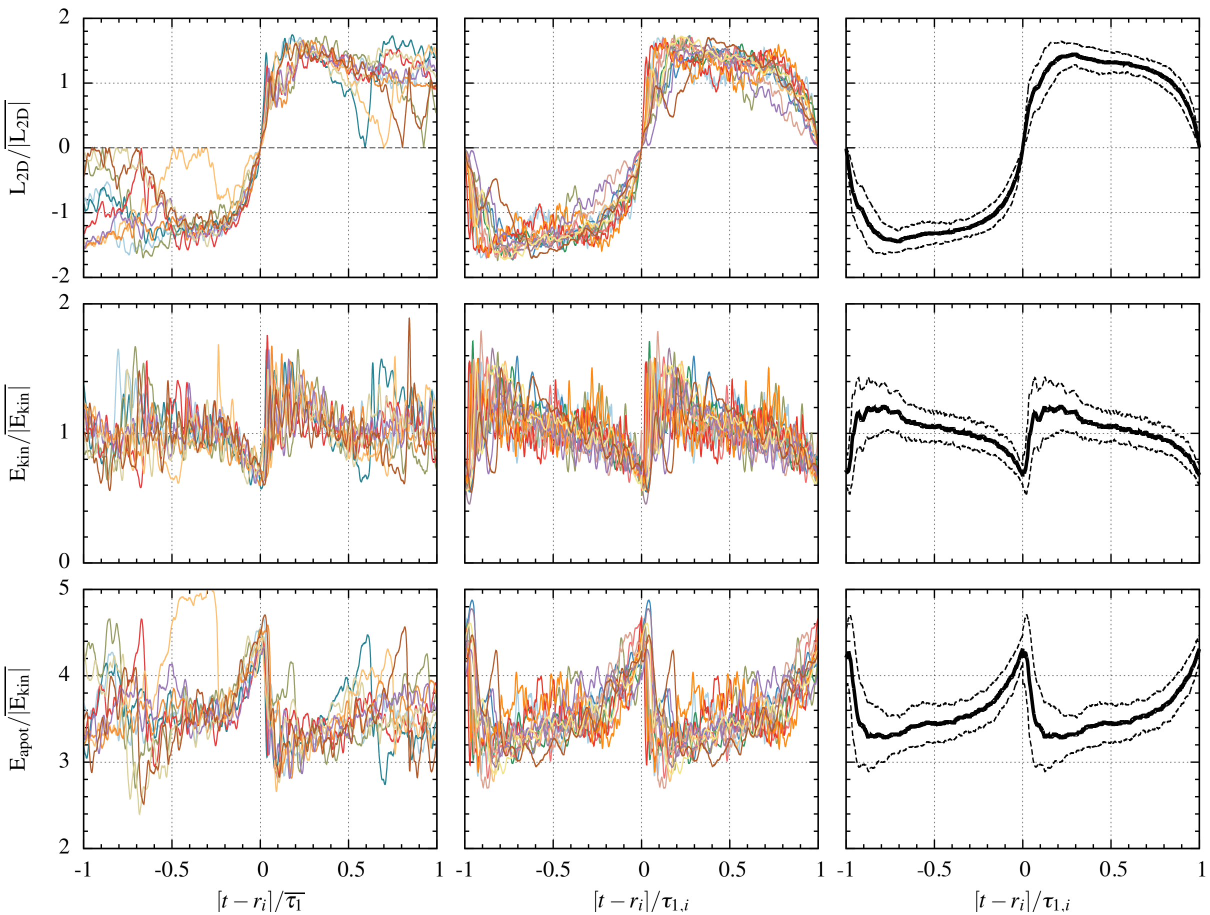
$y_r(\vec{x},t)$ corresponds to the height at the reference state

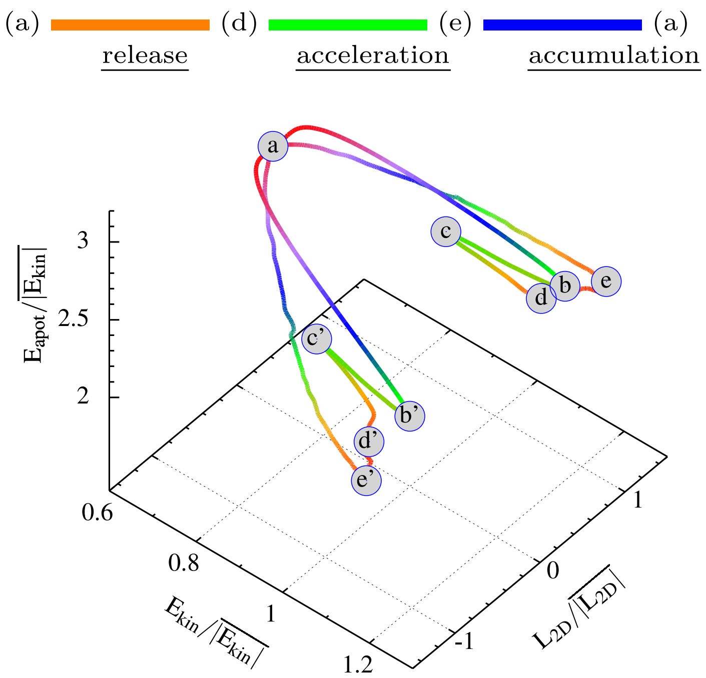
Statistical characterization of the CR regime

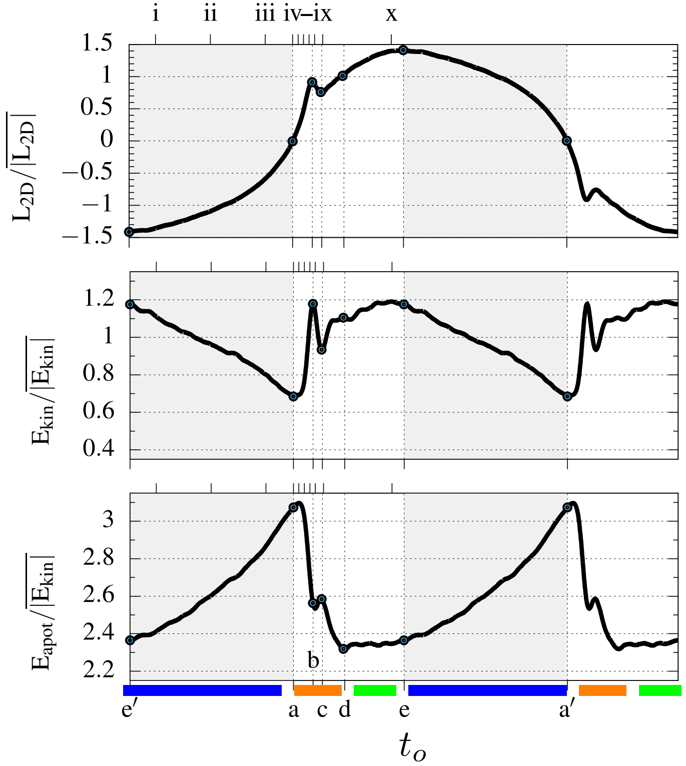
Similar features are observed for different realizations

Reversals are scattered but follow similar trends

Re-scale time based on τ_1 and average over the ensemble of reversals







ACCUMULATION of available potential / braking of central vortex

$$E_{apot}\uparrow \qquad E_{kin}\downarrow \qquad |L_{2D}|\downarrow$$

RELEASE of available potential and breakdown of the central vortex

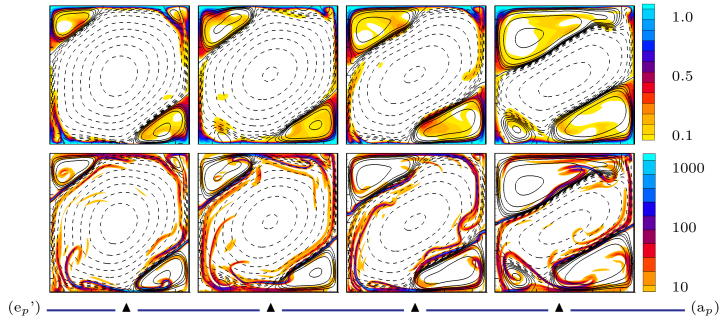
$$E_{apot}\downarrow\downarrow\downarrow \qquad E_{kin}\uparrow\uparrow\uparrow \qquad |L_{2D}|\uparrow$$

ACCELERATION of the central vortex

$$E_{apot}\simeq Const \qquad E_{kin}\uparrow \qquad |L_{2D}|\uparrow$$

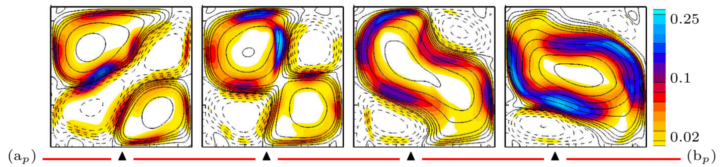
Phases of the generic reversal

ACCUMULATION



Field $-Pry_r\theta$ illustrates contributions to E_{apot}

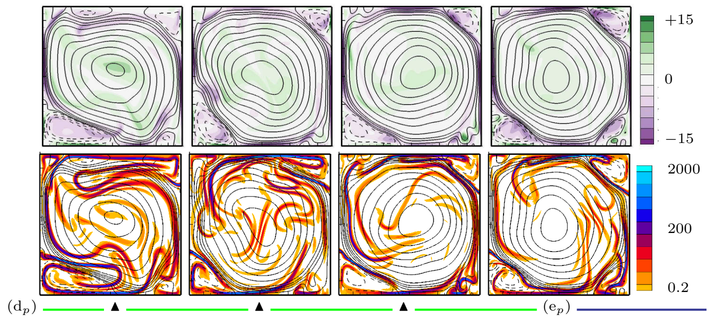
RELEASE



Stored available potential energy is transformed into motion

Field $\vec{u} \cdot \vec{u}$ illustrates contributions to $\mathrm{E_{kin}}$

ACCELERATION



flow organizes, number of plumes decreases

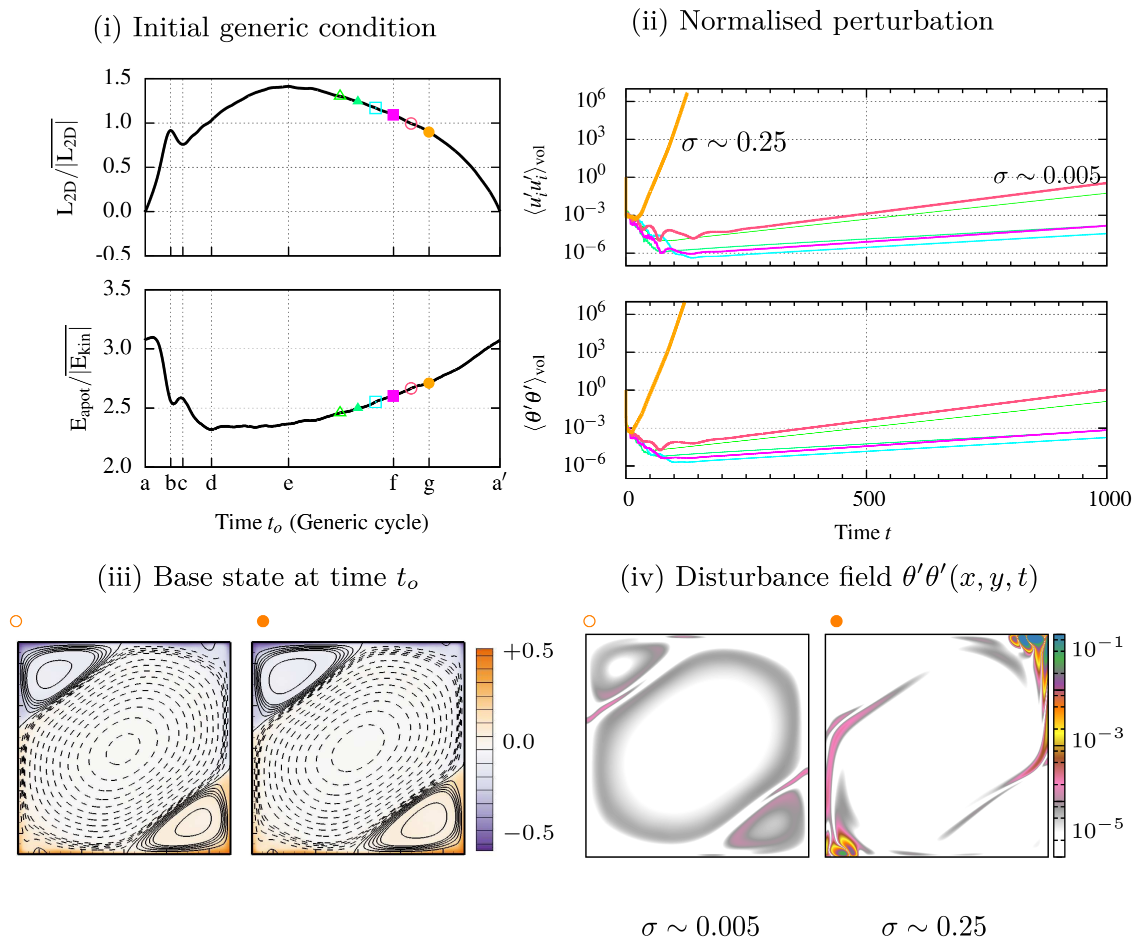
Field $\vec{\nabla} y_r \cdot \vec{\nabla}\theta$ illustrates the contours of thermal structures

Linear stability around the generic fields

Quasi-static approximation

$$\vec{u} = \vec{u}^o(\vec{x}, t_o) + \vec{u}'(\vec{x}, t)$$

$$\theta = \theta^o(\vec{x}, t_o) + \theta'(\vec{x}, t)$$



After reaching a threshold, growth rate σ increases!

2D Large scale dynamics

Two separate regimes : CR and EC regimes

An energetic approach identifies
a generic reversal cycle in 3 phases

Energy exchange in each phase is tied
to evolution of large-scale structures

A threshold state for transition has been
identified

"Reversal cycle in square Rayleigh-Bénard cells in turbulent regime"
Castillo-Castellanos, A.; Sergent, A. and Rossi, M.
Journal of Fluid Mechanics, Volume 808, pp. 614-640   (2016)