A decade of research with Gerris

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Basilisk/Gerris Users' Meeting, Princeton 2017

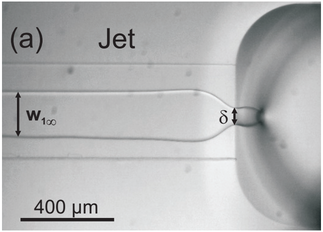
A decade of research with Gerris

Most recent ones:

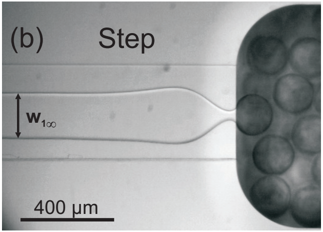
- **Liquid-solid interactions at nanoscale:** Kondic, Mahady (NJIT), Rack (UT, ORNL)
- **Variable surface tension:** Kondic, Seric (NJIT) (her talk is this afternoon)
- **Mixing of alcohol with water:** Seric (NJIT), Kim (KAIST), Stone (Princeton)
- **Ferrofluids:** Qiu, Feng (UBC)
- **Microfluidics:** Leshansky (Technion), Tabeling (ESPCI), Seemann (Saarbrucken)
- **Viscoelasticity:** Renardy (Virginia Tech)
- **Contact lines:** Zaleski, Popinet (UPMC)
  APS/DFD Focus Session: Modeling, Computations and Applications of Wetting/Dewetting
- **Breakup of liquid filament:** Nakrani, Dziedzic (NJIT)
- **Acknowledgement:** NSF Grants DMS-1320037 and CBET-1604351

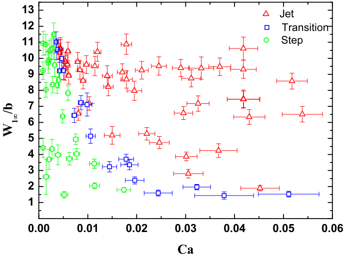
Capillary self-focusing: Generation of monodisperse emulsions

Ca$= \frac{\mu_1 U_{1\infty}}{\gamma} \approx 0.03$, $w_{1\infty}/b \approx 5$



Ca$= \frac{\mu_1 U_{1\infty}}{\gamma} \approx 0.01$, $w_{1\infty}/b \approx 5$





- Can we observe the capillary focusing?
- Can we predict the transition between two different emulsification mechanisms?

Volume-of-fluid (VoF) based Hele-Shaw solver

$$\frac{12\mu}{b^2}\mathbf{V} = -\nabla p^* + \gamma\kappa(x,y,t)\delta\hat{n},$$

$$p^* = p(x,y) + \frac{2\gamma}{b} f$$

Advection of the VoF function

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mathbf{V} f) = 0$$

At inlet:

$$\frac{\partial p_i^*}{\partial x} = -\frac{12\mu_i Q_i}{b^3 w_{i\infty}}, \quad i = 1, 2,$$

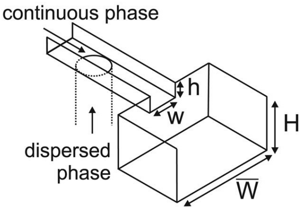
At outlet:

$$p^* = p_0 + A\frac{2\gamma}{b} f$$

[Hein et al., Microfluid. Nanofluid. 2015]

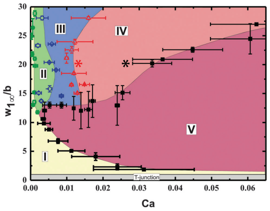
$$\nabla \cdot \left(\frac{b^2}{12\mu}\nabla p^*(x,y)\right) = \nabla \cdot \left(\frac{b^2}{12\mu}\mathbf{F}_{ST}\right)$$

$$\Delta t \ll \frac{24\mu_i}{\gamma b^2}(\Delta x)^3$$

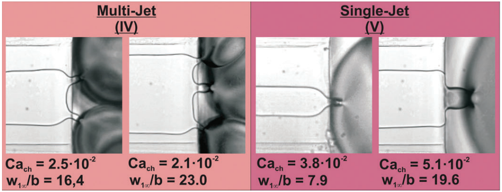


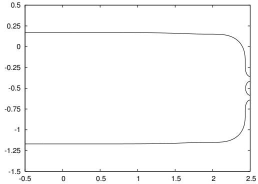
$w = 398\,\mu$m, $h = b = 31\,\mu$m,
$W = H = 1$ mm
$\mu_d = \mu_c = 2.1$ mPa.s, $\gamma = 0.035$ N/m
Ca$= U_d\mu_d/\gamma = 0.005$-$0.05$

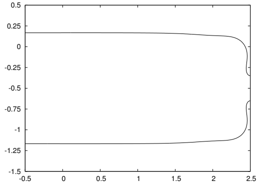
Generation of monodisperse emulsions



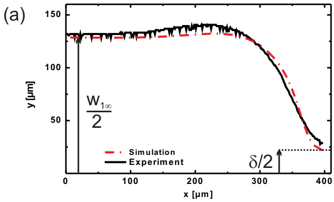
Step (Ca= 0.016)

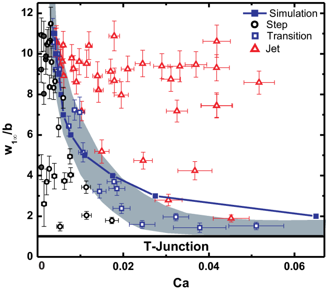






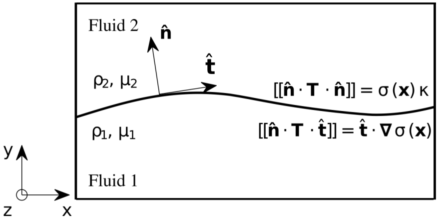
Ca$\approx$ 0.02, $w_{1\infty}/b \approx 8$, A=1





Variable surface tension flows: Marangoni flows

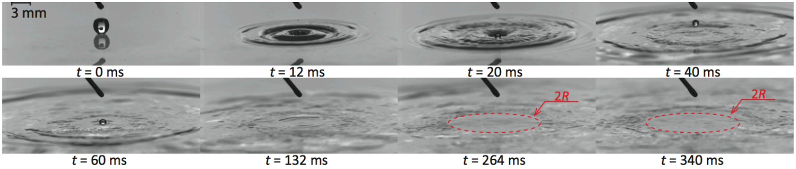
$$\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot (2\mu D) + \sigma\kappa\delta_s \hat{\mathbf{n}} + \nabla_s \sigma \delta_s$$
$$\nabla \cdot \mathbf{u} = 0$$

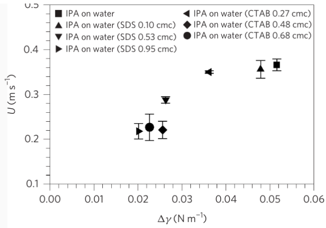


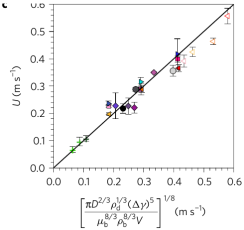
Seric et al. *J. Comput. Phys.*, 2018

Spreading of alcohol on water-air surface

H. Kim et. al., Nature Physics (2017)

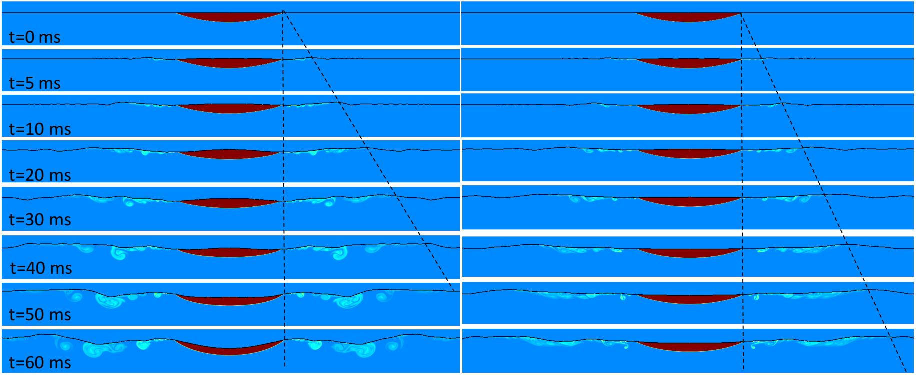






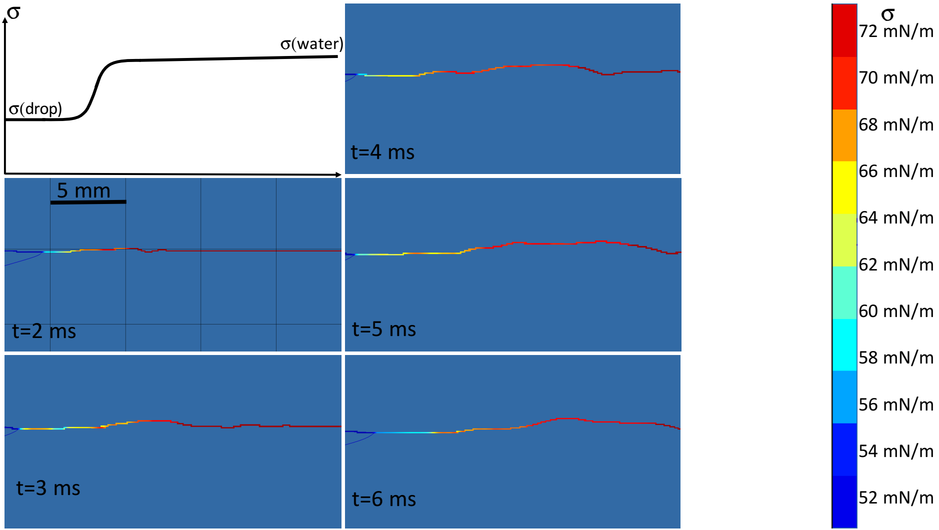
Marangoni-driven flow: liquid lens of diameter 30mm placed on water surface

$\Delta\sigma = 20$ mN/m $\qquad\qquad$ $\Delta\sigma = 10$ mN/m



Afkhami et. al., in preparation (2018)

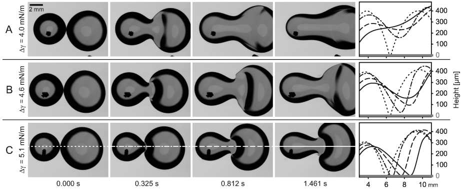
Dynamic surface tension: time evolution of surface tension, $\Delta\sigma = 20$ mN/m

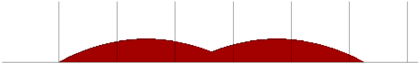


(Non-)Coalescence of sessile drops

S. Karpitschka and H. Riegler, Langmuir, **26**, 2010.

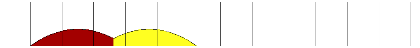
Marangoni number $M = 3\Delta\sigma/(2\bar{\sigma}\theta^2)$; $M_t \approx 2 \pm 0.225$





M = 0

M≈1.2



M≈1.8

Ferrofluidics flows

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + (\nabla \cdot \mathbf{u}\mathbf{u})) = \nabla \cdot \tilde{\tau} + \sigma \kappa \delta_s \mathbf{n} + \rho \mathbf{g}$$

$$\tilde{\tau} = p\mathbf{I} + \mu \dot{\gamma} + \tau_m$$

$$\dot{\gamma} = \nabla \mathbf{u} + \nabla \mathbf{u}^T$$

$$\tau_m = -\frac{\mu_m}{2} H^2 \mathbf{I} + \mu_m \mathbf{H}\mathbf{H}^T$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = 0$$

$$\mathbf{B}(\mathbf{x}, t) = \begin{cases} \mu_d \mathbf{H} & \text{in the ferrofluid} \\ \mu_o \mathbf{H} & \text{in the matrix,} \end{cases}$$

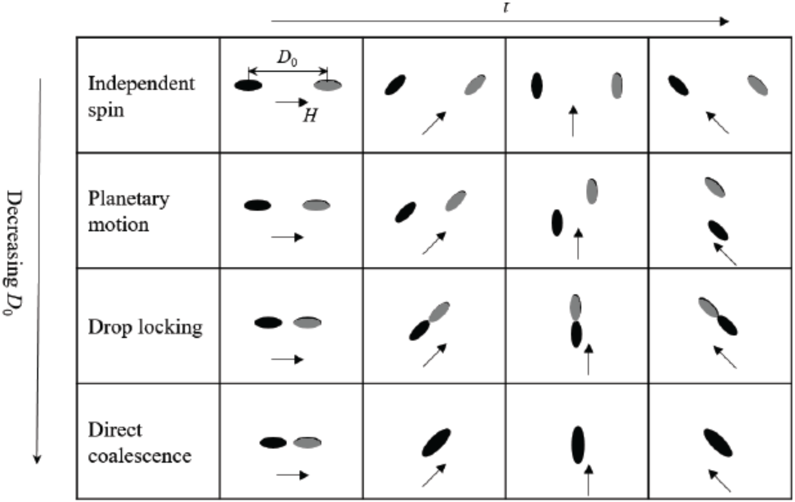
$$\mathbf{H} = \nabla \psi$$

$$\nabla \cdot (\mu \nabla \psi) = 0,$$

$$\mu_m = \mu_m(f)$$

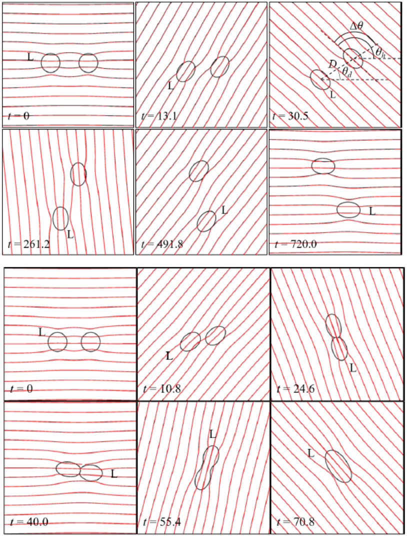
[J. M. Lopez-Herrera, S. Popinet, and M. A. Herrada. J Comput. Phys., 2011.]

Interaction of a Pair of Ferrofluid Drops in a Rotating Magnetic Field



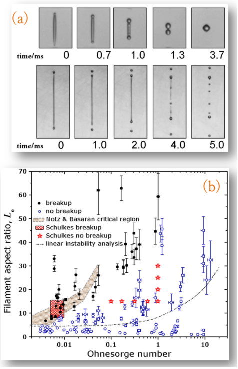
[Qui et al., JFM, submitted, 2018]

Interaction of a Pair of Ferrofluid Drops in a Rotating Magnetic Field

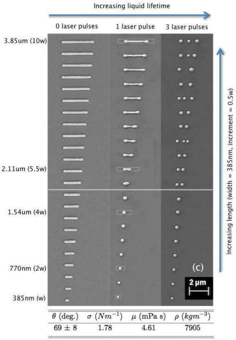


[Qui et al., JFM, submitted, 2018]

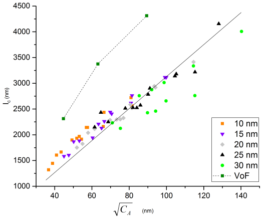
Instability of nanoscale liquid metal filaments



$$\mathrm{Oh} = \frac{\mu}{\sqrt{\rho \sigma R}}$$



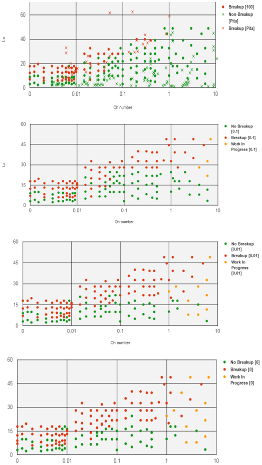
[Hartnett et al., Langmuir, 2015]

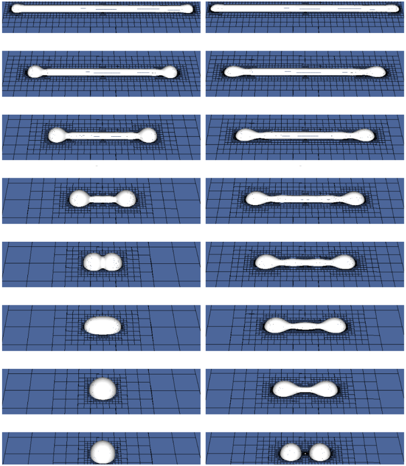


$$0.1 \leq \mathrm{Oh} \leq 0.2$$

$$10 \leq L_0 \leq 20$$

Breakup of Finite Size Liquid Filaments Involving Substrate Effects





Oh $= 0.15$, $L_0 = 23$

NJIT Math Modeling Undergrad Course Project, to be submitted to Computers & Fluids, 2018.

Link

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