Compressible schemes for multiphase flows on Basilisk

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We want to develop a generic solver for the Navier-Stokes/Euler equations for compressible/incompressible fluids

Desired properties of the numerical scheme

- Convergence to the classical incompressible formulation
- Conservative scheme
- Easy to generalize to multiphase flows
- Volume of Fluid method (sharp interface representation)

We focused our attention in *all mach formulations* proposed in previous works

- Yoon & Yabe [Comp. Phys. Comm, 1999]
- Kwatra et al [JCP, 2009]
- Shyue & Xiao [JCP, 2014], Xie et al [JCP, 2016]
- Jemison et al [JCP, 2014]
- others....

Platform used: *Basilisk*

FREE open source: www.basilisk.fr

Easy to take advantage of surface tension methods

Possibility to use different types of grids (adaptive/fixed)

Parallelized (MPI, openmp)

Single phase flow

Compressible flow formulation

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$

Momentum: $$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = -\nabla p$$

State equation:

$$\rho e_i = \frac{p}{\gamma - 1} + \frac{\Pi \gamma}{\gamma - 1}$$

Total Energy: $$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = -\nabla \cdot (\boldsymbol{u} p)$$

Compressible flow formulation

Advection step

Continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \rightarrow \rho^{n+1}$$

State equation:

$$\rho e_i = \frac{p}{\gamma - 1} + \frac{\Pi \gamma}{\gamma - 1}$$

Momentum:

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u}) = 0 \rightarrow (\rho u)^*$$

$$\frac{u^{n+1} - u^*}{\Delta t} = \frac{1}{\rho^{n+1}} \nabla p$$

Total Energy:

$$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = 0 \rightarrow (\rho e_T)^*$$

$$\frac{(\rho e_T)^{n+1} - (\rho e_T)^*}{\Delta t} = -\nabla \cdot (p \boldsymbol{u})$$

Compressible flow formulation

Advection step $\to$ Projection step

State equation:

$$\rho e_i = \frac{p}{\gamma - 1} + \frac{\Pi\gamma}{\gamma - 1}$$

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \to \rho^{n+1}$$

Momentum: $$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = 0 \to (\rho u)^*$$

$$\nabla \cdot \left(\frac{u^{n+1} - u^*}{\Delta t} = \frac{1}{\rho^{n+1}} \nabla p \right)$$

Total Energy: $$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = 0 \to (\rho e_T)^*$$

$$\frac{(\rho e_T)^{n+1} - (\rho e_T)^*}{\Delta t} = -\nabla \cdot (p \boldsymbol{u})$$

Equation for $\nabla \cdot u$: $$\frac{1}{\rho c^2} \frac{Dp}{Dt} = -\nabla \cdot \boldsymbol{u}$$ (internal energy)

Compressible flow formulation

Advection step $\rightarrow$ Projection step

State equation:

$$\rho e_i = \frac{p}{\gamma - 1} + \frac{\Pi\gamma}{\gamma - 1}$$

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \rightarrow \rho^{n+1}$$

Momentum: $$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u}) = 0 \rightarrow (\rho u)^*$$

$$\nabla \cdot \left(\frac{u^{n+1} - u^*}{\Delta t} = \frac{1}{\rho^{n+1}} \nabla p \right)$$

Total Energy: $$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = 0 \rightarrow (\rho e_T)^*$$

$$\frac{(\rho e_T)^{n+1} - (\rho e_T)^*}{\Delta t} = -\nabla \cdot (p \boldsymbol{u})$$

Equation for $\nabla \cdot u$: $$\frac{Dp}{Dt} = 0 \rightarrow p^{adv}$$

$$\frac{1}{\rho c^2} \frac{p^{n+1} - p^{adv}}{\Delta t} = -\nabla \cdot \boldsymbol{u}^{n+1}$$

Compressible flow formulation

Advection step $\rightarrow$ Projection step

Continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \rightarrow \rho^{n+1}$$

State equation:

$$\rho e_i = \frac{p}{\gamma - 1} + \frac{\Pi \gamma}{\gamma - 1}$$

Momentum:

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \boldsymbol{u}) = 0 \rightarrow (\rho u)^*$$

$$\frac{p^{n+1}}{\rho c^2 \Delta t} - \nabla \cdot \left(\frac{\Delta t}{\rho^{n+1}} \nabla p^{n+1} \right) = \frac{p^{adv}}{\rho c^2 \Delta t} - \nabla \cdot u^*$$

Total Energy:

$$\frac{\partial \rho e + 1/2 \rho \boldsymbol{u}^2}{\partial t} + \nabla \cdot (\rho e \boldsymbol{u} + 1/2 \rho \boldsymbol{u}^2) = 0 \rightarrow (\rho e_T)^*$$

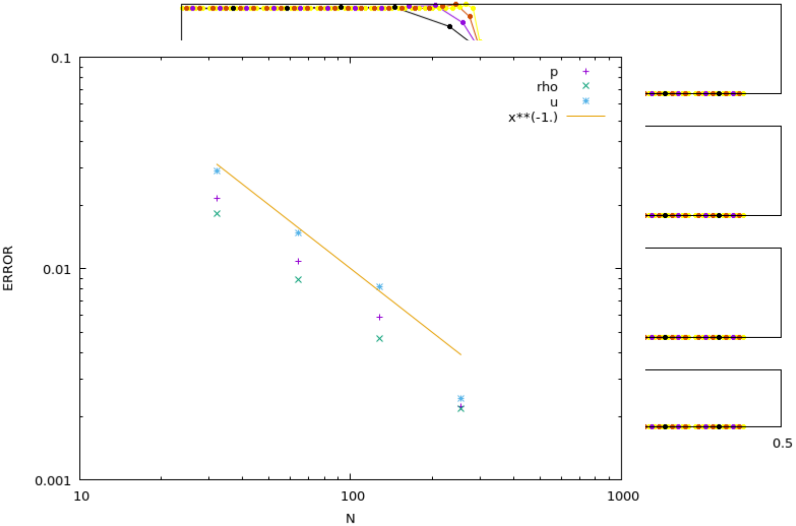
$$\frac{(\rho e_T)^{n+1} - (\rho e_T)^*}{\Delta t} = -\nabla \cdot (p \boldsymbol{u})$$

Equation for $\nabla \cdot u$:

$$\frac{Dp}{Dt} = 0$$

Tests for single phase flow

1D Shock wave propagation

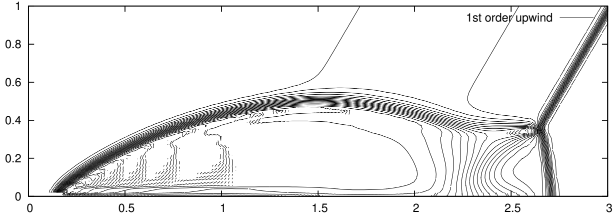


Influence of resolution

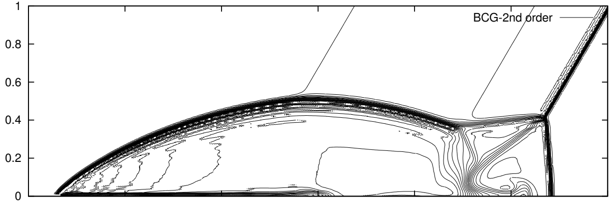
Shock spreads over 3 cells

Oblique shock wave

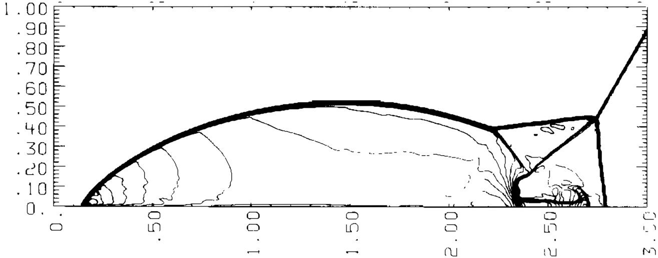
$$\Delta x = 4.88 \cdot 10^{-3}$$



Current: 1st order upwind

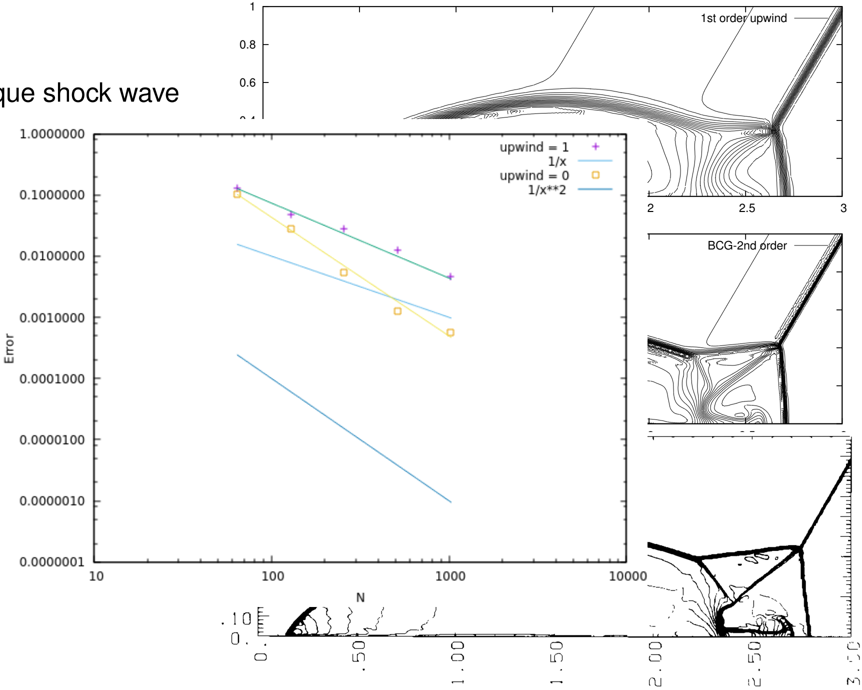


Current: 2nd order BCG

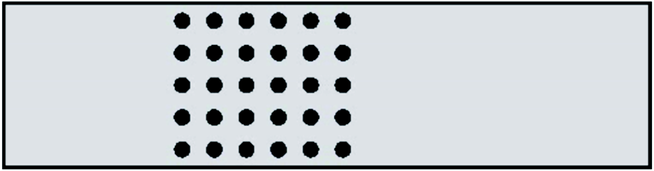


Review Woodward & Colella [JCP, 1984]

Oblique shock wave

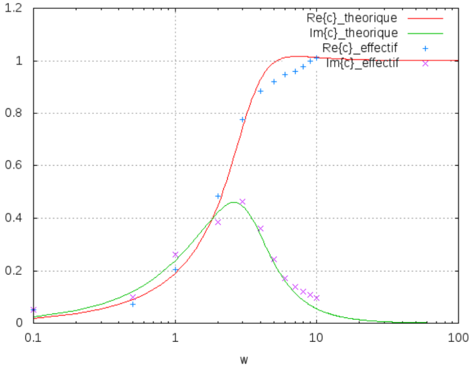


Linear propagation across a forest of cylinders



Foldy

$$\left(\frac{k_e}{k_0}\right)^2 = 1 + 4\imath \frac{N_0}{k_0^2} \sum_{n=0}^{\infty} A_n(k_0 R_0)$$



Multiphase flows

Numerical method: 1) Advection step

Continuity: $$\frac{\rho^{n+1} - \rho^n}{\Delta t} + \nabla \cdot (\rho u) = 0$$

Momentum: $$\frac{(\rho \boldsymbol{u})^* - \rho \boldsymbol{u}}{\Delta t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = 0$$

Total Energy: $$\frac{(\rho e_T)^* - (\rho e_T)^n}{\Delta t} + \nabla \cdot (\rho e_T \boldsymbol{u}) = 0$$

P advection: $$\frac{Dp}{Dt} = 0 \rightarrow EOS \rightarrow p^{adv}$$

Color function advection: $$\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla c = 0$$

Weymouth & Yue method [JCP, 2010]

Numerical method: 1) Advection step

Continuity: $$\frac{\rho^{n+1}-\rho^{n}}{\Delta t}+\nabla\cdot(\rho u)=0$$

Momentum: $$\frac{(\rho \boldsymbol{u})^{*}-\rho \boldsymbol{u}}{\Delta t}+\nabla\cdot(\rho \boldsymbol{u}\boldsymbol{u})=0$$

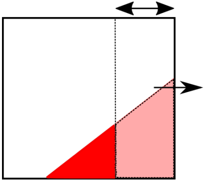
Total Energy: $$\frac{(\rho e_T)^{*}-(\rho e_T)^{n}}{\Delta t}+\nabla\cdot(\rho e_T \boldsymbol{u})=0$$

P advection: $$\frac{Dp}{Dt}=0\rightarrow EOS\rightarrow p^{adv}$$

Color function advection: $$\frac{\partial c}{\partial t}+\nabla\cdot(\boldsymbol{u}c)=c\nabla\cdot u$$

Weymouth & Yue method [JCP, 2010]

Numerical method: 1) Advection step



Continuity: $$\frac{\rho^{n+1}-\rho^n}{\Delta t}+\nabla\cdot(\rho u)=0$$

Momentum: $$\frac{(\rho \boldsymbol{u})^*-\rho \boldsymbol{u}}{\Delta t}+\nabla\cdot(\rho \boldsymbol{u}\boldsymbol{u})=0$$

Total Energy: $$\frac{(\rho e_T)^*-(\rho e_T)^n}{\Delta t}+\nabla\cdot(\rho e_T \boldsymbol{u})=0$$

P advection: $$\frac{Dp}{Dt}=0\rightarrow EOS\rightarrow p^{adv}$$

Color function advection: $$\frac{\partial c}{\partial t}+\nabla\cdot(\boldsymbol{u}c)=c\nabla\cdot u$$

VOF-like advection of conserved quantities:

- we know the geometrical fluxes of c: $F_{i+1/2}(c)=u_{i+1/2}c_{adv}$
- The flux of conservative quantities is based on these fluxes

$$F_{i+1/2}(c\rho_1)=\rho_{1,adv}u_{i+1/2}c_{adv}$$

Projection step

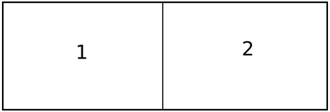
Continuity: ρ^{n+1}

Momentum: $$\frac{p^{n+1}}{\rho c^2 \Delta t} - \nabla \cdot \left(\frac{\Delta t}{\rho^{n+1}} \nabla p^{n+1} \right) = \frac{p^{adv}}{\rho c^2 \Delta t} - \nabla \cdot u^*$$

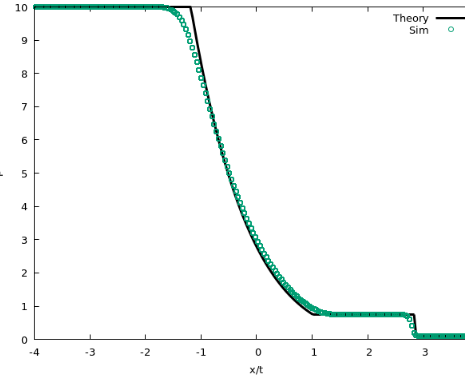
Total Energy: $$\frac{(\rho e_T)^{n+1} - (\rho e_T)^*}{\Delta t} = -\nabla \cdot (p\boldsymbol{u})$$

Test case: Sod problem

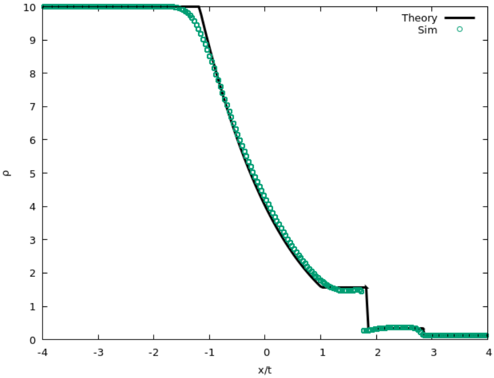
Test: Sod's problem (two different gases)



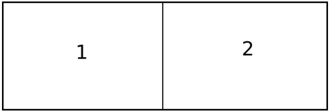
Pressure



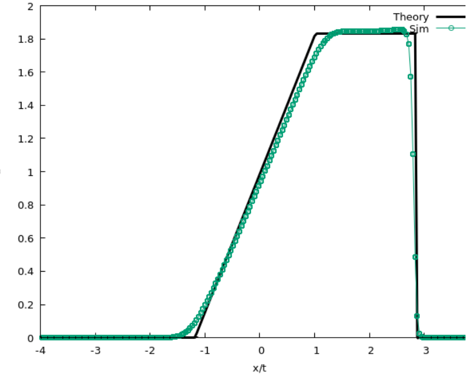
Density



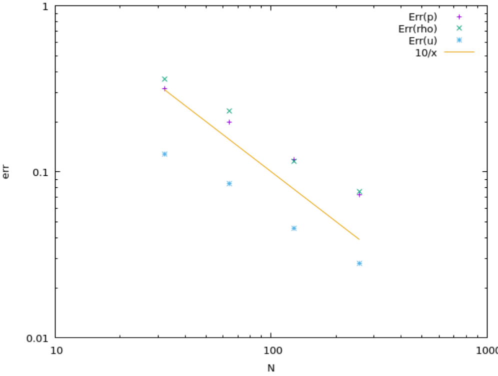
Test: Sod’s problem (two different gases)



Velocity



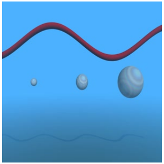
Convergence

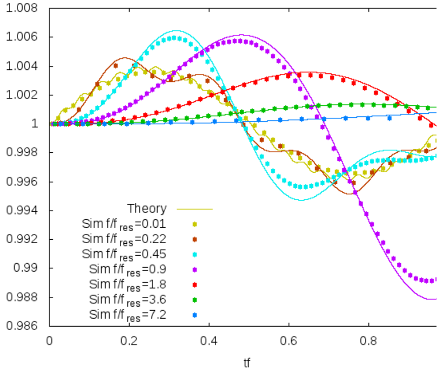


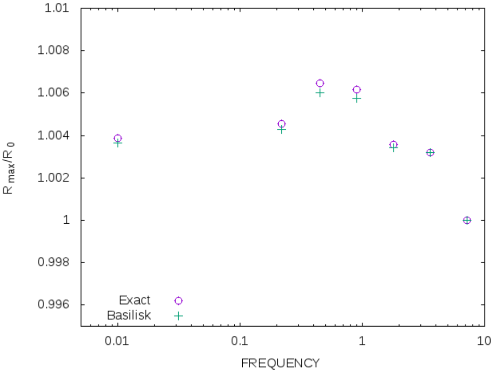
Preliminary results on *complex* problems:

Single bubble problems

Linear oscillation of an spherical bubble



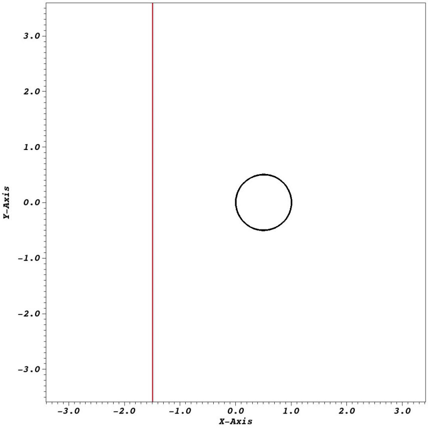




Example: 2D “air Bubble” collapse by a shock wave in water

$\rho_{g0}/\rho_l = 10^{-3}$

$p_{shock}/p_{g0} = 10^2$



Example: 2D “air Bubble” collapse by a shock wave in water

$\rho_{g0}/\rho_l = 10^{-3}$

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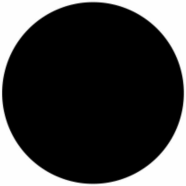
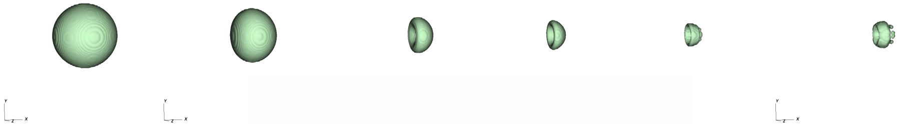
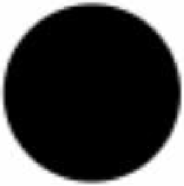


Example: 3D "Bubble" collapse by a shock wave

$\rho_{g0}/\rho_l = 10^{-2}\ p_{shock}/p_{g0} = 10^2$

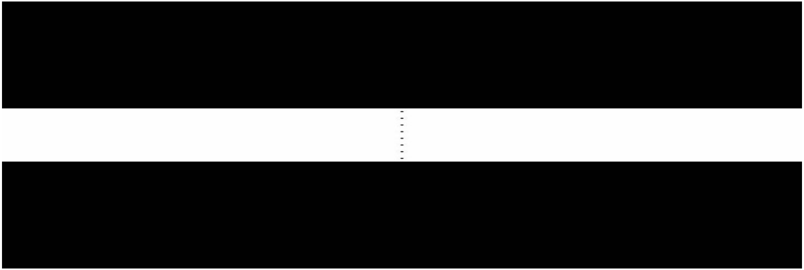




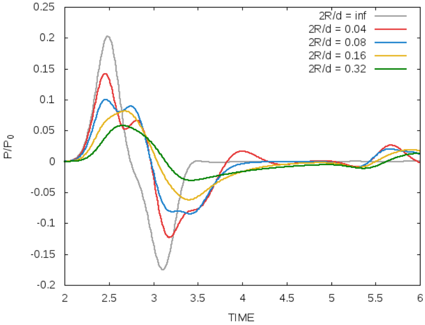
Preliminary results on *complex* problems:

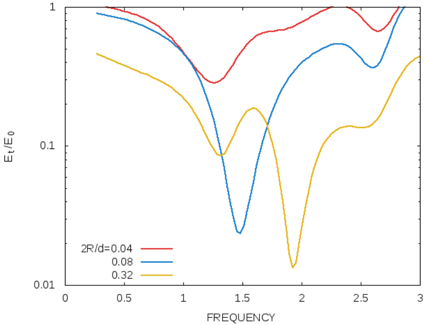
Bubble screens: bubble-bubble interactions

Linear transmission of bubble screens $\lambda/R_0 = 25$

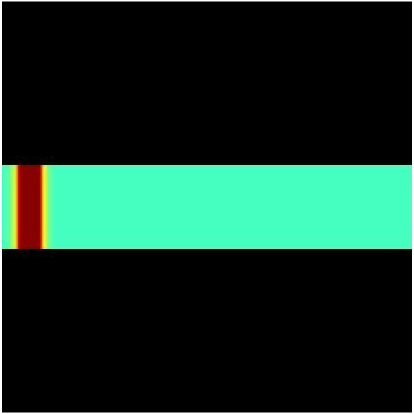


Linear transmission of bubble screens $\lambda/R_0 = 25$

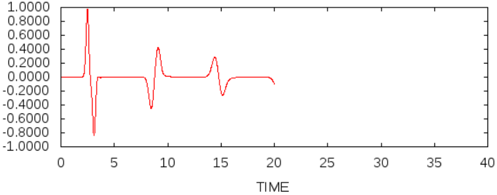




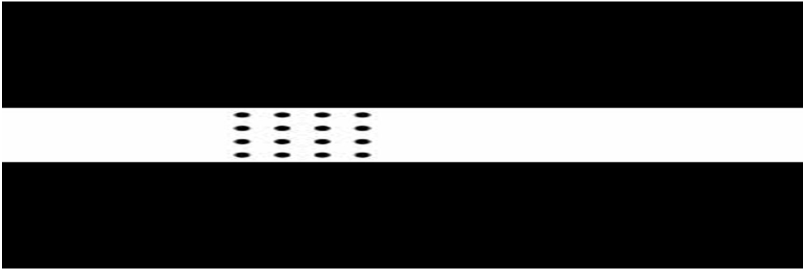
Non-Linear transmission of bubble screens $\lambda/R_0 = 5 \; c_{eff} = f(p)$



Reference

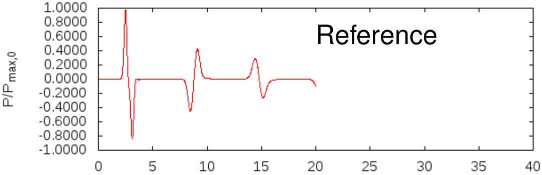


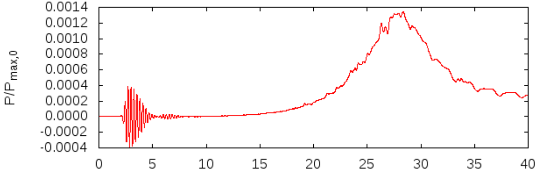
Non-Linear transmission of bubble screens $\lambda/R_0 = 5 \; c_{eff} = f(p)$



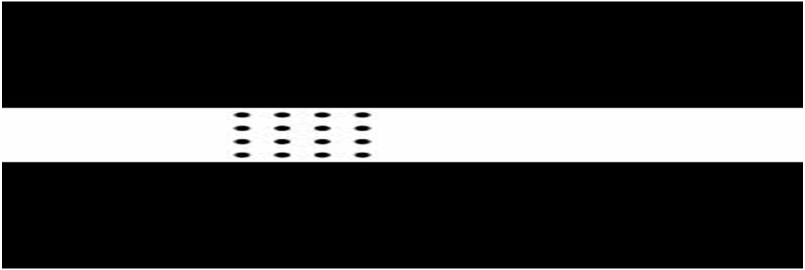
$\Delta p/p_0 = 30$

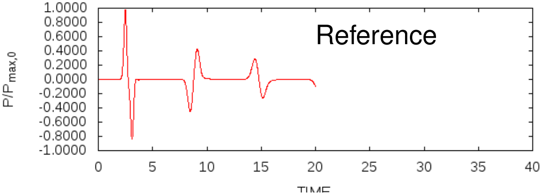
$c_{eff}/c_0 \approx 1/26$





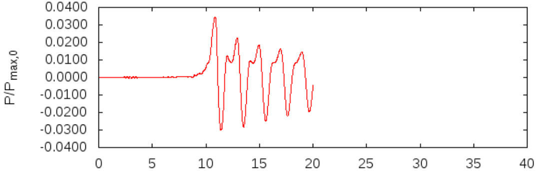
Non-Linear transmission of bubble screens $\lambda/R_0 = 5\ c_{eff} = f(p)$





$\Delta p/p_0 = 60$

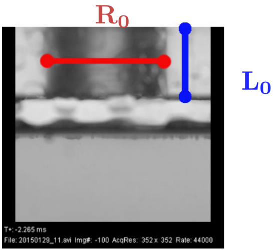
$c_{eff}/c_0 \approx 1/9$

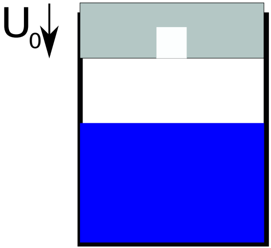


Preliminary results on *complex* problems:

Influence of gas compressibility in impact processes

Influence of gas compressibility on impacts





Forcing pressure: $P \approx \rho_l c_l U_{\text{impact}}$

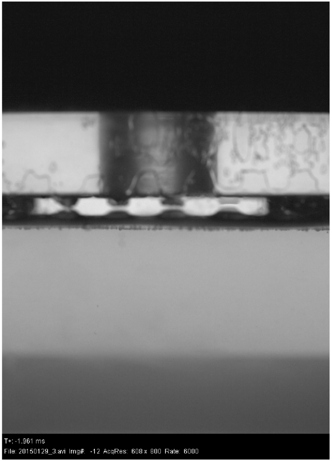
-$U_{\text{impact}} = f(H_0)$

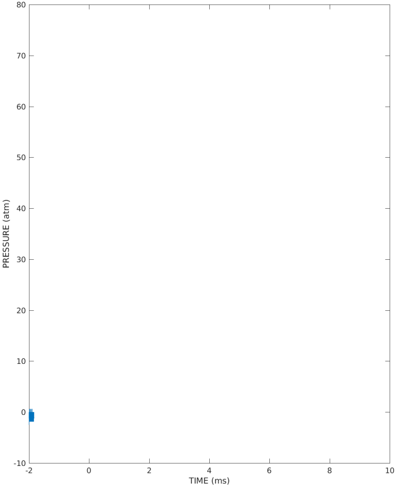
-Control cavity size

-Control gas/vapor ratio with p_0

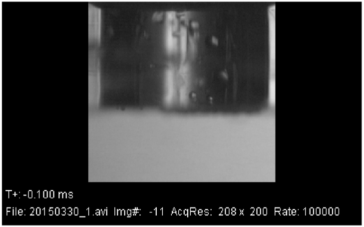
-Control collapse intensity with H_0

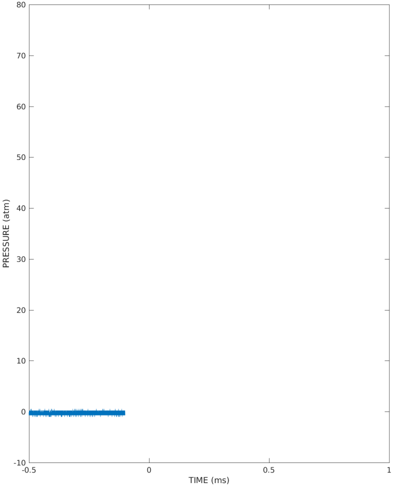
-We can measure P inside the cavity





Framerate: 6000

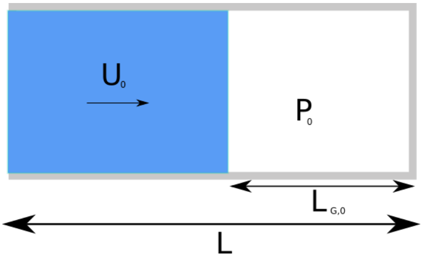




Framerate: 100000

Simplified problem

$U_c = U_0,\ \rho_c = \rho_L\ L_c = L_{g,0}$

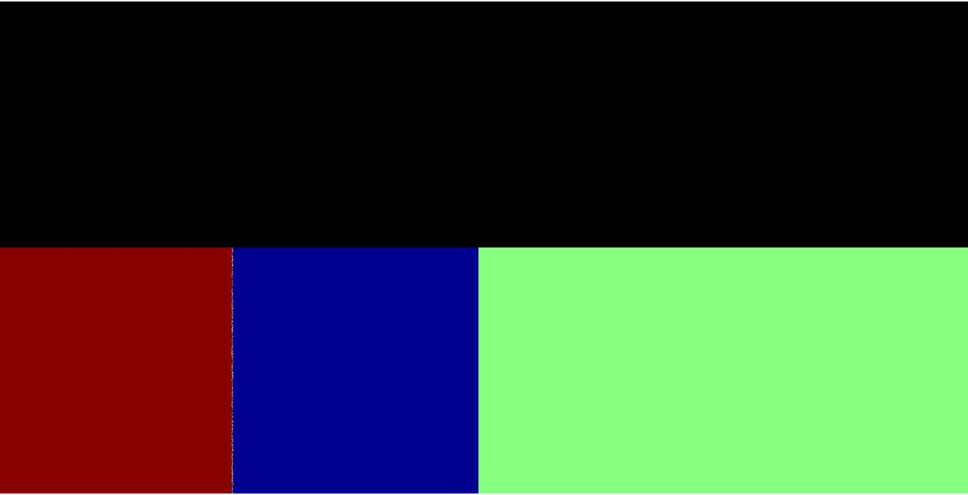


$$\rho_l \frac{\partial u_l}{\partial t} = -\frac{\partial p}{\partial x} \qquad u(0) = 1;\ x_I(0) = x_0$$

Defining $\chi = \dfrac{L - x_I}{L - x_{I,0}}$

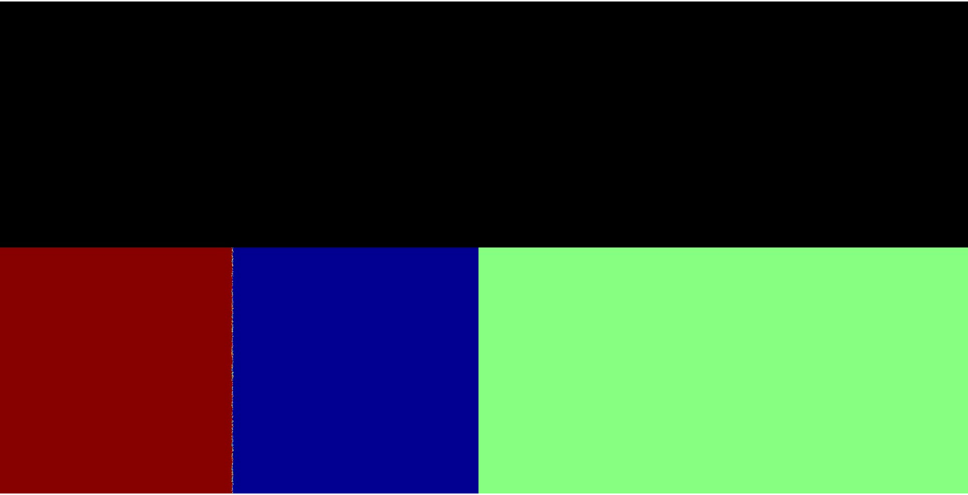
$$(\chi - \mathrm{LR})\, \mathrm{I}\chi_{\mathrm{tt}} = 1 - \frac{1}{\chi^\gamma} \qquad \chi(0) = 1;\ \chi_t(0) = -1$$

Solution depends on: I$= \dfrac{\rho U_0^2}{p_0}$, γ, LR $= L/L_{g,0}$



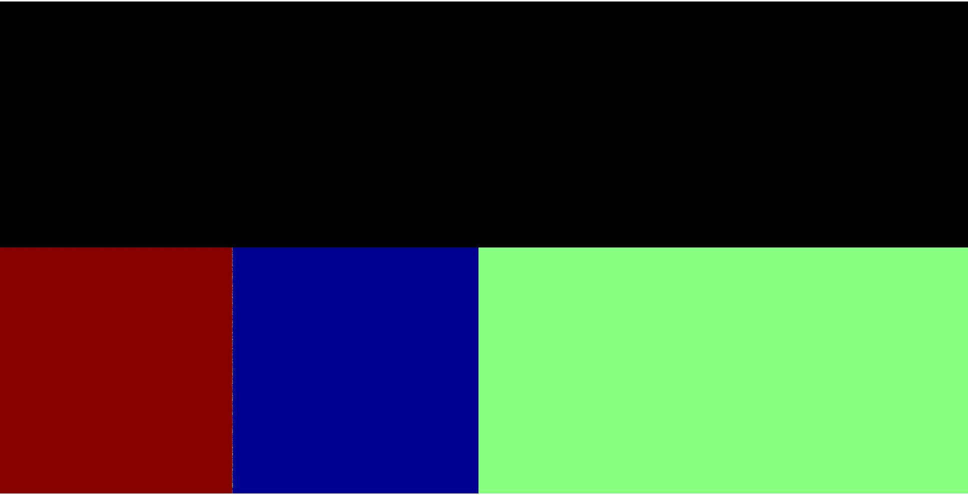
I=1 γ=1.4 LR = 2

$\mathrm{Re_L}$=1000 $$\lambda = 4\pi \left(\frac{4\mu^2}{(\rho_b^2 - \rho_w^2) g} \right)^{1/3} \approx 0.2$$



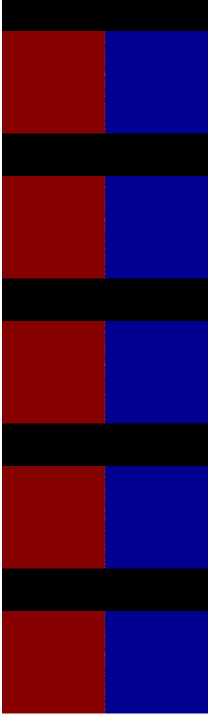
l=4 γ=1.4 LR = 2

$\mathrm{Re_L}$=2000 $$\lambda = 4\pi \left(\frac{4\mu^2}{(\rho_b^2 - \rho_w^2) g} \right)^{1/3} \approx 0.12$$



I=64 γ=1.4 LR = 2

$\mathrm{Re_L}$=8000 $$\lambda = 4\pi \left(\frac{4\mu^2}{(\rho_b^2 - \rho_w^2)g} \right)^{1/3} \approx 0.05$$



l=4

l=8

l=16

l=32

l=64

Conclusions:

-An implicit (all mach) formulation is implemented and tested in Basilisk

-A VOF approach is adopted for sharp interface representation

-Mutiphase component problems can be solved taking care of:

-EOS for mixture assuming uniform pressure assumption within the cell

-Defining fluxes consistent with the advection of the color function

-The schemes are applied to some real problems